35

$$\hat{H}\psi = [\vec{\alpha}\cdot\hat{\vec{p}} + \beta m]\psi = \underbrace{i\frac{\partial}{\partial t}}_{\hat{E}}\psi$$

DIRACOVA E.

$$i\frac{\partial\psi}{\partial t} = [-i\vec{\alpha}\cdot\vec{\nabla} + \beta m]\psi$$

$$i\beta\frac{\partial\psi}{\partial t} = [-i\beta\vec{\alpha}\cdot\vec{\nabla} + \underset{1}{\beta^2} m]\psi$$

$$\gamma^\mu = (\tfrac{\partial}{\partial t}, -\vec{\nabla})$$

$$i\left(\beta\frac{\partial}{\partial t} + \beta\vec{\alpha}\cdot\vec{\nabla} - m\right)\psi = 0$$

UVEDEM $\gamma^\mu = (\beta, \beta\vec{\alpha})$ $\rightarrow$ $\boxed{[i\gamma^\mu\partial_\mu - m]\psi = 0}$

MATRIKE γ^μ : $\gamma^0 = \beta$ $\quad \gamma^i = \beta\alpha_i$ $\quad i = 1,2,3$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$

METRIČNI TENZOR $\quad g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

REŠITEV DIR.E. ZA PROST DELEC IŠČEMO V OBLIKI

$$\psi = u(\vec{p})e^{-ip^\mu x_\mu}$$

$u(\vec{p})$ BISPINOR

$$[i\gamma^\mu\partial_\mu - m]\,u(\vec{p})\,e^{-ip^\mu x_\mu} = 0$$

$$\boxed{[\gamma^\mu p_\mu - m]\,u(\vec{p}) = 0}$$

6.4. ZA POLJUBEN ČETVEREC a_μ : $\gamma^\mu a_\mu \equiv \not{a}$

REŠEVANJE ENAČBE $u(\vec{p})$: ZAPIŠEMO H V ORIGINALNI OBLIKI

$$[\vec{\alpha}\cdot\vec{p} + \beta m]\,u(\vec{p}) = E\,u(\vec{p})$$

$$\begin{bmatrix} 0 & \vec{\sigma}\vec{p} \\ \vec{\sigma}\cdot\vec{p} & 0 \end{bmatrix} u(\vec{p}) + \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix} u(\vec{p}) = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix} u(\vec{p}) \qquad u(\vec{p}) = \begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

$$\begin{bmatrix} 0 & \vec{\sigma}\vec{p} \\ \vec{\sigma}\cdot\vec{p} & 0 \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} + \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

$$\begin{bmatrix} \vec{\sigma}\cdot\vec{p}\,u_B \\ \vec{\sigma}\cdot\vec{p}\,u_A \end{bmatrix} + \begin{bmatrix} m\,u_A \\ -m\,u_B \end{bmatrix} = \begin{bmatrix} E\,u_A \\ E\,u_B \end{bmatrix}$$

The X International Conference on Instrumentation for Colliding Beam Physics
Budker Institute of Nuclear Physics, Siberian Branch of Russian Academy of Science, Novosibirsk, Russia
February 28 - March 5, 2008

$$\vec{\sigma}\cdot\vec{p}\; u_B + m\, u_A = E\, u_A \qquad \Rightarrow \qquad \vec{\sigma}\cdot\vec{p}\; u_B = (E-m)\, u_A$$
$$\vec{\sigma}\cdot\vec{p}\; u_A + (-m)\, u_B = E\, u_B \qquad\qquad \vec{\sigma}\cdot\vec{p}\; u_A = (E+m)\, u_B$$

$$u_A^{(1)} = \begin{bmatrix}1\\0\end{bmatrix} = \chi^{(1)} \quad u_A^{(2)} = \begin{bmatrix}0\\1\end{bmatrix} = \chi^{(2)} \quad \text{SPINOR}$$

ČE $E>0$ $\quad u_B^{(s)} = \dfrac{\vec{\sigma}\cdot\vec{p}\, u_A^{(s)}}{E+m}$ $\quad \boxed{u^{(s)} = N \begin{bmatrix}\chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)}\end{bmatrix}}$ $\quad s=1,2$

ČE $E<0$ $\quad u_B^{(s)} = \chi^{(s)}$

$$u_A^{(s)} = \frac{\vec{\sigma}\cdot\vec{p}}{E-m}\, u_B^{(s)} = -\frac{\vec{\sigma}\cdot\vec{p}}{|E|+m}\, u_B^{(s)} \qquad \boxed{u^{(s)} = N\begin{bmatrix}-\frac{\vec{\sigma}\cdot\vec{p}}{|E|+m}\chi^{(s)} \\ \chi^{(s)}\end{bmatrix}}$$

ODKOD TA DEGENERACIJA, 2 REŠITVI Z $E<0$, 2 Z $E>0$?

POGLEJMO KOMUTATOR $\hat{H}$ IN $\hat{\vec{L}}$

$$\hat{H} = \left[\vec{\alpha}\cdot\hat{\vec{p}} + m\beta\right], \quad \vec{L} = \vec{r}\times\hat{\vec{p}}, \quad p_i = -i\frac{\partial}{\partial x_i}$$

$$[\hat{H},\hat{L}_1] = [\vec{\alpha}\cdot\vec{p} + \beta m,\, x_2\hat{p}_3 - x_3\hat{p}_2] = [\vec{\alpha}\cdot\vec{p},\, x_2\hat{p}_3 - x_3\hat{p}_2] + \underbrace{[\beta m,\, x_2 p_3 - x_3 p_2]}_{=0} =$$

UPOŠTEVAM $[x_i, p_j] = i\delta_{ij}$

$$= [\alpha_1\hat{p}_1 + \alpha_2\hat{p}_2 + \alpha_3\hat{p}_3,\, x_2\hat{p}_3 - x_3\hat{p}_2] = [\alpha_2\hat{p}_2, x_2\hat{p}_3] - [\alpha_3\hat{p}_3, x_3\hat{p}_2] =$$
$$= -i\alpha_2\hat{p}_3 + i\alpha_3\hat{p}_2 = -i[\alpha_2\hat{p}_3 - \alpha_3\hat{p}_2] = -i\,[\vec{\alpha}\times\hat{\vec{p}}]_1$$

$$[\hat{H},\hat{\vec{L}}] = -i\,(\vec{\alpha}\times\hat{\vec{p}})$$

$\hat{H}$ NE KOMUTIRA Z $\vec{L}$, L NI VEČ DOBRO KV. ŠTEVILO

UVEDEMO $\quad \vec{\Sigma} = \begin{bmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{bmatrix}$

$$[\vec{\alpha}\cdot\hat{\vec{p}}, \Sigma_1] = \left[\begin{bmatrix}0 & \vec{\sigma}\cdot\vec{p}\\ \vec{\sigma}\cdot\vec{p} & 0\end{bmatrix}, \begin{bmatrix}\sigma_1 & 0\\ 0 & \sigma_1\end{bmatrix}\right] = \begin{bmatrix}0 & \vec{\sigma}\cdot\vec{p}\,\sigma_1\\ \vec{\sigma}\cdot\vec{p}\,\sigma_1 & 0\end{bmatrix} -$$
$$-\begin{bmatrix}0 & (\sigma_1\,\vec{\sigma}\cdot\vec{p})\\ (\sigma_1\,\vec{\sigma}\cdot\vec{p}) & 0\end{bmatrix} = \begin{bmatrix}0 & [\vec{\sigma}, \sigma_1]\\ [\vec{\sigma}, \sigma_1] & 0\end{bmatrix}\cdot\hat{\vec{p}} = \begin{bmatrix}0 & -2i\sigma_3\\ -2i\sigma_3 & 0\end{bmatrix}\hat{p}_2 + \begin{bmatrix}0 & 2i\sigma_2\\ 2i\sigma_2 & 0\end{bmatrix}\hat{p}_3$$

PAULIJEVE MATRIKE $\quad \sigma_i^2 = I \quad [\sigma_1,\sigma_2] = 2i\sigma_3, \quad [\sigma_2,\sigma_3] = 2i\sigma_1, \quad [\sigma_3,\sigma_1] = 2i\sigma_2$

$$= 2i\left[-\alpha_3 \hat{p}_2 + \alpha_2 \hat{p}_3\right] = 2i\left(\vec{\alpha}\times\hat{\vec{p}}\right)_1 = \left[\vec{\alpha}\cdot\hat{\vec{p}}, \Sigma_1\right] \qquad [\beta m, \Sigma_i] = 0$$

$$[\hat{H}, \vec{\Sigma}] = 2i\,(\vec{\alpha}\times\hat{\vec{p}}) \qquad [\hat{H}, \hat{\vec{L}}] = -i\,(\vec{\alpha}\times\hat{\vec{p}})$$

$$\hat{\vec{J}} = \hat{\vec{L}} + \frac{1}{2}\vec{\Sigma} \qquad \boxed{[\hat{H}, \hat{\vec{J}}] = 0}$$

VIJAČNOST $\vec{\Sigma}\cdot\frac{\vec{p}}{p}$ $\quad \vec{\Sigma}\cdot\frac{\vec{p}}{p}\, u^{(1)} = +u^{(1)} \qquad \vec{\Sigma}\cdot\frac{\vec{p}}{p}\, u^{(2)} = -u^{(2)}$ DN

ALI JE TO $\frac{1}{2}\vec{\Sigma}$ RES SPIN? INTERAKCIJA Z MAG. POLJEM V NERELATIVISTIČNI LIMITI BO DALA OBEN OBLIKE $-\vec{\mu}\cdot\vec{B}$
(= PRISPEVEK K ENERGIJI)

INTERAKCIJA Z E.MAGNETNIM POLJEM UVEDEMO TAKO, DA $\quad \partial_\mu \to \partial_\mu - ieA_\mu$

$$A_\mu = (A_0, \vec{A}) \qquad \vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}A_0 \qquad \vec{B} = \vec{\nabla}\times\vec{A} \qquad p_\mu \to p_\mu - eA_\mu$$

$$\left[i\gamma^\mu(\partial_\mu - ieA_\mu) - m\right]\psi = 0 \quad \Rightarrow \quad \left[\beta E - \beta\vec{\alpha}\cdot\vec{p} + e\beta A_0 - e\beta\vec{\alpha}\cdot\vec{A} - m\right]u = 0$$

Z LEVE Z β
$$\left[E - \vec{\alpha}\cdot\vec{p} + eA_0 - e\vec{\alpha}\cdot\vec{A} - \beta m\right]u = 0$$
$$\left[\vec{\alpha}(\vec{p} + e\vec{A}) - eA_0 + \beta m\right]u = Eu$$

$$\begin{bmatrix} m - eA_0 & \vec{\sigma}\cdot(\vec{p} + e\vec{A}) \\ \vec{\sigma}\cdot(\vec{p} + e\vec{A}) & -m - eA_0 \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} = E\begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

$$\vec{\sigma}(\vec{p} + e\vec{A})\, u_B = (E - m + eA_0)\, u_A \qquad (1)$$
$$\vec{\sigma}(\vec{p} + e\vec{A})\, u_A = (E + m + eA_0)\, u_B \qquad (2)$$

ZA $E > 0$
$$u_B = (E + m + eA_0)^{-1}\,\vec{\sigma}\cdot(\vec{p} + e\vec{A})\cdot u_A \qquad \text{iz } (2)$$
$$\vec{\sigma}(\vec{p} + e\vec{A})(E + m + eA_0)^{-1}\,\vec{\sigma}\cdot(\vec{p} + e\vec{A})\, u_A = (E - m + eA_0)\, u_A \qquad \text{v } (1)$$

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V NERELATIVISTIČNI LIMITI $m \gg p$, $m \gg eA^0$ $\Rightarrow$ $E + m + eA_0 \approx 2m$

$E - m + eA_0 = E_{NR} + eA_0$

$$\Rightarrow \frac{1}{2m} \vec{\sigma}\cdot(\vec{p} + e\vec{A})\, \vec{\sigma}\cdot(\vec{p} + e\vec{A})\, u_A = (E_{NR} + eA_0)\, u_A$$

OZNAČIMO $\vec{p}\,' = \vec{p} + e\vec{A}$

UPOŠTEVAMO, DA ZA POLJUBNA $\vec{a}, \vec{b}$ VELJA

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$$

$$\Rightarrow (\vec{\sigma}\cdot\vec{p}\,')(\vec{\sigma}\cdot\vec{p}\,') = \vec{p}\,'\cdot\vec{p}\,' + i\vec{\sigma}\cdot(\vec{p}\,'\times\vec{p}\,')$$

NAVADNI VEKTOR: $\vec{p}\,' \times \vec{p}\,' = 0$ VENDAR $\vec{p}\,'$ VSEBUJE ODVODE IN POLJA, ZATO $\neq 0$

$$(\vec{p}\,'\times\vec{p}\,')\, u_A = (\vec{p} + e\vec{A})\times(\vec{p} + e\vec{A})\, u_A = \vec{p}\times\vec{p}\, u_A + [e\vec{p}\times\vec{A} + e\vec{A}\times\vec{p}]\, u_A + e^2 \underbrace{(\vec{A}\times\vec{A})}_{0} u_A$$

$$[e\hat{\vec{p}}\times\vec{A} + e\vec{A}\times\hat{\vec{p}}]\, u_A = -ie[\underbrace{\vec{\nabla}\times(\vec{A}u_A)}_{(\vec{\nabla}\times\vec{A}) - \vec{A}\times(\vec{\nabla}u_A)} + \vec{A}\times(\vec{\nabla}u_A)] = -ie(\vec{\nabla}\times\vec{A})\, u_A = -ie\vec{B}u_A$$

$$\Rightarrow \frac{1}{2m}\vec{\sigma}\cdot\vec{p}\,'\cdot\vec{\sigma}\cdot\vec{p}\,'\, u_A = (E_{NR} + eA_0)\, u_A$$

$$\frac{1}{2m}[p'^2 + e\vec{\sigma}\vec{B}]\, u_A = (E_{NR} + eA_0)\, u_A$$

$$\left[\frac{(\vec{p} + e\vec{A})^2}{2m} + \underbrace{\frac{e}{2m}\vec{\sigma}}_{-\vec{\mu}}\cdot\vec{B} - eA_0\right] u_A = E_{NR}\, u_A$$

$$\Rightarrow \frac{e\,\vec{\sigma}\cdot\vec{B}}{2m} = -\vec{\mu}\vec{B} \Rightarrow \vec{\mu} = -\frac{e}{2m}\vec{\sigma} =$$

$$= -\frac{e}{2m}\cdot 2\,\underbrace{\frac{\vec{\sigma}}{2}}_{\hat{\vec{S}}} \equiv -\frac{e}{2m}\, g_s\, \hat{\vec{S}}$$

$$\Rightarrow \boxed{g_s = 2}$$

VELIK USPEH DIRACOVE ENAČBE

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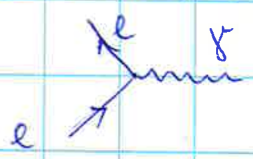
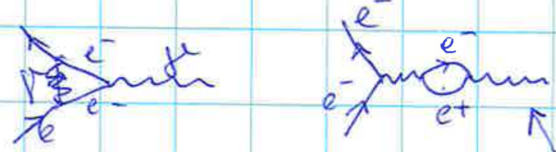
GIROMAGNETNO RAZMERJE TOČKASTEGA DELCA



$\Rightarrow g_s = 2$

MOŽNI TUDI PROCESI



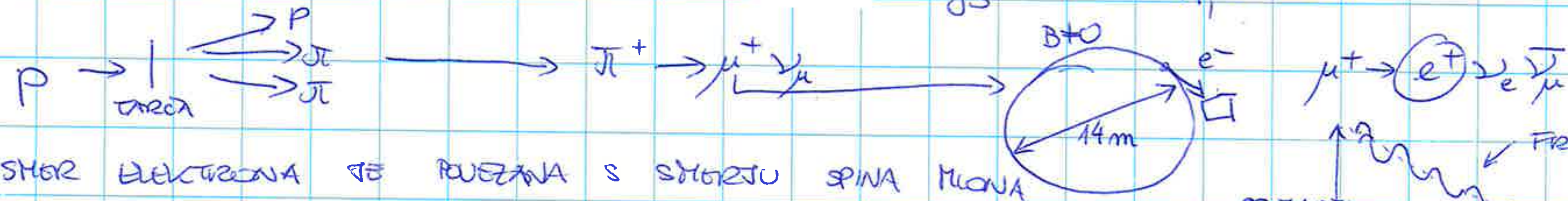
ZA MION

IZVREDNOTIMO NA TISOČE DIAGRAMOV

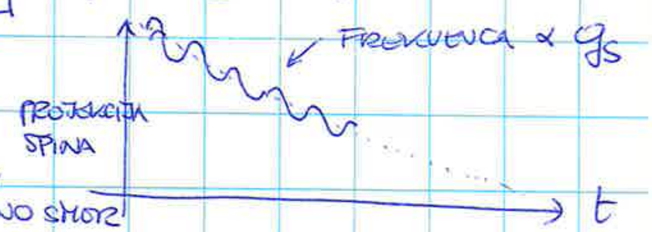
$\Rightarrow g_s \neq 2$

ANOMALNO ŽIROM. RAZMERJE

$g_s - 2 = 11{,}659214 \cdot 10^{-4}$



SMER ELEKTRONA JE POVEZANA S SMERJO SPINA MIONA



= ŠTEVILO e, KI LETIJO V DANO SMER

VROČA TEMA: EKSPERIMENT IN TEORIJA SE NE UJEMATA.

JUTRI 7.4. BO OBJAVLJEN REZULTAT NOVE, ŠE NATANČNEJŠE MERITVE.

VERJETNOSTNA GOSTOTA IN TOK ZA REŠITVE DIRACOVE E.

$$[i\gamma^\mu \partial_\mu - m]\psi = 0$$

$$i\gamma^0 \frac{\partial \psi}{\partial t} + i\gamma^k \frac{\partial \psi}{\partial x^k} - m\psi = 0 \qquad k = 1,2,3$$

$$\boxed{\gamma^{0+} = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}^T = \gamma^0 \qquad \gamma^{k+} = -\gamma^k}$$

$$-i\frac{\partial \psi^+}{\partial t}\gamma^{0+} + \frac{\partial \psi^+}{\partial x^k}(-i)\gamma^{k+} - m\psi^+ = 0 \Rightarrow -i\frac{\partial \psi^+}{\partial t}\gamma^0 + i\frac{\partial \psi^+}{\partial x^k}\gamma^k - m\psi^+ = 0 \quad |\gamma^0$$

$$-i\frac{\partial \psi^+}{\partial t}\gamma^0\gamma^0 + i\frac{\partial \psi^+}{\partial x^k}\gamma^k\gamma^0 - m\psi^+\gamma^0 = 0$$

$$i\frac{\partial \psi^+}{\partial t}\gamma^0\gamma^0 + i\frac{\partial \psi^+}{\partial x^k}\gamma^0\gamma^k + m\psi^+\gamma^0 = 0$$

KER $\gamma^0\gamma^k = -\gamma^k\gamma^0$

UVEDEM $\psi^+\gamma^0 = \overline{\psi}$

$$i\frac{\partial \overline{\psi}}{\partial t}\gamma^0 + i\frac{\partial \overline{\psi}}{\partial x^k}\gamma^k + m\overline{\psi} = 0$$

DIRACOVA E. ZA $\overline{\psi}$

$$\boxed{i\,\partial_\mu \overline{\psi}\gamma^\mu + m\overline{\psi} = 0}$$

$$\left.\begin{aligned} i\,\partial_\mu \overline{\psi}\gamma^\mu\psi + m\overline{\psi}\psi &= 0 \\ i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi &= 0 \end{aligned}\right\} \text{SEŠTEJEM}$$

$$\partial_\mu\overline{\psi}\gamma^\mu\psi + \overline{\psi}\gamma^\mu\partial_\mu\psi = 0 \Rightarrow \partial_\mu(\overline{\psi}\gamma^\mu\psi) = 0$$

KONTINUITETNA ENAČBA