NEPOLARIZIRANI PRESEK

S_1, S_2 : POLARIZIRANI e, μ

S_3, S_4 : MERIM POLARIZACIJO (SMER SPINA)

$$e^- \mu^- \to e^- \mu^-$$

$$S_1 \quad S_2 \qquad\quad S_3 \quad S_4$$

NEPOLARIZIRAN PRESEK: SIPALNI PRESEK ZA NEPOLARIZIRANE VPADNE DELCE IN ZA PRIMER, KO NE MERIM SPINOV V KONČNEM STANJU.

$$\sigma \propto |\mathcal{M}|^2$$

POLARIZIRAN PRESEK

$$\sigma \propto |\mathcal{M}_{s_1 s_2 s_3 s_4}|^2$$

NEPOLARIZIRAN PRESEK

$$\sigma \propto \frac{1}{(2s_a+1)(2s_b+1)} \sum_{s_1, s_2, s_3, s_4} |\mathcal{M}_{s_1, s_2, s_3, s_4}|^2$$

s_a, s_b ZA NAŠ PRIMER: $s_e = \frac{1}{2}$, $s_\mu = \frac{1}{2}$

$$\frac{1}{(2s_a+1)(2s_b+1)} = \frac{1}{4}$$

ZAKAJ

$$\sum |\mathcal{M}_{s_1, s_2, s_3, s_4}|^2 \quad \text{IN NE} \quad \cancel{\left|\sum \mathcal{M}_{s_1, s_2, s_3, s_4}\right|^2}$$

ZATO, KER LAHKO SPINE V ZAČETKU IN NA KONCU V PRINCIPU IZMERIMO. SEŠTEVANJE AMPLITUD PRIDE V POŠTEV, KADAR PROCESI NISO LOČLJIVI.

$$|\mathcal{M}|^2 \propto [\bar{u}(k')\gamma^\kappa u(k)][\underbrace{\bar{u}(k')}_{u^+(k')\gamma^0}\gamma^\lambda u(k)]^+ = [\bar{u}(k')\gamma^\kappa u(k)] \cdot [\bar{u}(k')\gamma^\lambda u(k)]^*$$

$$= [\bar{u}(k')\gamma^\kappa u(k)][u(k)^+ \underbrace{\gamma^{\lambda +}\gamma^{0+}}_{\gamma^{\lambda+}\gamma^0 = \gamma^0\gamma^\lambda} u(k')] =$$

$$= [\bar{u}(k')\gamma^\kappa u(k)][u(k)^+\gamma^0\gamma^\lambda u(k')] = [\bar{u}(k')\gamma^\kappa u(k)][\bar{u}(k)\gamma^\lambda u(k')]$$

$$\sum_{s,s'} [\bar{u}^{(s')}(k')\gamma^\kappa u^{(s)}(k)][\bar{u}^{(s)}(k)\gamma^\lambda u^{(s')}(k')] =$$

IMPLICITNA VSOTA PO PARIH INDEKSOV

$$= \sum_{s,s'} \left[\bar{u}_\alpha^{(s')}(k') \gamma^\kappa_{\alpha\beta} u_\beta^{(s)}(k)\right]\left[\bar{u}_\delta^{(s)}(k) \gamma^\lambda_{\delta\varepsilon} u_\varepsilon^{(s')}(k')\right] =$$

$$= \sum_{s,s'} \underbrace{u_\varepsilon^{(s')}(k') \bar{u}_\alpha^{(s')}(k')}_{[\not k' + m_e]_{\varepsilon\alpha}} \gamma^\kappa_{\alpha\beta} \underbrace{u_\beta^{(s)}(k) \bar{u}_\delta^{(s)}(k)}_{[\not k + m_e]_{\beta\delta}} \gamma^\lambda_{\delta\varepsilon}$$

$$= [\not k' + m_e]_{\varepsilon\alpha} \gamma^\kappa_{\alpha\beta} [\not k + m_e]_{\beta\delta} \gamma^\lambda_{\delta\varepsilon} =$$

$$= \left[[\not k' + m_e] \gamma^\kappa [\not k + m_e] \gamma^\lambda \right]_{\varepsilon\varepsilon} = \mathrm{Tr}\left[(\not k' + m_e)\gamma^\kappa(\not k + m_e)\gamma^\lambda\right]$$

ENAK RAZMISLEK ZA MIONE

$$\overline{|\mathcal{M}|^2} = \frac{1}{2}\cdot\frac{1}{2}\frac{e^2 e^2}{q^4}\cdot \mathrm{Tr}\left[(\not k' + m_e)\gamma^\kappa(\not k + m_e)\gamma^\lambda\right] \mathrm{Tr}\left[(\not p' + m_\mu)\gamma_\kappa(\not p + m_\mu)\gamma_\lambda\right]$$

TEOREMI O SLEDEH

EDEN OD NJIH

$$\mathrm{Tr}\left[(\not k' + m_e)\gamma^\kappa(\not k + m_e)\gamma^\lambda\right] = \cancel{4\left[k'^\kappa k^\lambda + k'^\lambda k^\kappa - (k' k\right.}$$

$$= 4\left[k'^\kappa k^\lambda + k'^\lambda k^\kappa - (k' \cdot k - m_e^2)\, g^{\kappa\lambda}\right]$$

$$\overline{|\mathcal{M}|^2} = 4\frac{e^4}{q^4}\left[(k'p')(kp) + (k'p)(kp') - m_e^2 p'p - m_\mu^2 k'k + 2m_e^2 m_\mu^2\right]$$

– LORENTZOVA INVARIANTA (KER SKALARNI PRODUKTI)

SIPALNI PRESEK $\quad \dfrac{d\sigma}{d\Omega} = \dfrac{dW_{fi}/d\Omega}{\rho_i v_i}$

$$\frac{dW_{fi}}{d\Omega} = \frac{2\pi}{\hbar}|T_{fi}|^2 \frac{d\rho_f}{d\Omega}$$

$$d^3N = V\frac{d^3p}{(2\pi\hbar)^3} = \frac{d^3p}{\rho(2\pi\hbar)^3}$$

$$\rho = \frac{1}{V}$$ NERELATIVISTIČNI PRIMER

RELATIVISTIČNI DELCI

$$d^3N = \frac{V}{2E}\frac{d^3p}{(2\pi\hbar)^3}$$

$$d^3N = \frac{V}{2E}\frac{d^3p}{(2\pi\hbar)^3}$$

$$\rho_e \propto \frac{d^3p}{E}$$

LORENTZOVA TRANSF. V SMERI X

$$dp_x' = \gamma(dp_x - \beta dE) \quad dp_y' = dp_y \quad dp_z' = dp_z$$

$$dE' = \gamma(dE - \beta dp_x)$$

$$\frac{d^3p'}{E'} = \frac{\gamma(dp_x - \beta dE)\,dp_y\,dp_z}{\gamma(E - \beta p_x)} = \frac{dp_x\left(1-\beta\frac{dE}{dp_x}\right)dp_y\,dp_z}{E-\beta p_x}$$

$$= \frac{dp_x\left(1-\beta\frac{p_x}{E}\right)dp_y\,dp_z}{E\left(1-\beta\frac{p_x}{E}\right)} =$$

$$E^2 = p_x^2 + p_y^2 + p_z^2 + m^2$$

$$E\,dE = p_x\,dp_x$$

$$= \frac{d^3p}{E}$$

LORENTZOVA INVARIANTA

$$\rho_i v_i = \underbrace{\frac{2E_a}{V} v_a}_{\text{UPADNI DELEC}} \underbrace{\frac{2E_b}{V}}_{\text{TARCA}}$$

PRIMER, KO DELEC b MIRUJE, a PA SE NA NJEM SIPLJE

ČE SE OBADVA GIBLJETA

$$\rho_i v_i = F = \frac{2E_a}{V}\frac{2E_b}{V}|\vec{v}_a - \vec{v}_b|$$

$$\frac{v}{c} = \beta = \frac{\gamma m v \cdot c}{\gamma m c\, c} = \frac{cp}{E}$$

$$\vec{v}_a - \vec{v}_b = \frac{|\vec{p}_a E_b - \vec{p}_b E_a|}{E_a E_b}$$

$$|\vec{p}_a E_b - \vec{p}_b E_a| = \sqrt{(p_a p_b)^2 - m_a^2 m_b^2}$$

$\Rightarrow$ TUDI TO JE RELATIVISTIČNA INVARIANTA

DIFERENCIALNI PRESEK

$$d\sigma = \frac{|\mathcal{M}|^2}{F}\,dQ$$

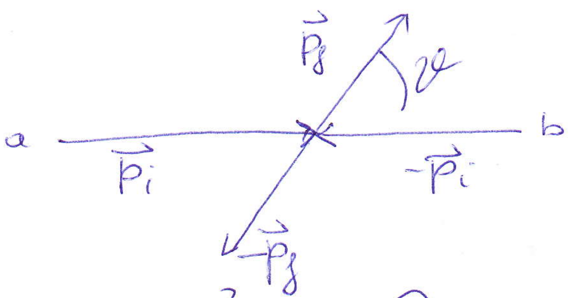
$$F = 4\sqrt{(p_a p_b)^2 - m_a^2 m_b^2}$$

$$dQ = (2\pi)^4 \delta^4(p' + k' - p - k) \cdot \frac{d^3p_c}{(2\pi)^3\,2E_c} \cdot \frac{d^3p_d}{(2\pi)^3\,2E_d}$$

$ab \to cd$

$$-i\mathcal{M} = [j^\mu_{ca}]\left(\frac{g_{\mu\nu}}{q^2}\right)[j^\nu_{db}]$$

V TEŽIŠČNEM SISTEMU (CENTER-OF-MASS, C.M.S.)



$$\vec p_a = \vec p_i = -\vec p_b$$
$$\vec p_c = \vec p_f = -\vec p_d$$

$$d^3p_c = p_f^2\, dp_f\, d\Omega \qquad d\Omega = 2\pi \sin\vartheta\, d\vartheta$$

$$\int \frac{d^3p_d}{2E_d}\underbrace{\delta^4(p_c+p_d-p_a-p_b)}_{\delta(E_c+E_d-E_a-E_b)\,\delta^3(\vec p_c+\vec p_d-\vec p_a-\vec p_b)} = \frac{1}{2E_d}\delta(E_c+E_d-E_a-E_b)$$

ENERGIJA V TEŽIŠČNEM SISTEMU $E = E_a + E_b$

$$dQ = \frac{1}{4\pi^2}\frac{p_f^2\, dp_f\, d\Omega}{4E_cE_d}\delta(E_c+E_d-E)$$

$$E = E_c + E_d = \sqrt{p_f^2+m_c^2}+\sqrt{p_f^2+m_d^2}$$

$$\frac{dE}{dp_f} = \frac{p_f}{E_c}+\frac{p_f}{E_d}$$

$$dp_f = dE\left(\frac{p_f}{E_c}+\frac{p_f}{E_d}\right)^{-1} = dE\,\frac{E_cE_d}{p_f\underbrace{(E_c+E_d)}_{E}}$$

$$dQ = \frac{1}{4\pi^2}\frac{p_f\, dE}{4(E_c+E_d)}\, d\Omega\, \delta(E_c+E_d-E)$$

$$\int dE \rightarrow dQ = \frac{1}{4\pi^2}\cdot\frac{p_f}{4E}\, d\Omega$$

$$F = 4p_iE$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2 p_f}{64\pi^2 p_i E^2}}$$

V ULTRARELATIVISTIČNI LIMITI $m_x \ll p_x$ $\quad p_i = p_f$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2E^2}}$$

UPORABIMO TA REZULTAT ZA $e^-\mu^- \to e^-\mu^-$, NEPOLARIZIRANO SIPANJE. V ULTRARELAT. LIMITI

$$k = \left(\frac{E}{2}, \vec{p}_i\right) \quad k' = \left(\frac{E}{2}, \vec{p}_f\right) \quad p = \left(\frac{E}{2}, -\vec{p}_i\right) \quad p' = \left(\frac{E}{2}, -\vec{p}_f\right)$$

$$k'p = \frac{E^2}{4} + \vec{p}_f \cdot \vec{p}_i = \frac{E^2}{4}(1+\cos\vartheta)$$

$$kp = \frac{E^2}{4} + \frac{E^2}{4} = \frac{E^2}{2}$$

$$\vec{p}_i = (p_i, 0, 0) = (E/2, 0, 0)$$
$$\vec{p}_f = \left(\frac{E}{2}\cos\vartheta, \frac{E}{2}\sin\vartheta, 0\right)$$

$$q^2 = (k'-k)^2 = (0, \vec{p}_f - \vec{p}_i)^2 =$$
$$= -(\vec{p}_f - \vec{p}_1)^2 = -\frac{E^2}{4}\left((\cos\vartheta - 1)^2 + \sin^2\vartheta\right) =$$
$$= -\frac{E^2}{2}(1-\cos\vartheta)$$

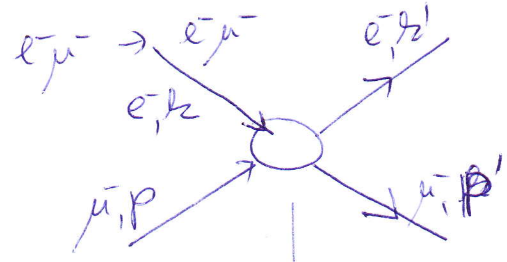
$$\overline{|\mathcal{M}|^2} = 4\frac{e^4}{q^4}\left[(k'p')(kp) + (k'p)(kp')\right] =$$
$$= 4\frac{e^4}{q^4}\left[\frac{E^4}{4} + \frac{E^4}{16}(1+\cos\vartheta)^2\right] =$$
$$= \frac{1}{4}\frac{e^4}{q^4}\left[4 + (1+\cos\vartheta)^2\right]E^4$$
$$= \frac{e^4\left[4 + (1+\cos\vartheta)^2\right]}{(1-\cos\vartheta)^2}$$

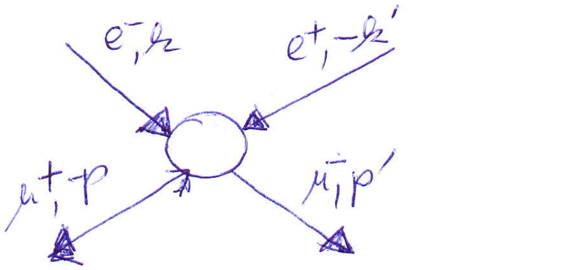
$$\frac{d\sigma}{d\Omega} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 E^2} = \frac{e^4\left(4 + (1+\cos\vartheta)^2\right)}{32\pi^2 E^2 (1-\cos\vartheta)^2}$$

PREVERITI DOMA

$e^+e^- \to \mu^+\mu^-$ IN KRIŽANJE

$e^-e^+ \to \mu^-\mu^+$





VHODNI		IZHODNI	
p_a	p_b	p_c	p_d
k	p	k'	p'
k	$-k'$	$-p$	p'

$$\overline{|\mathcal{M}|^2} = \frac{8e^4}{q^4}\left[(-pp')(-kk') + (k'p)(kp') + m_e^2 p'k' + m_\mu pk + 2m_e^2 m_\mu\right]$$

V ULTRARELATIVISTIČNI LIMITI

$$k = \left(\frac{E}{2}, \vec{p}_i\right),\quad k' = \left(\frac{E}{2}, \vec{p}_f\right),\quad p = \left(\frac{E}{2}, -\vec{p}_i\right),\quad p' = \left(\frac{E}{2}, -\vec{p}_f\right)$$

$$q^2 = (k'-k)^2 \xrightarrow[\text{PO KRIŽANJU}]{} (p+k)^2 = E^2$$

$$\frac{d\sigma}{d\Omega} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2E^2} = \frac{8e^4\left[(1-\cos\vartheta)^2 + (1+\cos\vartheta)^2\right]\frac{E^4}{4\cdot 4}}{64\pi^2E^2\,E^4} =$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{4\cdot 32\pi^2E^2}\left(2 + 2\cos^2\vartheta\right) = \frac{e^2}{4\cdot 16\pi^2E^2}\left(1+\cos^2\vartheta\right)$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega = \frac{e^4}{4\cdot 16\pi^2E^2}\int_{-1}^{+1}\left(1+\cos^2\vartheta\right)2\pi\cdot d(\cos\vartheta) =$$

$$= \frac{e^4}{4\cdot 8\pi E^2}\left(\cos\vartheta + \frac{1}{3}\cos^3\vartheta\right)\Big|_{-1}^{+1} = \frac{e^2}{8\pi E^2}\left(2+\frac{2}{3}\right) =$$

$$\sigma = \frac{e^4}{4\cdot 3\pi E^2} = \frac{e^4}{12\pi E^2} \qquad \sigma \propto \frac{1}{E^2}$$