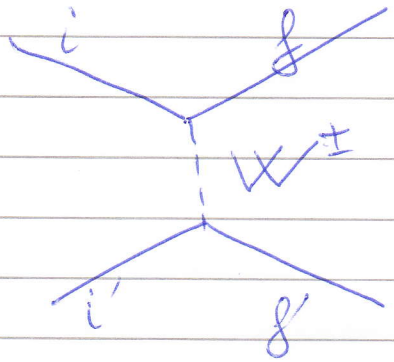
$\vec{\Sigma}\hat{p}\,u_R = +u_R$ za ULTRARELATIVISTIČNE DELCE

V ULTRAREL. LIMITI: ROČNOST ≡ SUČNOST

KONČNA OBLIKA MATRIČNEGA ELEMENTA ZA ŠIBEK PROCES



$$-i\mathcal{M} = \left[\frac{g_W}{\sqrt{2}}\bar{u}_f \gamma^\kappa (1-\gamma^5) u_i\right]\left(-\frac{g_{\kappa\nu}}{M_W^2 - q^2}\right)$$

$$\left[\frac{g_W}{\sqrt{2}}\bar{u}_{f'} \gamma^\nu (1-\gamma^5) u_{i'}\right]$$

RAZPAD: $q^2 \ll M_W^2$ $\quad$ $(MeV)^2 \ll (83\,GeV)^2$

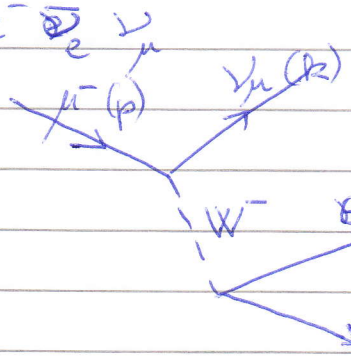
$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{2M_W^2} \Rightarrow \boxed{G_F = \frac{g_W^2}{\sqrt{2}M_W^2}}$$

A. SALAM, S. GLASHOW, S. WEINBERG
→ ELEKTROŠIBKA INTERAKCIJA
→ STANDARDNI MODEL

ENA OD NAPOVEDI: NEVTRALNI ŠIBKI TOKOVI: Z^0

PRIMERI ŠIBKEGA PROCESA PRI OSNOVNIH DELCIH

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$



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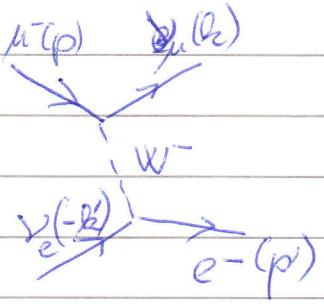


RAZPAD → RAZPADNA ŠIRINA $\Gamma = \frac{\hbar}{\tau}$

$$d\Gamma = \frac{|\mathcal{M}|^2}{2E} dQ$$

FAZNI PROSTOR

$$dQ = \frac{d^3p'}{(2\pi)^3 2E'} \cdot \frac{d^3k}{(2\pi)^3 2\omega} \cdot \frac{d^3k'}{(2\pi)^3 2\omega'} (2\pi)^4 \delta^4(p - p' - k - k')$$

$\uparrow e$ $\uparrow \nu_\mu$ $\uparrow \bar{\nu}_e$

RAČUNAMO $\frac{d\Gamma}{dE'}$ - SPEKTER ELEKTRONOV

NAJPREJ INTEGRAL PO k, UPORABIMO IDENTITETO

$$\int \frac{d^3k}{2\omega} = \int d^4k\, \Theta(\omega)\, \delta(k^2) \qquad \Theta(\omega) = \begin{cases} 1, \omega > 0 \\ 0, \omega < 0 \end{cases}$$

$$dQ = \frac{1}{(2\pi)^5} \frac{d^3p'}{2E'} \frac{d^3k'}{2\omega'} \int d^4k\, \Theta(\omega)\, \delta(k^2)\, \delta^4(p - p' - k - k')$$

$$= \frac{1}{(2\pi)^5} \frac{d^3p'}{2E'} \frac{d^3k'}{2\omega'} \Theta(E - E' - \omega')\, \delta((p - p' - k')^2)$$

MATRIČNI ELEMENT

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left[\bar{u}(k) \gamma^\mu (1 - \gamma^5) u(p)\right] \left[\bar{u}(p') \gamma_\mu (1 - \gamma^5) u(-k')\right]$$

$$= \frac{G_F}{\sqrt{2}} \left[\bar{u}(k) \gamma^\mu (1 - \gamma^5) u(p)\right] \left[\bar{u}(p') \gamma_\mu (1 - \gamma^5) v(k')\right]$$

$v(k')$ BISPINOR ZA ANTIDELEC

$u^{(1,2)} e^{-ipx}$ $E > 0$

$u^{(3,4)} e^{-(-ipx)} \equiv v^{(2,1)} e^{ipx}$ $E > 0$

DIRACOVA ENAČBA ZA v: ZA u: $(\not{p} - m) u = 0$

ZA $\bar{v}$: $(\not{p} + m) \bar{v} = 0$ $\not{p} = \gamma_\mu p^\mu$

POLNOSTNA RELACIJA

ZA v: $\sum_{s=1,2} v^{(s)}(p) \bar{v}^{(s)}(p) = \not{p} - m$ ZA u: $\sum_{s=1,2} u^{(s)}(p) \bar{u}^{(s)}(p) = \not{p} + m$

$$|\mathcal{M}|^2 = \frac{1}{2} \frac{G_F^2}{2} \sum \left[\bar{u}(k) \gamma^\mu (1 - \gamma^5) u(p)\right] \left[\bar{u}(p) \gamma_\mu (1 - \gamma^5) v(k')\right]$$
$$\left[\bar{u}(k) \gamma^\alpha (1 - \gamma^5) u(p)\right]^\dagger \left[u(p') \gamma_\alpha (1 - \gamma^5) v(k')\right]^\dagger$$

$\alpha^* = \alpha^\dagger$ (α JE A KOMPL. ŠTEVILO)

$$\left[\bar{u}(k)\gamma^2(1-\gamma^5)u(p)\right]^+ = \left[u^+(k)\gamma^0\gamma^2(1-\gamma^5)u(p)\right]^+ =$$

$$= \underbrace{u^+(p)}_{u^+\gamma^0}(1-\gamma^5)^+\gamma^{2+}\gamma^{0+}u(k) = u^+(p)(1-\gamma^5)\gamma^{2+}\gamma^0 u(k) =$$

$$= -\bar{u}(p)(1+\gamma^5)\gamma^{2+}u(k) = -\bar{u}(p)\gamma^{2+}(1-\gamma^5)u(k) =$$

$$= \bar{u}(p)\gamma^2(1-\gamma^5)u(k)$$

PODOBNO:

$$\left[\bar{u}(p')\gamma_2(1-\gamma^5)v(k')\right]^+ = \bar{v}(k')\gamma_2(1-\gamma^5)u(p')$$

$$|\overline{\mathcal{M}}|^2 = \frac{G_F^2}{4}\sum_{\substack{\text{SPINA}\\ \mu,\nu_\mu}}\left[\bar{u}(k)\gamma^\mu(1-\gamma^5)u(p)\right]\left[\bar{u}(p)\gamma^6(1-\gamma^5)u(k)\right]\cdot$$

$$\cdot\sum_{\substack{\text{SPINA}\\ e,\nu_e}}\left[\bar{u}(p')\gamma_\mu(1-\gamma^5)v(k')\right]\left[\bar{v}(k')\gamma_2(1-\gamma^5)u(p')\right]$$

POLNOSTNA RELACIJA →

TAKO KOT PRI E.M. INTERAKCIJI: VSOTA PO SPINIH → SLED. MATRICE

$$|\overline{\mathcal{M}}|^2 = \frac{G_F}{4}\mathrm{Tr}\left[(\not{k}+m_{\nu_\mu})\gamma^\mu(1-\gamma^5)(\not{p}+m_\mu)\gamma^2(1-\gamma^5)\right]$$

$$\mathrm{Tr}\left[(\not{p}'+m_e)\gamma_\mu(1-\gamma^5)(\not{k}'-m_{\nu_e})\gamma^6(1-\gamma^5)\right]$$

V NADALJEVANJU: UPOŠTEVAMO $m_{\nu_e}\simeq 0$, $m_{\nu_\mu}\simeq 0$

$m_e \ll m_\mu$

PRODUKTA

SLED LIHEGA ŠTEVILA MATRIK γ $= 0$, $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$

$$|\overline{\mathcal{M}}|^2 = \frac{G_F^2}{4}\mathrm{Tr}\left[\not{k}\gamma^\mu(1-\gamma^5)\not{p}\gamma^2(1-\gamma^5)\right]$$

$$\mathrm{Tr}\left[\not{p}'\gamma_\mu(1-\gamma^5)\not{k}'\gamma_2(1-\gamma^5)\right]$$

+ IZRAČUN O SLEDI $\quad |\overline{\mathcal{M}}|^2 = \frac{G_F^2}{4}256(kp')(pk')$

V MIROVNEM SISTEMU MIONA : $p = (m_\mu, 0)$

$$(p - k')^2 = (p' + k)^2 = \underbrace{p'^2}_{m_e^2 \sim 0} + \underbrace{k^2}_{m_{\nu_\mu}^2 \sim 0} + 2p'k$$

$$\Rightarrow p'k = \frac{1}{2}(p - k')^2$$

$$\overline{|\mathcal{M}|}^2 = 32 G_F^2 (p - k')^2 (p'k') = 32 G_F^2 \underbrace{(m_\mu - \omega' - \vec{k}')^2}_{(m_\mu - \omega')^2 - \vec{k}'^2 =} m_\mu \omega' =$$

$$= 32 G_F^2 m_\mu^2 (m_\mu - 2\omega')$$

$$= (m_\mu^2 - 2m_\mu\omega' + \underbrace{\omega'^2 - \vec{k}'^2}_{=0}$$

$$d\Gamma = \frac{\overline{|\mathcal{M}|}^2}{2E} dQ = \quad \leftarrow \Theta(\omega) = 1$$

$$= \frac{1}{2m_\mu} 32 G_F^2 m_\mu^2 (m_\mu - 2\omega') \frac{1}{(2\pi)^5} \frac{d^3p'}{2E'} \cdot \frac{d^3k'}{2\omega'} \delta\left((p - p' - k')^2\right)$$

$$d^3p' = 4\pi E'^2 dE'$$

$$d^3k' = \not{4}\pi \omega'^2 d\omega' \, d(\cos\vartheta)$$

ϑ: kot med elektronom in ν_e, os z v smeri e^-

$$\delta((p - p' - k')^2) = \cdots = \delta(m_\mu^2 - 2m_\mu E' - 2m_\mu\omega' + 2E'\omega'(1 - \cos\vartheta))$$

$$= \delta(\cdots + 2E'\omega'\cos\vartheta) = \frac{1}{2E'\omega'} \delta(\cdots + \cos\vartheta)$$

$$d\Gamma = \frac{G_F^2}{2\pi^3} m_\mu \omega' (m_\mu - 2\omega') dE' d\omega'$$

$$\cos\vartheta = \frac{m_\mu^2 - 2m_\mu E' - 2m_\mu\omega'}{2E'\omega'} + 1$$

$$-1 \leq \cos\vartheta \leq 1$$

$$-2 \leq \frac{m_\mu^2 - 2m_\mu E' - 2m_\mu\omega'}{2E'\omega'} \leq 0$$

$$\omega'(4E' - m_\mu) \geq \frac{m_\mu}{2}(2E' - m_\mu)$$

$$\boxed{\omega' \geq \frac{m_\mu}{2} - E'}$$

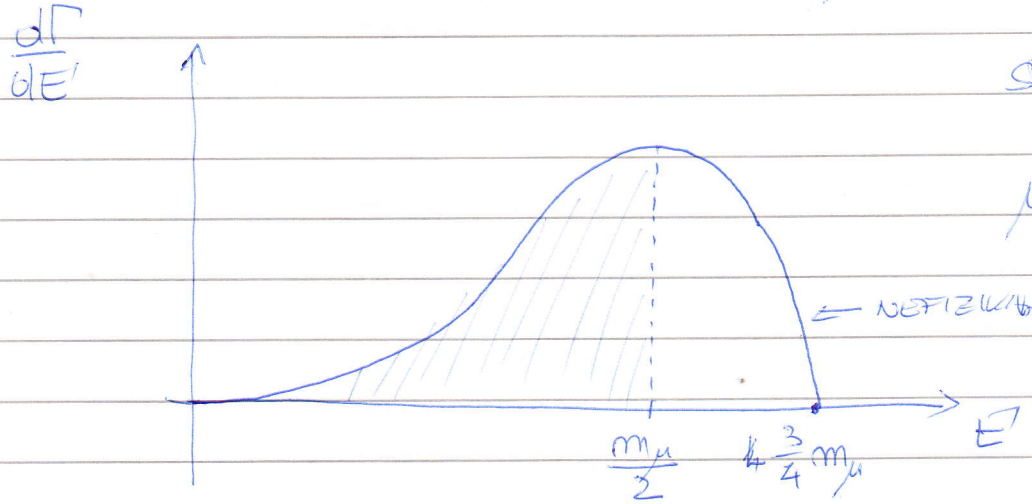
ČE BI BILA PRI RAZPADU DVA DELCA V KONČNEM STANJU (BREZMASNA) $E' = \frac{m_\mu}{2}$

3 DELCI → $E' \leq \frac{m_\mu}{2}$

⇒ LEVA MEJA → $\boxed{\omega' \leq \frac{m_\mu}{2}}$

$$d\Gamma = \frac{G_F^2}{2\pi^3} m_\mu dE' \int_{\frac{m_\mu}{2}-E'}^{\frac{m_\mu}{2}} \omega'(m_\mu - 2\omega')\, d\omega' =$$

$$\frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)$$



$$\Gamma = \int \frac{d\Gamma}{dE'} dE' = \frac{G_F^2}{12\pi^2} \int_0^{\frac{m_\mu}{2}} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right) dE' =$$

$$= \frac{G_F^2 m_\mu^5}{192\pi^3}$$

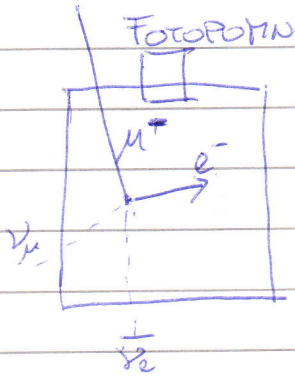
$$\boxed{\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}}$$

$$\boxed{\tau_\mu = \frac{\hbar}{\Gamma} = 2{,}1\ \mu s}$$

RAZPADNI ČAS MIONA → G_F

POSKUS

POSKUS (FIZIKALNI EKSPERIMENTI 1)

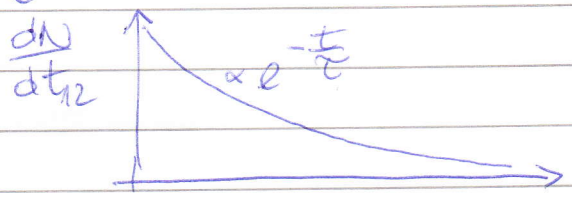


$pp \to pp\pi^+\pi^-$

$\pi^+ \to \mu^+ \nu_\mu$

μ^-: SVETLOBNI SUNEK ①

e^- — " — ②



RAZVEJITVENO RAZMERJE
(BRANCHING FRACTION, BR)

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$ $\quad \Gamma_1(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$

$$\Gamma_\mu = \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) + \underbrace{\Gamma_2 + \Gamma_3 + \dots}_{\text{OSTALI RAZPADNI KANALI}}$$

$$BR(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma_\mu}$$

RAZVEJITVENO RAZMERJE ZA DOLOČEN RAZPAD

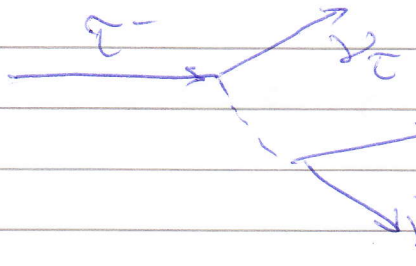
PRI TEM KONKRETNEM PRIMERU (MION):

$$\Gamma_2 + \Gamma_3 + \dots = 0$$

$$\Gamma_\mu = \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$$

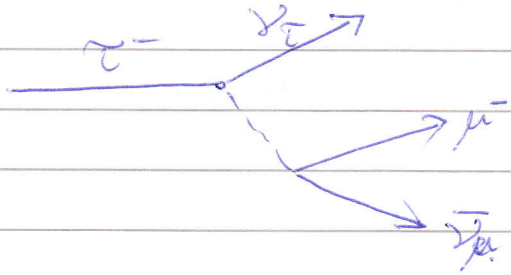
RAZPADNA ŠIRINA ZA LEPTON τ

EDEN OD RAZPADOV TAK KOT PRI MIONU

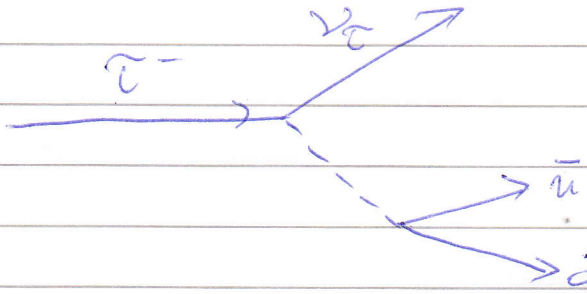


$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \frac{G_F^2 m_\tau^5}{192\pi^3}$$

$m_\tau \doteq 1{,}8\ \mathrm{GeV}$



$$\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = \frac{G_F^2 m_\tau^5}{192\pi^3}$$



$$\Gamma(\tau^- \to \nu_\tau \bar{u} d) \sim 3\frac{G_F m_\tau^5}{192\pi^3}$$

$\tau^- \to \nu_\tau \pi^-$, $\nu_\tau \pi^- \pi^0$, ...

$$\Gamma_\tau = \sum \Gamma_i \doteq 5\,\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$$

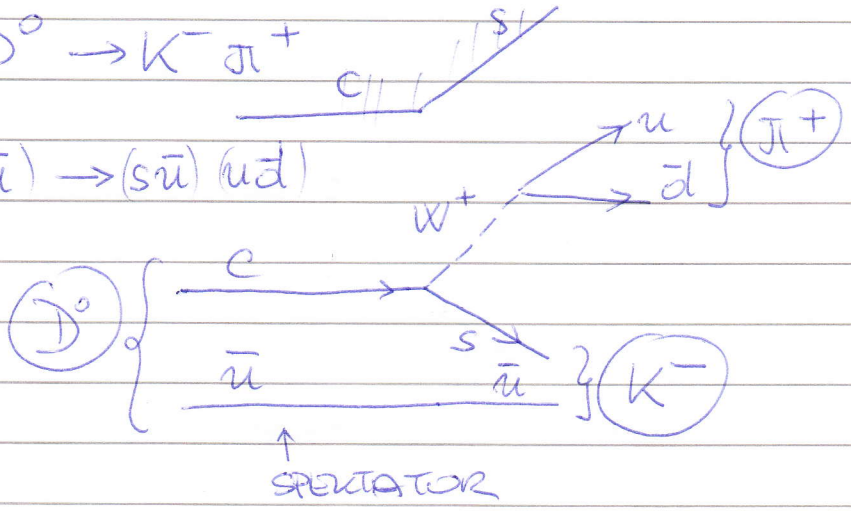
$$\boxed{\frac{\tau_\tau}{\tau_\mu} = \frac{1}{5}\left(\frac{m_\mu}{m_\tau}\right)^5}$$

DN τ_τ

ŠIBKI RAZPADI MEZONOV

$D^0 \to K^- \pi^+$

$(c\bar{u}) \to (s\bar{u})(u\bar{d})$



NAIVNO: $\Gamma(D^0 \to K^- \pi^+) \propto m_D^5$

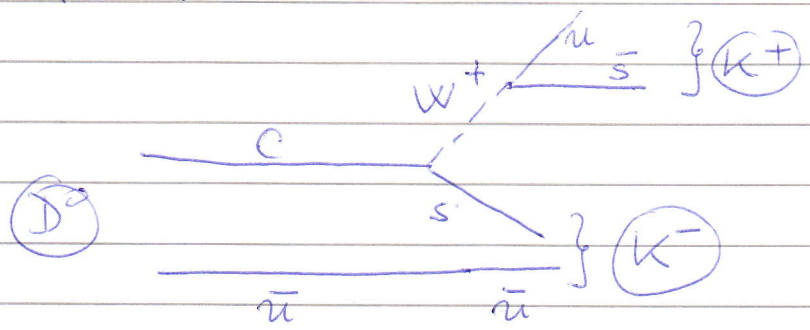
(ČE DELCI Z ZANEMARLJIVIMI MASAMI)

KONČNI

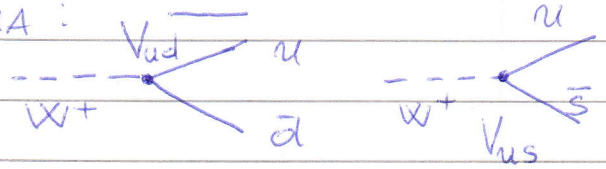
$m_{D^0} \sim 1{,}86$ GeV $\quad m_K \sim 0{,}5$ GeV $\quad m_\pi \sim 0{,}14$ GeV

BOLJŠE: $\Gamma(D^0 \to K^- \pi^+) \propto (m_{D^0} - m_{K^-})^5$

KAJ PA RAZPAD $D^0 \to K^+ K^-$?



RAZLIKA MED RAZPADOMA:



V_{ud}, V_{us} ELEMENTA MATRIKE

MATRIKA CABBIBA - KOBAYASHIJA IN MASKAWE (CKM)

3×3 MATRIKA

$V_{us} \sim 0{,}2\, V_{ud}$