GOSTOTA STANJ

$$d^6 N_f = \frac{d^3 r\, d^3 p}{h^3} \; ; \; d^3 N_f = V_N \frac{d^3 p}{h^3}$$

$$d\rho_f = \frac{d^3 N_f}{dE_f} = V_N \frac{p_f^2\, dp_f\, d\Omega}{h^3\, dE_f}$$

$$\frac{d\rho_f}{d\Omega} = V_N \frac{p_f^2\, dp_f}{h^3\, dE_f} = \qquad E_f = \frac{p_f^2}{2m}\,, \quad dE_f = \frac{p_f\, dp_f}{m}$$

$$= V_N \frac{m\, p_f}{(2\pi \hbar)^3} \qquad \frac{d\rho_f}{d\Omega}(E_i) = V_N \frac{m^2 v_i}{(2\pi\hbar)^3}$$

MATRIČNI ELEMENT ZA PREHOD $i \rightarrow f$

$$V_{fi} = \int \psi_f^*(\vec{r})\, V(\vec{r})\, \psi_i(\vec{r})\, d^3 r$$

$$\frac{d\sigma}{d\Omega} = \frac{dW_{fi}/d\Omega}{\rho_i v_i} \qquad \left[\frac{d\sigma}{d\Omega}\right] = \frac{s^{-1}}{m^{-3}\, m s^{-1}} = m^2$$

MATRIČNI ELEMENT:

$$V_{fi} = \int \psi_f^*(\vec{r})\, V(\vec{r})\, \psi_i(\vec{r})\, d^3 r =$$

$$= \frac{1}{V_N} \int e^{-i\vec{k}_f \vec{r}}\, V(\vec{r})\, e^{i\vec{k}_i \vec{r}}\, d^3 r =$$

$$= \frac{1}{V_N} \int e^{i(\vec{k}_i - \vec{k}_f)\cdot\vec{r}}\, V(\vec{r})\, d^3 r \qquad \vec{q} = (\vec{k}_i - \vec{k}_f)$$

$$= \frac{1}{V_N} \int e^{i\vec{q}\vec{r}}\, V(\vec{r})\, d^3 r$$

NASLEDNJI KORAK: UPORABIMO GREENOVO FORMULO

$$\int_V [u\, \nabla^2 v - v \nabla^2 u]\, d^3 r = \oint [u \vec{\nabla} v - v \vec{\nabla} u]\, d\vec{S}$$

$$u = e^{i\vec{q}\cdot\vec{r}} \qquad v = V(\vec{r}) = e\,\mathcal{U}(\vec{r})$$

$$\nabla^2 \mathcal{U}(\vec{r}) = -\frac{\rho_e(\vec{r})}{\varepsilon_0}$$

$\rho_e$ : GOSTOTA EL. NABOJA

$$\int \rho_e(\vec{r})\, d^3r = Ze$$

$$V(\vec{r}),\ \vec{\nabla} V(\vec{r}) \xrightarrow[r\to\infty]{} 0$$

$$\int u \nabla^2 v\, d^3r - \int v \nabla^2 u\, d^3r = 0 \rightarrow$$

$$\int e\, e^{i\vec{q}\cdot\vec{r}} \nabla^2 \mathcal{U}(\vec{r})\, d^3r = \int V(\vec{r}) \left(-q^2\right) e^{i\vec{q}\cdot\vec{r}}\, d^3r$$

$$V_{fi} = \frac{1}{V_N} \int e^{i\vec{q}\cdot\vec{r}} V(\vec{r})\, d^3r = -\frac{1}{q^2} e \frac{1}{V_N} \int e^{i\vec{q}\cdot\vec{r}} \nabla^2 \mathcal{U}(\vec{r})\, d^3r$$

$$\boxed{V_{fi} = \frac{e}{\varepsilon_0 q^2} \cdot \frac{1}{V_N} \underbrace{\int e^{i\vec{q}\cdot\vec{r}} \rho_e(\vec{r})\, d^3r}_{F(\vec{q})}} = \frac{e}{\varepsilon_0 q^2} \cdot \frac{1}{V_N} F(\vec{q})$$

$F(\vec{q})$ = OBLIKOVNI FORM FAKTOR

$$\frac{d\sigma}{d\Omega} = \frac{dW_{fi}/d\Omega}{\rho_i v_i} = \frac{2\pi}{\hbar} \frac{|V_{fi}|^2}{\rho_i v_i} \cdot V_N \frac{m^2 v_i}{(2\pi\hbar)^3} =$$

$$\rho_i = \frac{1}{V_N}$$

$$= \frac{2\pi}{\hbar} \cdot V_N^2 \frac{m^2}{(2\pi\hbar)^3} \cdot \frac{e^2}{\varepsilon_0^2 q^4} \frac{1}{V_N^2} \left|F(\vec{q})\right|^2 =$$

$$= \frac{2\pi}{\hbar} \left[\frac{me}{\varepsilon_0 q^2}\right]^2 \frac{1}{(2\pi\hbar)^3} \cdot \left|F(\vec{q})\right|^2$$

ELASTIČNO SIPANJE $E_i = E_f$, $q^2 = (\vec{k}_i - \vec{k}_f)^2 = k_i^2 + k_f^2 + 2k_ik_f\cos\vartheta$

$$k_i^2 = k_f^2 = \frac{p^2}{\hbar^2} \Rightarrow q^2 = 4\frac{p^2}{\hbar^2} \sin^2\frac{\vartheta}{2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left[\frac{me}{8\pi\varepsilon_0 p^2}\right]^2 \frac{1}{\sin^4\frac{\vartheta}{2}} \left|F(\vec{q})\right|^2}$$

če $\lambda \gg R_{\text{JEDRO}}$ $\quad p \ll \frac{\hbar}{R}$

→ ZA TAK DELEC JE JEDRO TOČKASTO

$$\rightarrow \rho(\vec{r}) = Ze\,\delta(\vec{r}-\vec{r}_0)$$

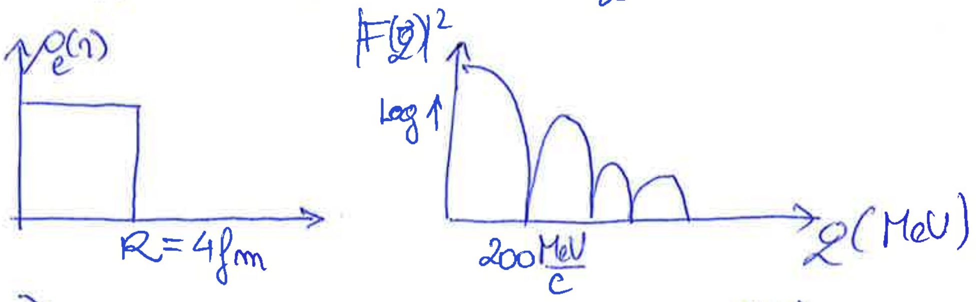
$$F(\vec{q}) = Ze \int e^{i\vec{q}\cdot\vec{r}}\,\delta(\vec{r}-\vec{r}_0)\,d^3r = Ze\,e^{i\vec{q}\vec{r}_0}$$

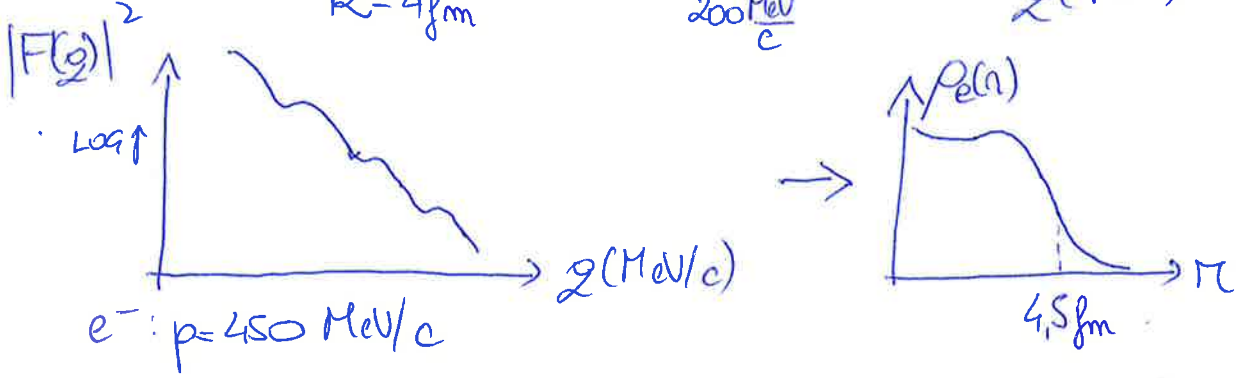
$$|F(\vec{q})|^2 = Z^2e^2$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \left[\frac{Ze^2 m}{8\pi\varepsilon_0 p^2}\right]^2 \frac{1}{\sin^4\frac{\vartheta}{2}}$$ RUTHERFORD

če $\vec{q}\cdot\vec{r} \ll 1 \rightarrow e^{i\vec{q}\vec{r}} = 1 + i\vec{q}\vec{r} \ldots$

PRIMERI $\rho_e(\vec{r})$




## SPIN JEDRA

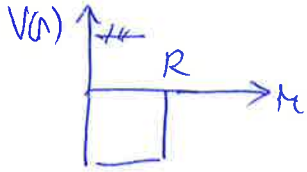
REŠUJEMO SCHRÖDINGERJEVO ENAČBO ZA JEDRO

NUKLEONI SE GIBLJEJO V POPREČNEM POTENCIALU JEDRU → REŠUJEM SCHRÖD. E. ZA POSAMEZEN NUKLEON

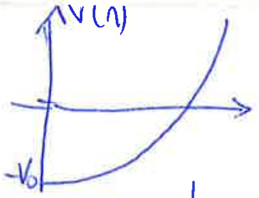
$$\hat{H}\psi = E\psi \qquad 3D: \ \psi = R(r)\,Y_{lm}(\vartheta,\varphi) \qquad R(r) = \frac{u(r)}{r}$$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{\hbar^2 l(l+1)}{2m_n r^2}\right]u(r) = E\,u(r)$$

$V(r)$

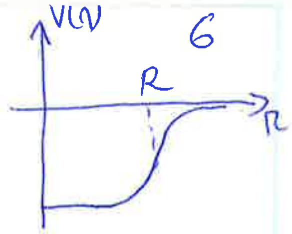



$\rightarrow$




$V(r) = -V_0 + \frac{1}{2} m\omega^2 r^2$

$\rightarrow$




SAXON-WOODS

$$V(r) = \frac{-V_0}{1 + e^{(r-R)/a}}$$

HARMONSKI POTENCIAL

$$E_{n,l} = \left(2n + l - \frac{1}{2}\right)\hbar\omega \qquad \text{OSNOVNO STANJE } \frac{3}{2}\hbar\omega$$

OPIS JEDRA:

- IZBEREM POTENCIAL
- REŠIM SCHRÖD. E.
- IZRAČUNAM ENERGIJSKE NIVOJE
- MAGIČNA ŠTEVILA: ZAKLJUČENE LUPINE SE UJEMAJO Z EKSPERIMENTOM

(MAGIČNA ŠTEVILA: ŠTEVILO Z ALI N, KJER JEDRO BISTVENO BOLJ VEZANO KOT SEMIEMPIRIČNA FORMULA! 2, 8, 20, 28, 50 ...)

- ČE NE UJEMANJE → NOV POTENCIAL

HARMONSKI POTENCIAL: 2, 8, 20, 40, ...

REŠITEV! DODATNI ČLEN $E' = -2\eta\, \hat{\vec{l}} \cdot \hat{\vec{s}}$

$\Rightarrow$ POSLEDICA! REŠITVE $l, l_3, s, s_3 \rightarrow j, j_3, l, s$

$$\hat{\vec{j}} = \hat{\vec{l}} + \hat{\vec{s}} \qquad \hat{j}^2 = (\hat{\vec{l}} + \hat{\vec{s}})^2 = \hat{\vec{l}}^2 + \hat{\vec{s}}^2 + 2\hat{\vec{l}} \cdot \hat{\vec{s}}$$

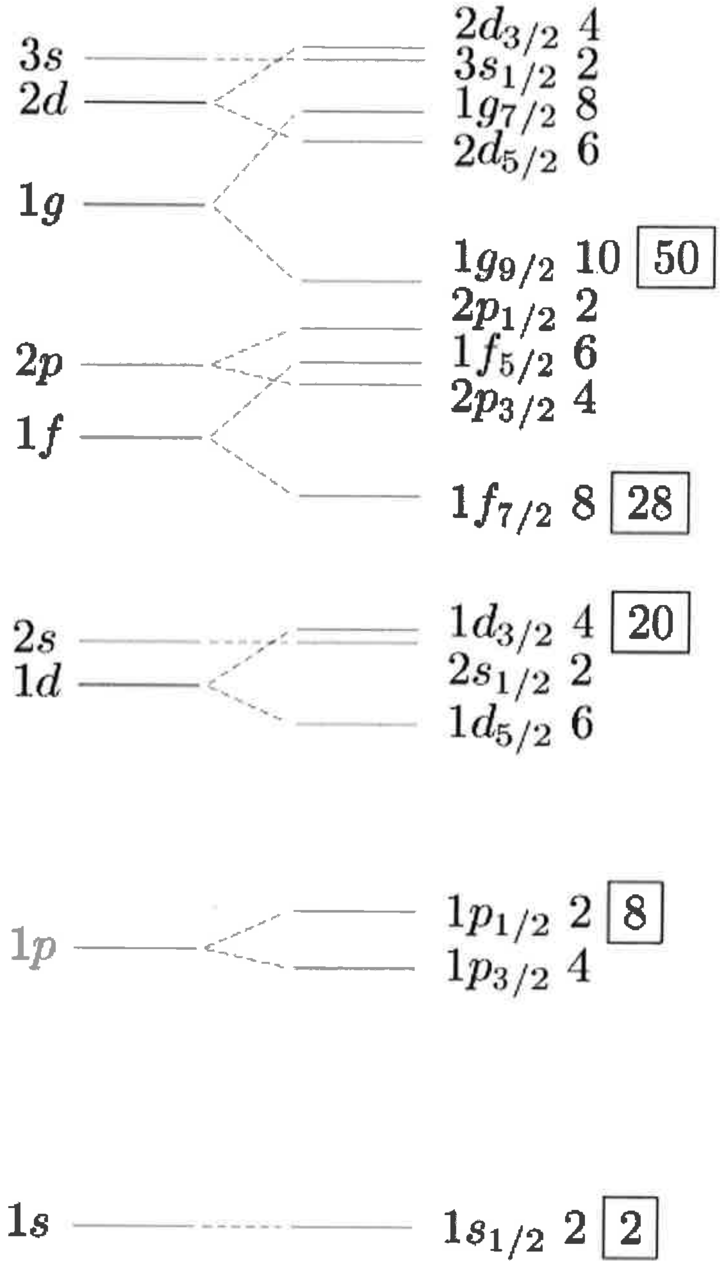
$$\hat{\vec{l}} \cdot \hat{\vec{s}} = \frac{1}{2}\left(\hat{\vec{j}}^2 - \hat{\vec{l}}^2 - \hat{\vec{s}}^2\right)$$

$$\langle j, j_3, l, s | \hat{\vec{l}} \cdot \hat{\vec{s}} | j, j_3, l, s \rangle = \hbar^2 \frac{1}{2}\left(j(j+1) - l(l+1) - s(s+1)\right)$$

$$s = \frac{1}{2}, \quad j = l \pm \frac{1}{2}$$

$$j = l + \frac{1}{2} \qquad \langle \hat{\vec{l}} \cdot \hat{\vec{s}} \rangle = \hbar^2 \frac{1}{2}\left(\left(l + \frac{1}{2}\right)\left(l + \frac{3}{2}\right) - l(l+1) - \frac{3}{4}\right)$$

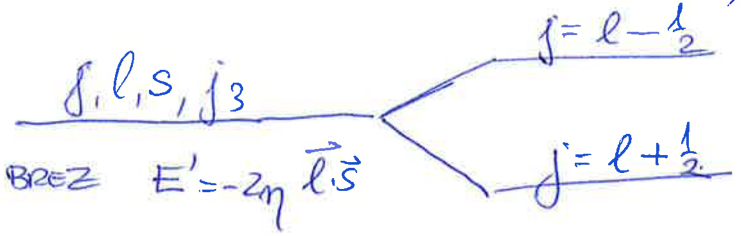
$$= \frac{\hbar^2}{2} \cdot l$$


3s
2d
1g
2p
1f
2s
1d
1p
1s
2d3/2 4
3s1/2 2
1g7/2 8
2d5/2 6
1g9/2 10 50
2p1/2 2
1f5/2 6
2p3/2 4
1f7/2 8 28
1d3/2 4 20
2s1/2 2
1d5/2 6
1p1/2 2 8
1p3/2 4
1s1/2 2 2

$$j = l - \frac{1}{2} \qquad \langle \vec{l}\cdot\vec{s} \rangle = \hbar^2 \frac{1}{2}\left( (l-\tfrac{1}{2})(l+\tfrac{1}{2}) - l(l+1) - \tfrac{3}{4} \right) =$$

$$= -\frac{\hbar^2}{2}(l+1)$$

$$\langle \vec{l}\cdot\vec{s} \rangle = \begin{cases} \frac{1}{2}\hbar^2 l & j = l + \frac{1}{2} \\ -\frac{1}{2}\hbar^2 (l+1) & j = l - \frac{1}{2} \end{cases}$$




$$\Delta E_{l+\frac{1}{2},\, l-\frac{1}{2}} = 2\eta \hbar^2 \frac{1}{2}(2l+1)$$

→ VELIK RAZCEP ZA VELIKE $l$

RECIMO $l=3$ (ORBITALA $f$) IN $l=4$ ($g$)