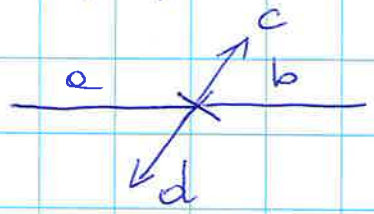
20.4. SIPANJE $ab \to cd$

$$d\mathcal{G} = \frac{|\mathcal{M}|^2}{F}\, dQ \qquad F = 4\sqrt{(p_a p_b)^2 - m_a^2 m_b^2}$$

$$dQ = (2\pi)^4 \delta^4(p_c + p_d - p_a - p_b)\, \frac{d^3p_c}{(2\pi)^3 2E_c}\, \frac{d^3p_d}{(2\pi)^3 2E_d}$$

V TEŽIŠČNEM SISTEMU

$\vec{p}_a = -\vec{p}_b = \vec{p}_i$, $\vec{p}_c = -\vec{p}_d = \vec{p}_f$



$$d^3p_c = p_f^2\, dp_f\, d\Omega$$

$$\int \frac{d^3p_d}{2E_d} \frac{\delta^4(p_c + p_d - p_a - p_b)}{\delta(E_c + E_d - E_a - E_b)\,\delta^3(\vec{p}_c + \vec{p}_d - \vec{p}_a - \vec{p}_b)} = \frac{1}{2E_d}\,\delta(E_c + E_d - E_a - E_b)$$

ENERGIJA V TEŽIŠČNEM SISTEMU $E = E_a + E_b$

$$dQ = \frac{1}{4\pi^2}\cdot \frac{p_f^2\, dp_f\, d\Omega}{2E_c\, 2E_d}\,\delta(E_c + E_d - E)$$

$$E = E_c + E_d = \sqrt{p_f^2 + m_c^2} + \sqrt{p_f^2 + m_d^2}$$

$$\frac{dE}{dp_f} = \frac{2p_f}{2\sqrt{p_f^2 + m_c^2}} + \frac{2p_f}{2\sqrt{p_f^2 + m_d^2}} = \frac{p_f}{E_c} + \frac{p_f}{E_d}$$

$$dQ = \frac{1}{4\pi^2}\frac{p_f\, dE}{4(E_c + E_d)}\, d\Omega\,\delta(E_c + E_d - E)$$

$$dp_f = dE\left(\frac{p_f}{E_c} + \frac{p_f}{E_d}\right)^{-1} = dE\,\frac{E_c E_d}{p_f (E_c + E_d)}$$

$$\int dE \;\to\; dQ = \frac{1}{16\pi^2}\frac{p_f}{E}\, d\Omega$$

$$F = 4 p_i E$$

$$\boxed{\frac{d\mathcal{G}}{d\Omega} = \frac{|\mathcal{M}|^2 p_f}{64\pi^2 p_i E^2}}$$

V ULTRARELATIVISTIČNI LIMITI: $m_a, m_b \ll p_i$, $m_c, m_d \ll p_f$

$$\Rightarrow p_i = p_f$$

$$\boxed{\frac{d\mathcal{G}}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 E^2}}$$

IZRAZ ZA $d\sigma/d\Omega$ UPORABIMO ZA $e^-\mu^- \to e^+\mu^+$ NEPOLARIZIRANA SIPANJE 46

V ULTRARELATIVISTIČNI LIMITI

$k = (\frac{E}{2}, \vec{p}_i)$, $k' = (\frac{E}{2}, \vec{p}_f)$

$p = (\frac{E}{2}, -\vec{p}_i)$, $p' = (\frac{E}{2}, -\vec{p}_f)$

$p_a = k$, $p_c = k'$, $p_b = p$, $p_d = p'$ V TEŽIŠČNI SIST.

$$\overline{|\mathcal{M}|^2} = 8\frac{e^4}{q^4}\left[(k\cdot p')(k'\cdot p) + (k'\cdot p)(k\cdot p')\right] =$$

$k\cdot p = \frac{E^2}{4} + \frac{E^2}{4} = \frac{E^2}{2} = k'\cdot p'$

$$= 8\frac{e^4}{q^4}\left[\frac{E^4}{4} + \frac{E^4}{4\cdot 4}(1+\cos\vartheta)^2\right] =$$

$$= \frac{8}{4\cdot 4}\frac{e^4 4\left[4 + (1+\cos\vartheta)^2\right]}{(1-\cos\vartheta)^2} = \frac{e^4\left[4+(1+\cos\vartheta)^2\right]}{(1-\cos\vartheta)^2}\cdot 2$$

$$\frac{d\sigma}{d\Omega} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 E^2} = \frac{e^4\left(4+(1+\cos\vartheta)^2\right)}{32\pi^2 E^2 (1-\cos\vartheta)^2} =$$

$$= \alpha^2 (\hbar c)^2 \frac{1}{2}\frac{1}{E^2}\frac{(4+(1+\cos\vartheta)^2)}{(1-\cos\vartheta)^2}.$$

$k\cdot p' = k'\cdot p = \frac{E^2}{4} - \vec{p}_f\cdot(-\vec{p}_i) = \frac{E^2}{4} + \vec{p}_i\cdot\vec{p}_f = \frac{E^2}{4}(1+\cos\vartheta)$

$\vec{p}_i = (p_i, 0, 0) = (\frac{E}{2}, 0, 0)$

$\vec{p}_f = (p_f\cos\vartheta, p_f\sin\vartheta, 0) = \frac{E}{2}(\cos\vartheta, \sin\vartheta, 0)$

$\vec{p}_i\cdot\vec{p}_f = \frac{E^2}{4}\cos\vartheta$

$q^2 = (k'-k)^2 = (0, \vec{p}_f + \vec{p}_i)^2 = 0 - (\vec{p}_f+\vec{p}_i)^2 =$

$= -\frac{E^2}{4}\left[(\cos\vartheta - 1)^2 + \sin^2\vartheta\right] = -\frac{E^2}{4}2(1-\cos\vartheta)$

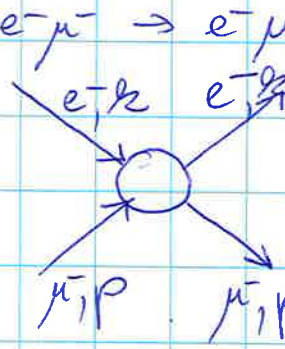
$\alpha = \frac{e^2}{4\pi\varepsilon_0(\hbar c)} = \frac{1}{137}$

$\hbar c = 200$ MeV fm

KRIŽANJE, PROCES $e^+e^- \to \mu^+\mu^-$

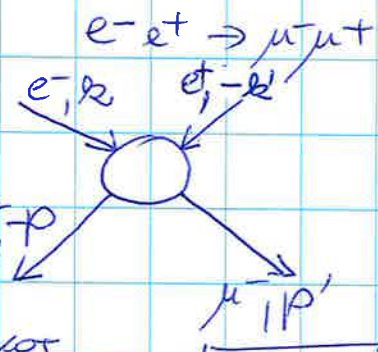
KAKO SEA POVEZANA $e^-\mu^- \to e^-\mu^-$ IN $e^+e^- \to \mu^+\mu^-$?

e^- NADOMESTIMO S e^+, ZAMENJAMO SMER GIB. KOLIČINE, $\mu^- \to \mu^+$ PODOBNO



	VPADNI DELCI		IZHODNI DELCI	
	p_a	p_b	p_c	p_d
$\longrightarrow$	k	p	k'	p'
	k	$-k'$	$-p$	p'

$\longleftarrow$



TO ISTO KOT ZA $e^-\mu^- \to e^-\mu^-$, LE $\boxed{p \leftrightarrow -k'}$

$$\overline{|\mathcal{M}|^2} = 8\frac{e^4}{q^4}\left[(-p\cdot p')(-k\cdot k') + (+p)\cdot(-k)\,(k\cdot p') + m_e^2\, p'k' + m_\mu^2\, pk + 2m_e^2 m_\mu^2\right]$$

$k = \left(\frac{E}{2}, \vec{p}_i\right)$, $k' = \left(\frac{E}{2}, \vec{p}_f\right)$, $p = \left(\frac{E}{2}, -\vec{p}_i\right)$, $p' = \left(\frac{E}{2}, -\vec{p}_f\right)$

$q^2 = (k'-k)^2 \to (p+k)^2 = E^2$

$pp' = \frac{E^2}{4}(1-\cos\vartheta)$

$kk' = \frac{E^2}{4}(1-\cos\vartheta)$

V ULTRAREL. LIMITI:

$$\frac{d\sigma}{d\Omega} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 E^2} = \frac{8e^4 \frac{1}{4}\frac{1}{4}E^4}{64\pi^2 E^2 E^4}\left[(1-\cos\vartheta)^2 + (1+\cos\vartheta)^2\right] =$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{e^4}{8\cdot 16\pi^2}\frac{1}{E^2}2\left[1+\cos^2\vartheta\right] = \frac{e^4}{64\pi^2}\frac{1}{E^2}\left(1+\cos^2\vartheta\right)}$$

$$\sigma = \int d\Omega \left(\frac{d\sigma}{d\Omega}\right) = 2\pi\int_{-1}^{1} d(\cos\vartheta)\,\frac{e^4}{64\pi^2}\frac{1}{E^2}\left(1+\cos^2\vartheta\right) = \frac{1}{32\pi}\frac{e^4}{E^2}\left(\cos\vartheta + \frac{1}{3}\cos^3\vartheta\right)\Big|_{-1}^{+1}$$

$2\pi\, d(\cos\vartheta)$

$$= \frac{1}{32\pi}\frac{e^4}{E^2}\left(2+\frac{2}{3}\right) = \frac{e^4}{12\pi E^2} = \left(\frac{e^2}{4\pi}\right)^2 \frac{4\pi}{3}\frac{1}{E^2}$$

$$\boxed{\sigma = \alpha^2 (\hbar c)^2 \frac{4\pi}{3}\frac{1}{E^2}}$$

$$\sigma = \alpha^2 (\hbar c)^2 \frac{4\pi}{3} \frac{1}{E^2} = \frac{1}{137^2} (0{,}2\,\mathrm{GeV})^2 \cdot \mathrm{fm}^2 \cdot \frac{4\pi}{3} \frac{1}{E^2}$$

$$= \frac{0.04}{137^2} \cdot \frac{4\pi}{3} \frac{(\mathrm{GeV})^2}{100\,(\mathrm{GeV})^2} \cdot \mathrm{fm}^2 =$$

$$= \frac{1}{100} \cdot \frac{0.4 \cdot 10^{-2}}{(1{,}37 \cdot 10^2)^2} \cdot \frac{4\pi}{3} \frac{1}{100}$$

$$\sigma = 0.9 \cdot 10^{-9}\,\mathrm{b} = \underline{0.9\ \mathrm{nb}}$$

PRI ENERGIJI $E = 10\,\mathrm{GeV}$

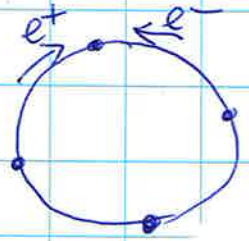
$1\,\mathrm{b} = (10\,\mathrm{fm})^2 = 10^{-28}\,\mathrm{m}^2$

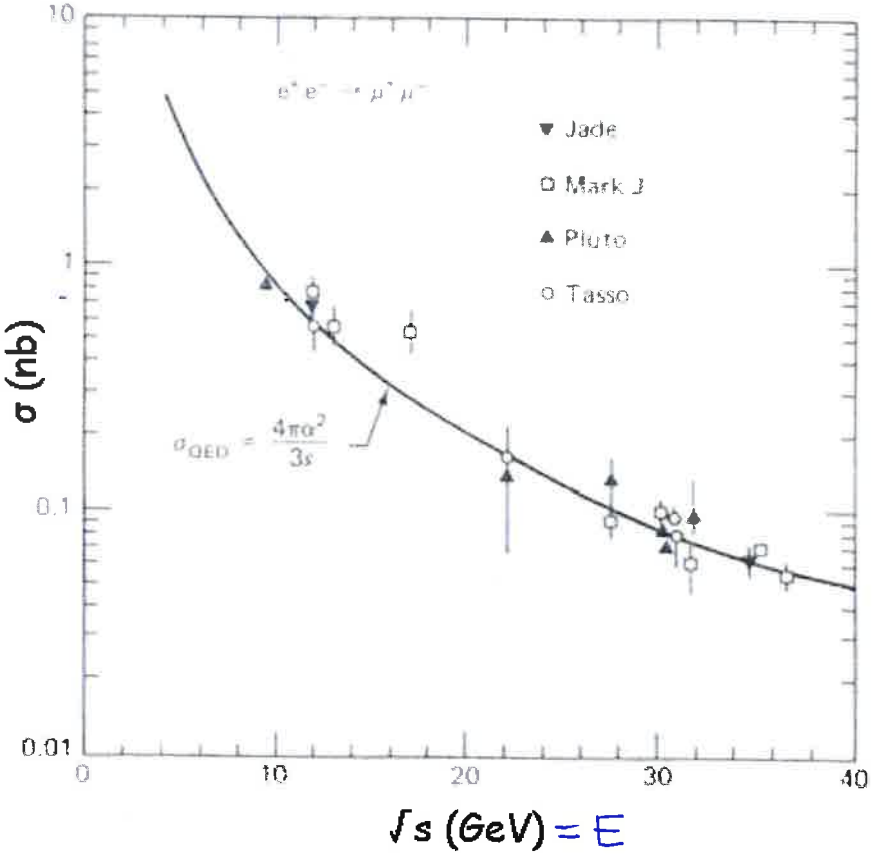
$$\boxed{s \equiv E^2}$$

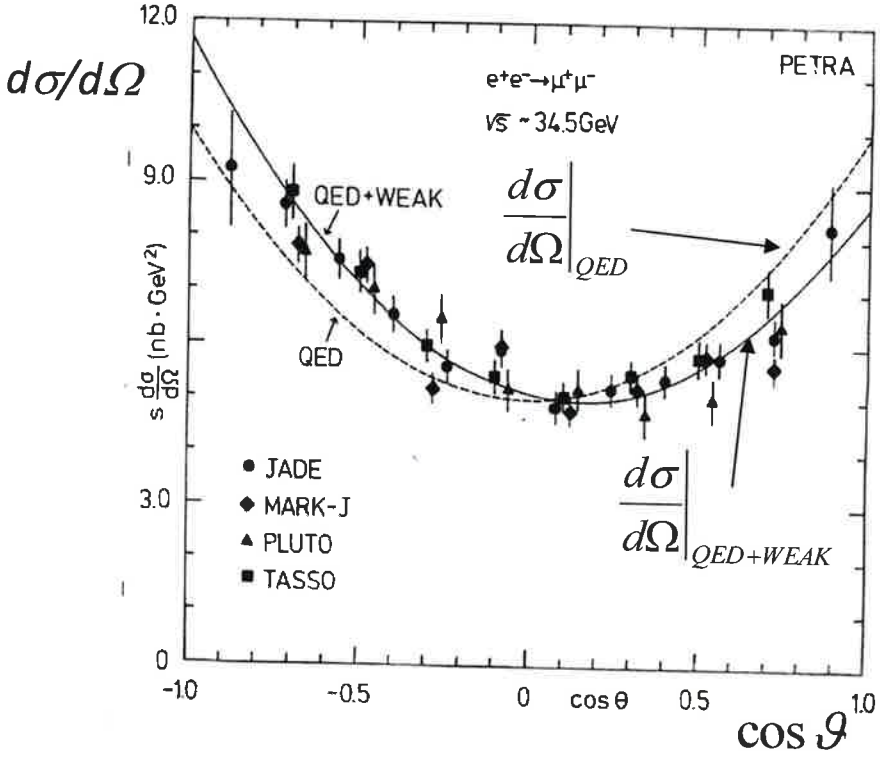
MERITVE NA TRKALNIKU PETRA V DESY V HAMBURGU

MERILI SO TUDI DIFERENC. SIP. PREČ.

$\frac{d\sigma}{d\Omega} \propto (1 + \cos^2\vartheta)$ GLEJ STR. 47







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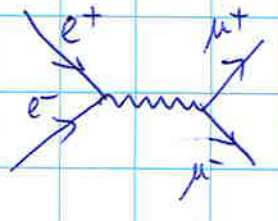

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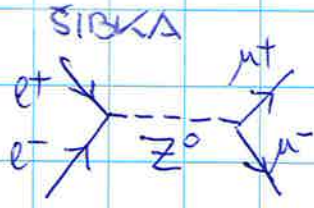
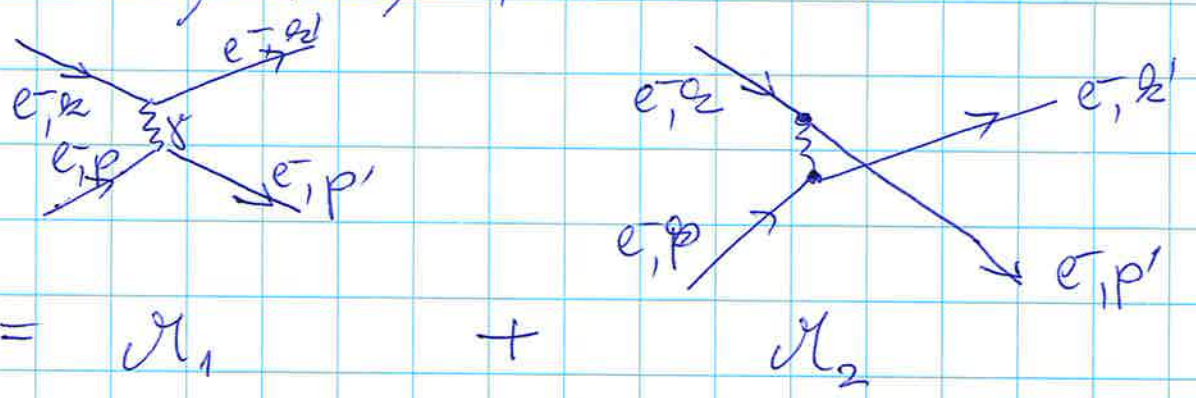


SIPANJE $e^-e^- \to e^-e^-$

PODOBNO KOT $e^-\mu^- \to e^-\mu^-$, LE DELCA IDENTIČNA

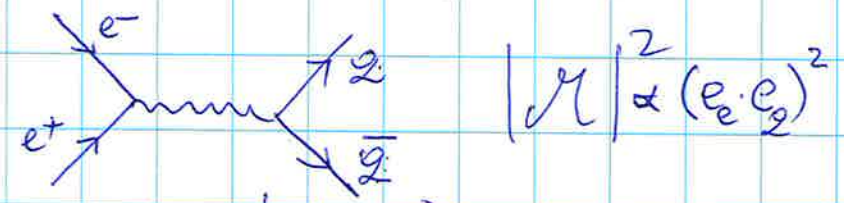


KER NE VEM, PO KATERI POTI JE POTEKLA REAKCIJA, SEŠTEJEM AMPLITUDI!

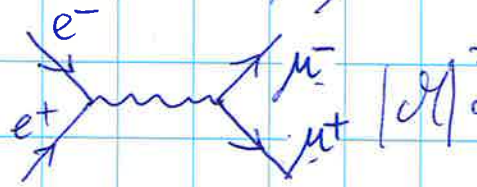
$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$$

$$\mathcal{M}_1 \propto \frac{1}{q^2} e^2 [\bar{u}(q') \gamma^\kappa u(q)][\bar{u}(p') \gamma_\kappa u(p)] \qquad \mathcal{M}_2 \propto \frac{1}{2} [\bar{u}(p') \gamma^\kappa u(q)][\bar{u}(q') \gamma_\kappa u(p)] e^2$$

SIPANJE $e^-e^+ \to q\bar{q}$ PODOBNO KOT $e^-e^+ \to \mu^-\mu^+$



$$|\mathcal{M}|^2 \propto (e_e e_q)^2$$



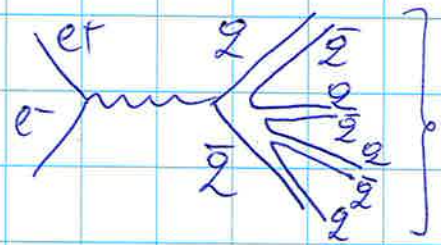
$$|\mathcal{M}|^2 \propto (e_e e_\mu)^2 = e_0^4$$

$$\frac{\sigma_{e^+e^- \to q\bar{q}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} = \frac{|\mathcal{M}_{q\bar{q}}|^2}{|\mathcal{M}_{\mu\mu^+}|^2} \propto \frac{e_e^2 e_q^2}{e_e^2 e_\mu^2} = \left(\frac{e_q}{e_0}\right)^2$$

ZBEREMO V BARVE

$$\Rightarrow \boxed{\frac{\sigma_{e^+e^- \to q\bar{q}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} = 3\left(\frac{e_q}{e_0}\right)^2}$$

KVARKI NISO PROSTI $\to$



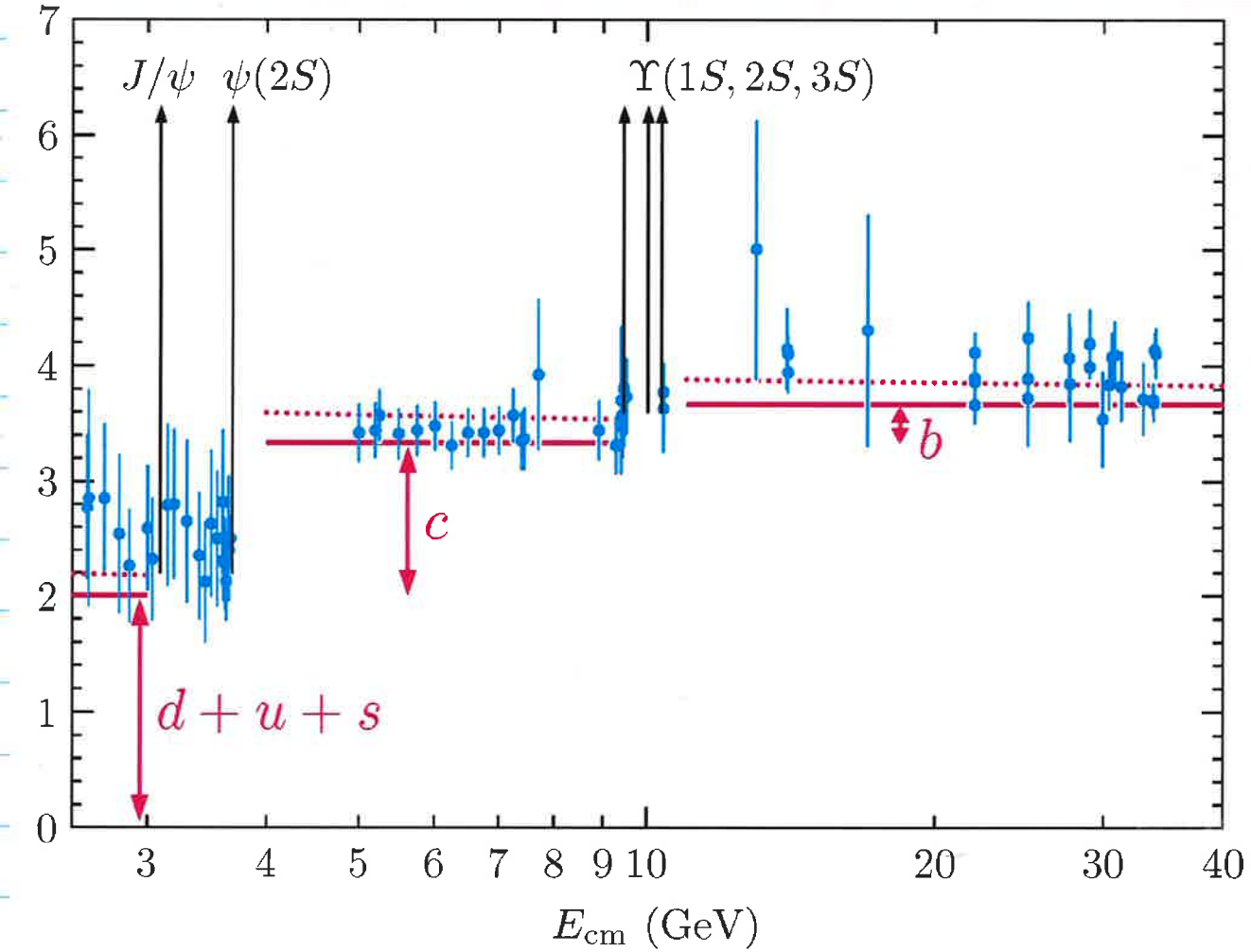
$$\frac{\sigma(e^+e^- \to \text{HADRONI})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q 3\left(\frac{e_q}{e_0}\right)^2$$

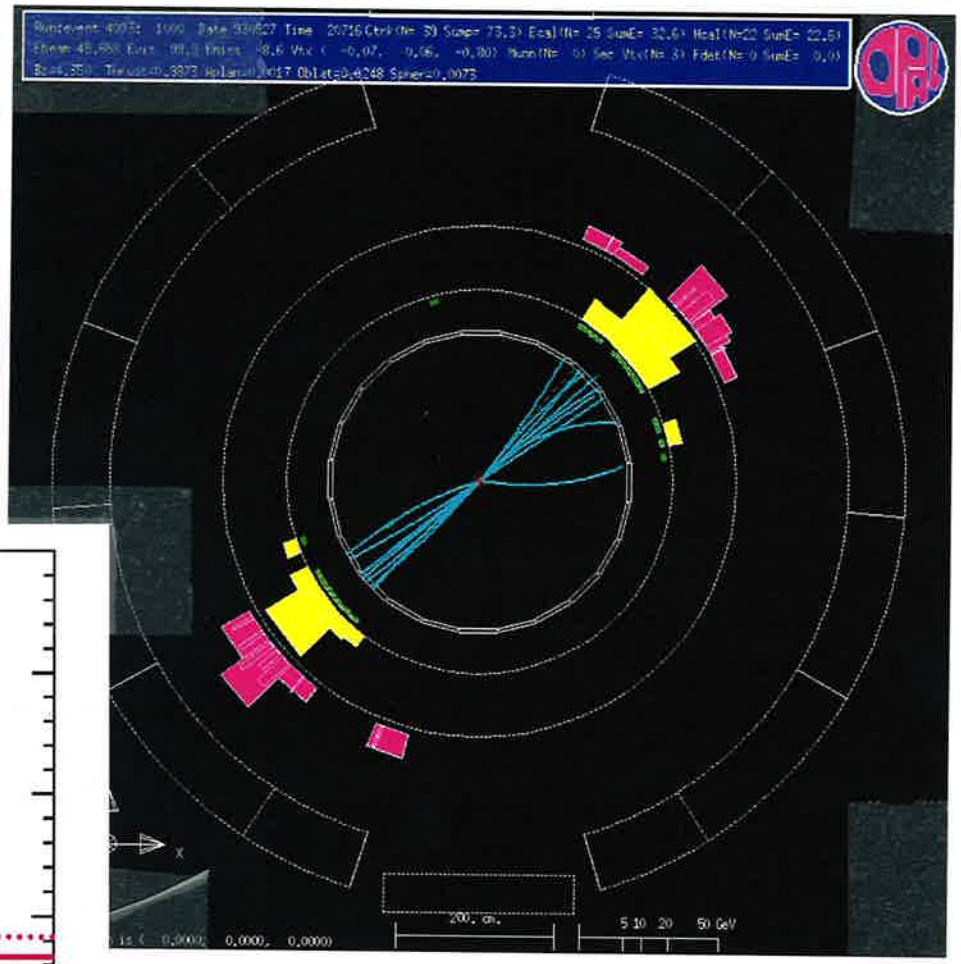
$$\frac{\sigma(e^+e^- \to \text{HADRONI})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q 3\left(\frac{e_q}{e_0}\right)^2$$

~~ZA~~ ENERGIJAH $E < 3$ GeV : $q = u, d, s$

$$\sum_q 3\left(\frac{e_q}{e_0}\right)^2 = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$$

$3 < E < 10$ GeV $\quad \Sigma = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = \frac{10}{3} = 3{,}3$ ↓ KVARK c





$E > 10$ GeV : $\Sigma = \frac{10}{3} + 3\cdot\frac{1}{9} = \frac{11}{3}$ ↑ KVARK b

KAJ PA SKOKI V σ PRI PUŠČICAH?

- RESONANCE, VEZANA STANJA

$c\bar{c}$: J/ψ, $\psi(2S)$

$b\bar{b}$: $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$