2.3.2021

8

SPIN JEDRA

POSEBNO ENOSTAVNO ZA JEDRA Z NESPARJENIM NUKLEONOM V NAJVIŠJI PODLUPINI.

PRIMER: ^{17}O : $8p + 9n$

p: $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$ VSE PODLUPINE POLNE

n: $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$ POLNE, EN n V $1d_{5/2}$.

NESPARJEN n: $l=2$, $s=\frac{1}{2}$, $j=\frac{5}{2}$ $\Rightarrow$ ^{17}O: $\underline{\underline{l=2, s=\frac{1}{2}, j=\frac{5}{2}}}$

DOMAČA NALOGA: SPIN JEDER ^{13}C, ^{39}K,

MAGNETNI MOMENT JEDRA

KLASIČNO

$$\vec{\mu} = IS\vec{n}, \quad I = \frac{dq}{dt} = \frac{e}{t_0} = \frac{e \cdot v}{2\pi r} = \frac{ep}{2\pi r m}$$

$$\vec{\mu} = \frac{ep}{2\pi r m} \cdot \pi r^2 \vec{n} = \frac{e}{2m} \vec{r} \times \vec{p} = \frac{e}{2m} \vec{l}$$

KVANTNOMEHANSKO $\hat{\vec{\mu}} = \frac{e}{2m} \hat{\vec{l}}$ $\qquad \hat{\vec{\mu}}^2 \psi = \left(\frac{e}{2m}\right)^2 \hat{\vec{l}}^2 \psi = \left(\frac{e\hbar}{2m}\right)^2 l(l+1)\psi$

$\frac{e\hbar}{2m_e} = \mu_B$ BOHROV MAGNETON

ZA DELEC S SPINOM: NAIVNO $\hat{\vec{\mu}} = \frac{e}{2m}(\hat{\vec{l}} + \hat{\vec{s}})$ $\underline{\underline{\text{NI TAKO}}}$

PRISPEVEK SPINA K MAGNETNEMU MOMENTU $\hat{\vec{\mu}}_s = g_s \frac{e}{2m} \hat{\vec{s}}$

g_s = GIROMAGNETNO RAZMERJE

ZA TOČKASTE FERMIONE S $s=\frac{1}{2}$: $\underline{g_s = 2}$ $\qquad$ (ZA l: $\underline{g_l = 1}$)

$$\hat{\vec{\mu}} = \frac{e}{2m}\left(g_\ell \hat{\vec{\ell}} + g_s \hat{\vec{s}}\right) \underset{\uparrow}{=} \frac{e}{2m}\left(\vec{\ell} + 2\vec{s}\right) = \frac{e}{2m}\left(\vec{j} + \vec{s}\right)$$

TOČKAST FERMION

p, n FERMIONA S $s = \frac{1}{2}$, TODA $\underline{g_{s,p} = 5{,}6}$ $\underline{g_{s,n} = -3{,}8}$

ODSTOPANJE OD 2 (OZIROMA ∅): ZARADI STRUKTURE NUKLEONOV.

$$\mu_{s,p} = g_{s,p}\,\mu_N \sqrt{s(s+1)}\,, \quad \mu_{s,n} = g_{s,n}\,\mu_N \sqrt{s(s+1)}\,; \quad \mu_N = \frac{e\hbar}{2m_N}$$

$$\mu = \langle JJ|\hat{\mu}_z|JJ\rangle \;; \quad |JJ\rangle = |\ell, s, J, J_3 = J\rangle$$

EFEKTIVNO GIROMAGNETNO RAZMERJE g: $\hat{\mu} = g\frac{e}{2m_N}\vec{J}$

MAGNETNI MOMENT JEDRA: POSEBNO ENOSTAVNO ZA JEDRA Z ENIM SAMIM NUKLEONOM NA ZUNANJI PODLUPINI S ℓ, s, j $(J=j)$

$$\mu = \langle jj|\hat{\mu}_z|jj\rangle \quad \text{OZIROMA} \quad \mu = \frac{e}{2m_N}\langle jj|\, g_\ell \hat{\ell} + g_s \hat{s}\,|jj\rangle$$

EFEKTIVNO GIROMAGNETNO RAZMERJE

$$\langle jm|\hat{\vec{\mu}}\cdot\hat{\vec{j}}|jm\rangle = \langle jm|\, g\frac{e}{2m}\hat{j}^2|jm\rangle = g\,\mu_N\,\hbar\, j(j+1)$$

$$\langle jm|\, g_\ell \hat{\vec{\ell}}\cdot\hat{\vec{j}} + g_s \hat{\vec{s}}\cdot\vec{j}\,|jm\rangle = \mu_N\hbar\left[g_\ell \frac{j(j+1)+\ell(\ell+1)-s(s+1)}{2} + g_s\frac{j(j+1)-\ell(\ell+1)+\frac{3}{4}}{2}\right]$$

$$\langle jm|\vec{\ell}\cdot\vec{j}|jm\rangle = \langle jm|\vec{\ell}(\vec{\ell}+\vec{s})|jm\rangle = \hbar^2\ell(\ell+1) + \langle jm|\vec{\ell}\vec{s}|jm\rangle$$

$$\langle jm|\vec{s}\cdot\vec{j}|jm\rangle = \langle jm|\vec{s}(\vec{\ell}+\vec{s})|jm\rangle = \hbar^2\underbrace{\tfrac{1}{2}\left(\tfrac{1}{2}+1\right)}_{3/4} + \langle jm|\vec{\ell}\vec{s}|jm\rangle$$

$\langle \vec{\ell}\vec{s}\rangle$ GLEJ LIST 7

$$\langle \vec{\ell}\cdot\vec{j}\rangle = \frac{1}{2}\hbar^2\left[j(j+1) + \ell(\ell+1) - s(s+1)\right]$$

$$\langle \vec{s}\cdot\vec{j}\rangle = \frac{1}{2}\hbar^2\left[j(j+1) - \ell(\ell+1) + s(s+1)\right]$$

$$g = g_l \frac{j(j+1) + l(l+1) - 3/4}{2j(j+1)} + g_s \frac{j(j+1) - l(l+1) + \frac{3}{4}}{2j(j+1)}$$

$g_{l,p} = 1$ $g_{s,p} = 5{,}6$
$g_{l,n} = 0$ $g_{s,n} = -3{,}8$

JEDRA Z LIHIM ŠTEVILOM p (lih Z), sodim N

$$\frac{\mu}{\mu_N} = \begin{cases} j + 2{,}3 & ; \quad j = l + \frac{1}{2} \\ \frac{j(j+\frac{3}{2})}{j+1} - 2{,}8\frac{j}{j+1} & ; \quad j = l - \frac{1}{2} \end{cases}$$

JEDRA Z LIHIM ŠTEVILOM n, sodim Z

$$\frac{\mu}{\mu_N} = \begin{cases} -1{,}9 & ; \quad j = l + \frac{1}{2} \\ 1{,}9\frac{j}{j+1} & ; \quad j = l - \frac{1}{2} \end{cases}$$

JEDRSKI RAZPADI

RAZPAD α

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

${}^{4}_{2}He$ = DELEC α

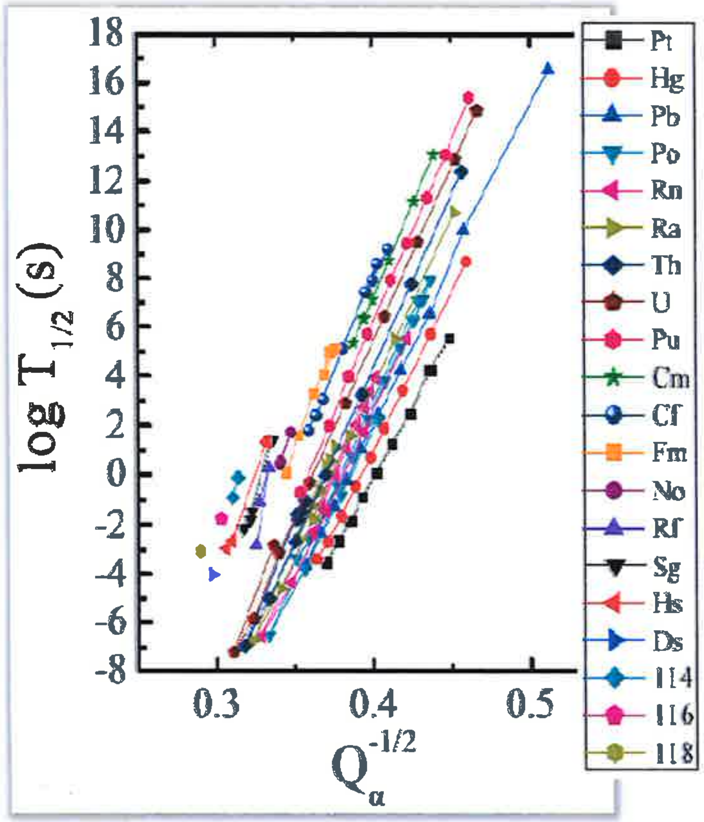
VEČINO KINETIČNE ENERGIJE ODNESE DELEC α (DN: $T_\alpha = ?$)

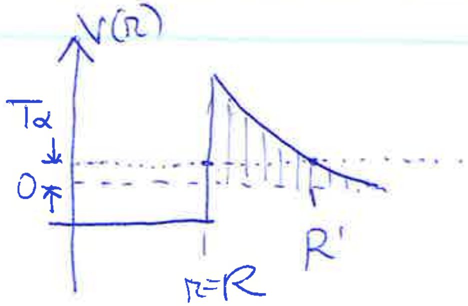
$$T_\alpha + T_Y = (m_X - m_Y - m_\alpha)c^2$$

IZ SEMIEMPIRIČNE MAS. FORMULE: KINEMATIČNO MOŽNO OD $A = 155$

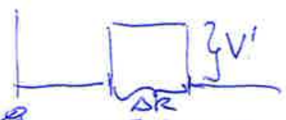
VERJETNOST ZNATNA OD $A = 207$ DALJE.

EKSPERIMENT: GEIGER-NUTTALOVO PRAVILO $\log T_{1/2} \propto \frac{1}{\sqrt{T_\alpha}}$





$W_\alpha \propto e^{-g}$ 11



VERJETNOST ZA TUNELIRANJE:

$$\left[\frac{4kk}{k^2+k^2}\right]^2 e^{-2k\Delta R}$$

$k = \sqrt{\frac{2mT_\alpha}{\hbar^2}}$ $\quad k = \sqrt{\frac{2m_\alpha (V'-T_\alpha)}{\hbar^2}}$

$$\prod_i e^{-2k_i\Delta R} = \to e^{-2\int_R^{R'} k(r)\,dr}$$

$$\Rightarrow g \doteq \sqrt{\frac{2m}{\hbar^2}}\sqrt{V(R)}\, R\left[\pi\sqrt{\frac{V(R)}{T_\alpha}} - 2\right]$$

$$W_\alpha \propto e^{-g}, \quad \ln W_\alpha \propto -k_1\frac{R\,V(R)}{\sqrt{T_\alpha}} + k_2 R\sqrt{V(R)}$$

RAZPAD β:

$${}^A_Z X \to {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e \qquad \beta^- \qquad n \to p e^- \bar{\nu}_e$$

$${}^A_Z X \to {}^{A}_{Z-1}Y + e^+ + \nu_e \qquad \beta^+ \qquad p \to n e^+ \nu_e$$

$(m_n - m_p)c^2 = 1{,}29$ MeV

$m_e c^2 = 0{,}511$ MeV

PROST NEVTRON: $\tau_n = 880$ s

β^- POGOJ: $m_X c^2 - m_Y c^2 - m_e c^2 \geq 0$

$$Z m_p c^2 + (A-Z) m_n c^2 + W_X - (Z+1) m_p c^2 - (A-Z-1) m_n c^2 - W_Y - m_e c^2 \geq 0$$

$$W_X - W_Y \geq -0{,}78 \text{ MeV} \Rightarrow |W_Y| - |W_X| \geq 0{,}78 \text{ MeV}$$

POGOJ ZA β^+ : $|W_Y| - |W_X| \geq 1.80$ MeV

VERJETNOST ZA RAZPAD β

$$W_{fi} = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho_f(E_i)$$

FERMIJEVA IZPELJAVA $V_{fi} = G_F \int \psi_f^* \varphi_e^* \varphi_\nu^* \psi_i \, d^3r$

G_F: FERMIJEVA SKLOPITVENA KONSTANTA

φ_e, φ_ν RAVNI VAL $\quad \varphi_e = \frac{1}{\sqrt{V}} e^{i\vec{k}_e \vec{r}} \quad \varphi_\nu = \frac{1}{\sqrt{V}} e^{i\vec{k}_\nu \vec{r}}$

$$V_{fi} = G_F \frac{1}{V} \int \psi_f^* \psi_i e^{-i\vec{k}\vec{r}} d^3r \qquad \vec{k} = \vec{k}_e + \vec{k}_\nu \qquad k_e = \frac{p_e}{\hbar} = \frac{\sqrt{E_e^2 - m^2c^4}}{\hbar c}$$

$$kr \sim \frac{1\,\mathrm{MeV}}{\underbrace{\hbar c}_{\sim 200\,\mathrm{MeV\,fm}}} \cdot 5\,\mathrm{fm} \sim \frac{1}{40}$$

$$\Rightarrow \quad V_{fi} = G_F \frac{1}{V} \int \psi_f^* \psi_i \left(1 - i\vec{k}\cdot\vec{r} + \frac{(\vec{k}\vec{r})^2}{2!} + \dots\right) d^3r =$$

$$= G_F \frac{1}{V} \left[\int \psi_f^* \psi_i \, d^3r - i \int \psi_f^* \psi_i \, \vec{k}\cdot\vec{r} \, d^3r + \dots \right]$$

ČE PRVI INTEGRAL $\neq 0 \Rightarrow$ DOVOLJENI RAZPAD β

— " — $= 0$, DRUGI PA $\neq 0 \Rightarrow$ ENKRAT PREPOVEDANI RAZPAD

IN TAKO NAPREJ,

$\frac{1}{l!} (\vec{k}\cdot\vec{r})^l$ IMA KOTNO ODVISNOST $Y_{lm}(\vartheta, \varphi)$

The X International Conference on Instrumentation for Colliding Beam Physics
Budker Institute of Nuclear Physics, Siberian Branch of Russian Academy of Science, Novosibirsk, Russia
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SPEKTER e^-/e^+ PRI RAZPADIH β DOVOLJENIH $\left(\frac{dN}{dE_e}\right)$

$d^3p_e = 4\pi p_e^2 dp_e$

$$d^6N_f = V^2 \frac{d^3p_e\, d^3p_\nu}{(2\pi\hbar)^6} \qquad d^5\rho_f = \frac{d^6N_f}{dE_f} \qquad d^2N_f = 16\pi^2V^2 \frac{p_e^2 dp_e\; p_\nu^2 dp_\nu}{(2\pi\hbar)^6}$$

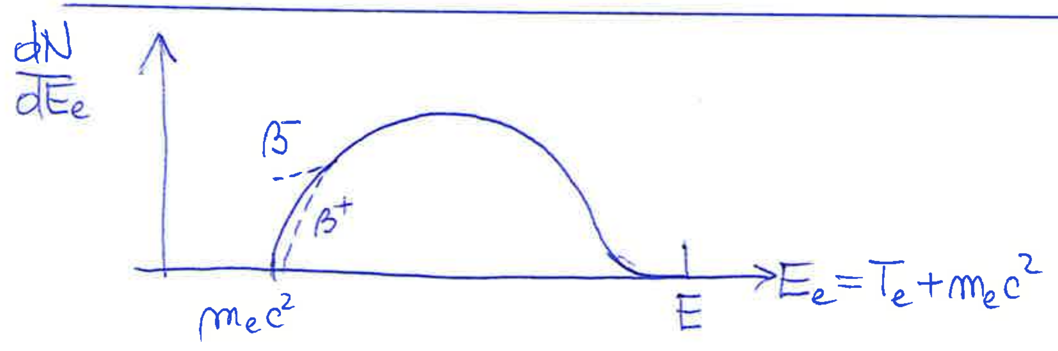
$$E_i = m_x c^2 = m_y c^2 + \underbrace{E_e + E_\nu}_{E} ; \quad E_e^2 = p_e^2c^2 + m_e^2c^4 , \; E_\nu = cp_\nu ; \; p_\nu = \frac{E-E_e}{c}$$

$$dp_\nu = \frac{dE}{c} \qquad d^2N_f \propto p_e^2\, dp_e\, (E-E_e)^2\, dE$$

$$\frac{d\rho_f}{dp_e} = \frac{d^2N_f}{dE_f\, dp_e} \propto p_e^2 (E-E_e)^2 \qquad \frac{dW_{fi}}{dp_e} \propto p_e^2 (E-E_e)^2$$

$$\frac{dW_{fi}}{dE_e} = \frac{dW_{fi}}{dp_e} \frac{dp_e}{dE_e} \propto p_e E_e (E-E_e)^2 = \sqrt{E_e^2 - m_e^2c^4}\, E_e (E-E_e)^2$$

$$2E_e dE_e = 2p_e dp_e \qquad \frac{dp_e}{dE_e} = \frac{E_e}{p_e}$$



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