素粒子宇宙起源研究機構
Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

OSNOVNI DELCI

SPIN $\frac{1}{2}$ IN SO TOREJ FERMIONI

LEPTONI e^- μ^- τ^-
ν_e ν_μ ν_τ

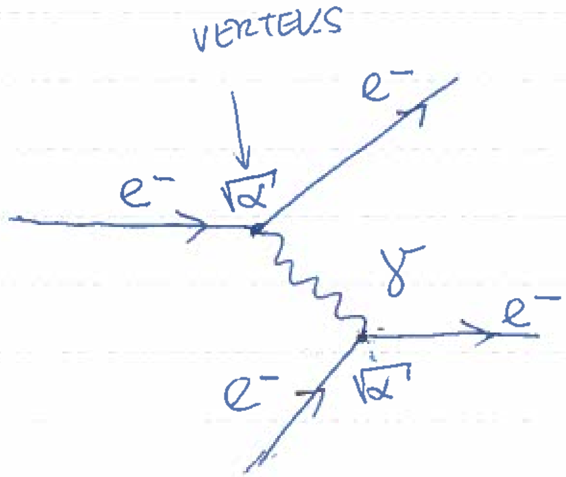
KVARKI u c t $+\frac{2}{3}e_0$
d s b $-\frac{1}{3}e_0$

KVARKI NASTOPAJO SAMO V VEZANIH STANJIH
MEZONI $q\bar{q}$ BARIONI qqq

INTERAKCIJE

ELEKTROMAGNETNA	FOTON
ŠIBKA	ŠIBKI BOZON $W^{\pm}, Z^0$
MOČNA	GLUONI

ELEKTROMAGNETNA INTERAKCIJA



$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

$$\gamma : \; E_\gamma \neq p_\gamma c$$

VIRTUALNI FOTON

$$e^- e^- \to e^- e^-$$

$$\frac{e^2}{4\pi\varepsilon_0} = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cdot \hbar c = \alpha \hbar c \qquad \alpha = \frac{1}{137}$$

KONSTANTA FINE STRUKTURE

$$|V_{fi}| \propto \frac{e^2}{4\pi\varepsilon_0} = \alpha \hbar c$$

$$E = T + V \qquad \text{KLASIČNA VEZA}$$

$$\hat{E}\psi = \hat{T}\psi + V\psi \qquad \text{KVANTNA MEHANIKA}$$

$$E^2 = p^2c^2 + m^2c^4$$

$$\hat{E}^2\psi = \left(\hat{p}^2c^2 + m^2c^4\right)\psi$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t} \qquad \hat{p} = -i\hbar \vec{\nabla}$$

$$-\hbar^2 \frac{\partial^2}{\partial t^2}\psi = -\hbar^2 \nabla^2 c^2 \psi + m^2 c^4 \psi$$

VAL. ENAČBA ZA PROST DELEC

KLEIN-GORDONOVA ENAČBA

$$\boxed{\nabla^2 \psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \frac{m^2c^2}{\hbar^2}\psi = 0}$$

$$m = 0 \qquad \nabla^2\psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi = 0$$

VALOVNA ENAČBA ZA EM VALOVANJE

STACIONARNA REŠITEV $\mathcal{U}$ $\qquad \frac{\partial \mathcal{U}}{\partial t} = 0 \qquad \mathcal{U} = \mathcal{U}(r)$

$$\nabla^2 \mathcal{U} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \mathcal{U}}{\partial r}\right) = 0 \Rightarrow \mathcal{U} = \frac{g}{r}$$

$$\text{Z.}: \quad g = \frac{e^2}{4\pi\varepsilon_0} \Rightarrow \mathcal{U} = \text{ELEKTROST. POT.}$$

$$\nabla^2 \mathcal{U} = \frac{m^2c^2}{\hbar^2}\mathcal{U} \qquad \mathcal{U} = \frac{g}{r} e^{-\frac{r}{R}}$$

$$R = \frac{\hbar}{mc}$$

~~KONEC~~

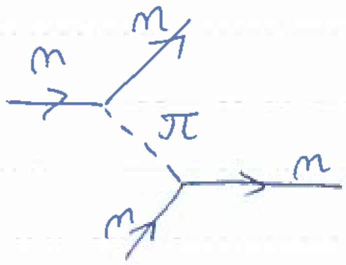
$m \neq 0 \Rightarrow$ POTENCIAL S KONČNIM DOSEGOM

HIDEKI YUKAWA: MOČNA INTERAKCIJA (KONČNI DOSEG)

$\Rightarrow$ NOSILCI MASIVNI DELCI: DOSEG $\to$ MASA

MEZONI

OCENA ~ 100 MeV

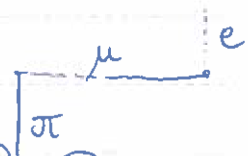


$\frac{e^{-r/R}}{r}$ YUKAWIN POTENCIAL

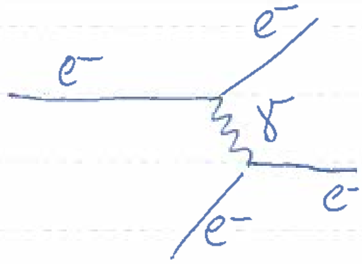
$$m_\pi c^2 \approx \frac{\hbar c}{R} \sim \frac{200 \text{ MeV fm}}{\text{nekaj fm}} \sim 100 \text{ MeV}$$

mion: $m_\mu c^2 = 104$ MeV

pion $m_\mu c^2 \sim 140$ MeV



$\pi^- \to \mu^- \bar{\nu}_\mu$
$\hookrightarrow e^- \bar{\nu}_e \nu_\mu$



$$f = \int U(r)\, e^{+i\vec{q}\cdot\vec{r}}\, d^3 r$$

ZA $U(r) = \frac{g}{r} e^{-\frac{r}{R}}$

$$f = \int \frac{g}{r} e^{+i\vec{q}\vec{r}} e^{-\frac{r}{R}} \underbrace{d^3 r}_{2\pi r^2 dr} =$$

$w = -i\underbrace{\vec{q}\vec{r}}_{qr} + \frac{r}{R}$

$w = r\left(\frac{1}{R} - iq\right)$

$$= g 2\pi \int_0^\infty e^{-w} \frac{w}{\left(\frac{1}{R} - iq\right)^2} dw =$$

$$= \frac{g}{\left(\frac{1}{R} - iq\right)^2} 2\pi \underbrace{\int_0^\infty e^{-w}\, w\, dw}$$

$$\sigma \propto |f|^2 \propto \frac{g^2}{\left|\frac{1}{R} - iq\right|^2} = \frac{g^2}{\frac{1}{R^2} + q^2} = \frac{g^2}{\frac{m^2c^2}{\hbar^2} + q^2} =$$

$$= \frac{g^2}{m^2c^4 + q^2\hbar^2} \cdot \hbar^2 c^2$$

$$\sigma \propto \frac{g^2}{m^2c^4 + q^2\hbar^2}$$

EM: $m = 0$

$g = \frac{e^2}{4\pi\varepsilon_0}$

$\sigma \propto \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \frac{1}{q^4}$

$$\sigma = \left[\frac{me}{8\pi\varepsilon_0 p^2}\right]^2 \frac{1}{\sin^4\frac{\theta}{2}} |F(q)|^2$$

POSLEDICA PRI ŠIBKI INTERAKCIJI

$m_W \sim 83\, GeV/c^2$

$$\sigma_W \propto \frac{g^2}{m_W^2 c^4 + q^2\hbar^2} \xrightarrow[\hbar^2 q^2 \ll m_W^2 c^4]{} \frac{g^2}{m_W^2 c^4}$$

SENČENJE NABOJA IN POLARIZACIJA VAKUUMA

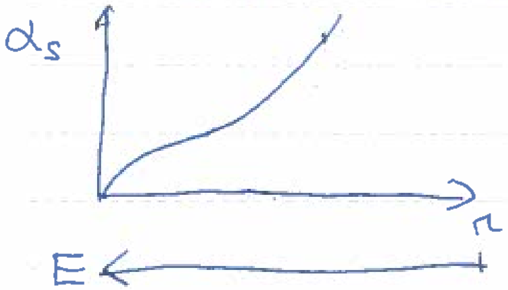
BLIZU e^-: VIDIM PRAVI NABOJ (GOLI)

NA VEČJI ODDALJENOSTI: NABOJ JE DEL SENČEN.

MOČNA INTERAKCIJA: ANTI-SENČENJE

1973 GROSS, POLITZER, WILCZEK

ASIMPTOTSKA SVOBODA



EM. INT. : $\alpha \approx \frac{1}{137}$ AMPLITUDA $f = \sum_{i=0}^{\infty} f_i \sim$

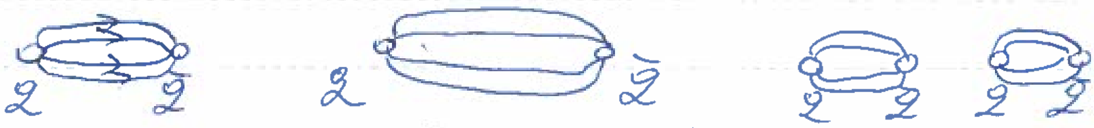
$\sim f_1 \alpha + f_2 \alpha^2 + \dots$

MOČNA INT. $\alpha \sim 1$ PRI NIZKIH E

PERTURBACIJSKI RAČUN PRI NIZKIH ENERGIJAH ODPOVE

VISOKE ENERGIJE $\alpha_s \ll 1 \rightarrow$ PERTURBACIJA DELUJE!

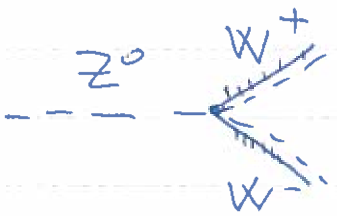
POTENCIAL MED KVARKI



ENERGIJA SISTEMA SE POVEČUJE

→ PROSTIH KVARKOV NI!

ŠIBKA INTERAKCIJA



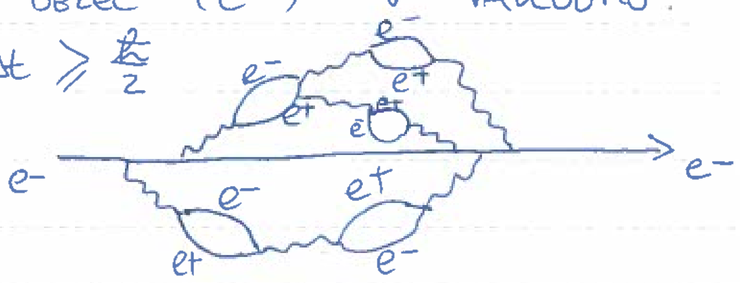
α_W Z RAZDALJO RASTE

Z ENERGIJO PADA

SKLOPITVENE KONSTANTE NISO ZARES KONSTANTE, AMPAK SO ODVISNE OD ENERGIJE, PRI KATERI POTEKA PROCES.

OSNOVNI DREVEC (e^-) V VAKUUMU:

$\Delta E \Delta t \geq \frac{\hbar}{2}$

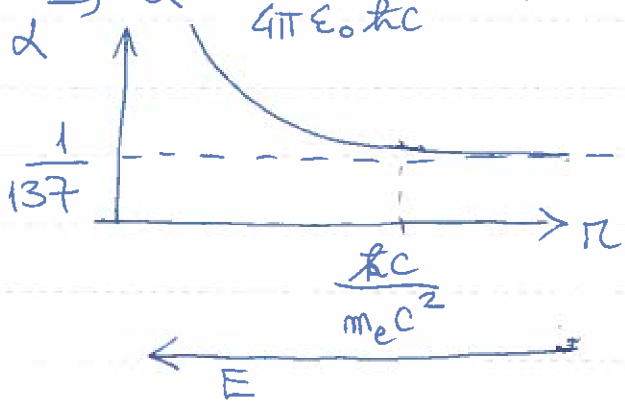


$$\Delta t \sim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{m_e c^2} \qquad c\Delta t \sim \frac{\hbar c}{m_e c^2} = \frac{200\ \text{MeV fm}}{0.5\ \text{MeV}} \sim 400\ \text{fm}$$

OD $\sim 10^3$ fm SE VREDNOST NABOJA NE BO SPREMINJALA. $e(r) = e_0 = +1{,}6 \cdot 10^{-19}$ As

→ POLARIZACIJA VAKUUMA.

$\Rightarrow \alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c}$ JE FUNKCIJA RAZDALJE



$\cancel{et}$ $\alpha(r) \to \alpha(E)$

$\Delta p \Delta x \geq \frac{\hbar}{2}$

$E\ r \sim \hbar$

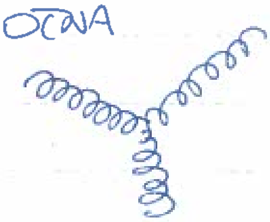
α_s, α_W : ODVISNOST OD ENERGIJE DRUGAČNA

EM

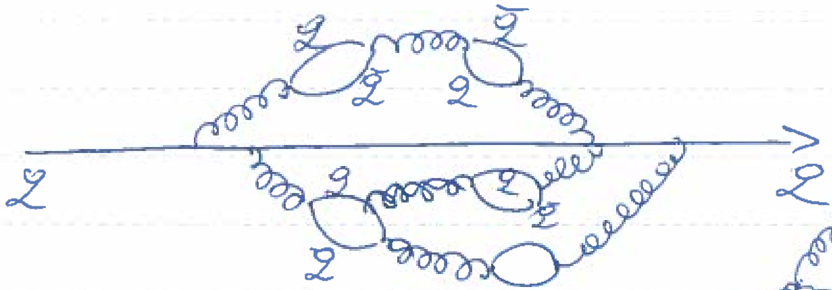
NE GRE!

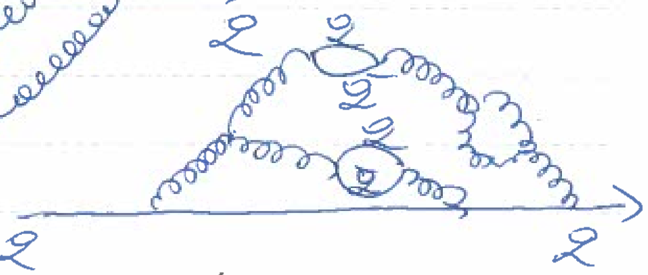
FOTONI NIMAJO NABOJA

MOČNA



GLUONI IMAJO NABOJ

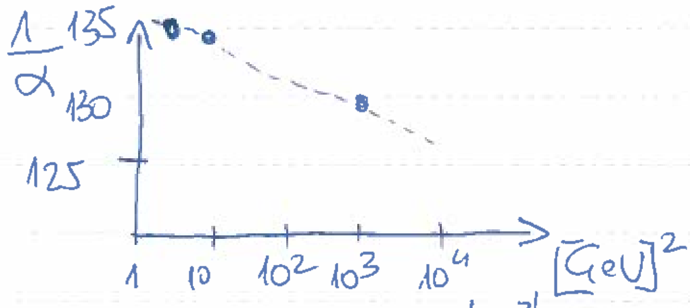


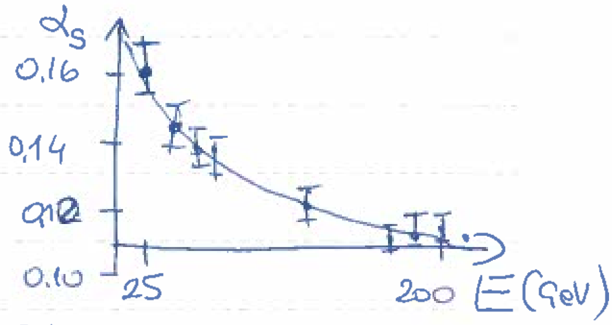


VELIKOST $\alpha_s, \alpha, \alpha_W$ PRI ~ 100 GeV

$\alpha \sim \frac{1}{128}$ PRI 100 GeV, $\frac{1}{137}$ PRI ~ 1 MeV

$\alpha_s \sim 20\,\alpha$ PRI 100 GeV

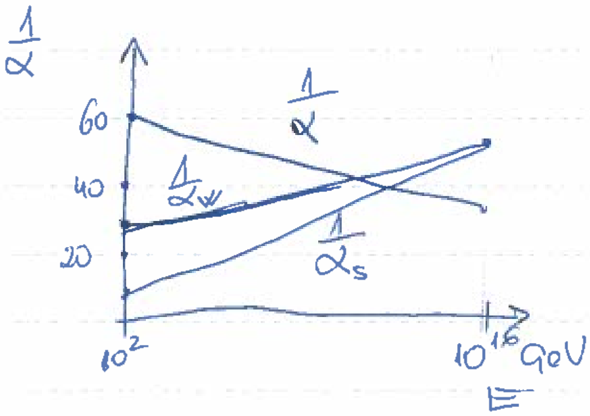


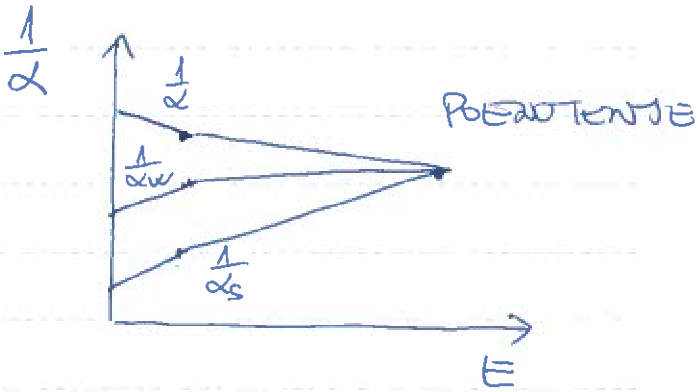


$|Q^2|$ = CETVEREC PRENOSA GIBAL KOLICINE

$(p_j^4 - p_i^4)$

$\alpha_W \sim \alpha$ $\qquad G \propto \frac{g^2}{m_W^2 c^4}$ PRI NIZKIH ENERGIJAH





SUPERSIMETRIČNI PARTNERJI OBIČAJNIH DELCEV SPIN $\frac{1}{2} \rightarrow 0$

e^- SELEKTRON

SIMETRIJE IN OHRANITVENI ZAKONI

OPAZLJIVKE, KI SE OHRANJAJO

STANJE $|\psi(t)\rangle$, ob $t=0$: $|\psi(t=0)\rangle$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat H |\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-\frac{i\hat H t}{\hbar}} |\psi(0)\rangle$$

$$|\psi(t)\rangle = \hat U(t)\, |\psi(0)\rangle \qquad \hat U = e^{-\frac{i\hat H t}{\hbar}}$$

OPOMBA: ČE $|\psi(t)\rangle$ LASTNA F.: $\hat H |\psi\rangle = E|\psi\rangle$

$$e^{-\frac{i\hat H t}{\hbar}} |\psi(0)\rangle = \left(1 \mp \frac{i\hat H t}{\hbar} + \dots\right)|\psi(0)\rangle = \left(1 \mp \frac{iEt}{\hbar} + \dots\right)|\psi(0)\rangle = e^{-\frac{iEt}{\hbar}} |\psi(0)\rangle$$

OPAZLJIVKA x, KI SE NAJ OHRANJA

$$\langle \psi(t)|x|\psi(t)\rangle = \langle \psi(0)|\hat U^+ x\, \hat U(t)|\psi(0)\rangle = \langle \psi(0)|\, x_0\, |\psi(0)\rangle$$

$$\hat U^+ x\, \hat U = x_0$$

$\hat U$ UNITAREN $\qquad \hat U^+ \hat U = I \Rightarrow x = \hat U x_0 \hat U^+$

$$\frac{\partial x}{\partial t} = \frac{\partial \hat U}{\partial t} x_0 U^+ + \hat U x_0 \frac{\partial U^+}{\partial t}$$

$$\frac{\partial \hat U}{\partial t} = -\frac{i\hat H}{\hbar} \hat U \qquad \frac{\partial \hat U^+}{\partial t} = \frac{i\hat H}{\hbar} \hat U^+$$

$$\frac{\partial x}{\partial t} = -\frac{i\hat H}{\hbar} \underbrace{\hat U x_0 \hat U^+}_{x} + \hat U x_0 \underbrace{\,}_{\hat U^+ U} \frac{i\hat H}{\hbar} U^+ = 0$$

2. ČLEN $\quad x\, \hat U \frac{i\hat H}{\hbar} \hat U^+ =$

$$\hat U \hat H \hat U^+ = e^{-\frac{i\hat H t}{\hbar}} \hat H e^{+\frac{i\hat H t}{\hbar}} =$$

$$\left(1 - \frac{i\hat H}{\hbar} t + \frac{(i\hat H t)^2}{2\hbar^2} - \dots\right) \hat H \left(1 + \frac{i\hat H}{\hbar} t + \frac{(it\hat H)^2}{2\hbar^2}\right) =$$

$$= \hat U \hat U^+ \hat H = \hat H$$

$$\frac{\partial x}{\partial t} = -i\frac{\hat H}{\hbar} x + \frac{i x H}{\hbar} = -\frac{i}{\hbar}[\hat H, x] = 0$$

x KOMUTIRA S $\hat H \Rightarrow$ PRIČAKOVANA VRED. x

KONSTANTNA $\rightarrow$ SE OHRANJA

PRIMER: 3. KOMPONENTA VRTILNE KOLIČINE

če $[\hat{H}, \hat{J}_z] = 0 \Rightarrow$ PRIČAKOVANA VREDNOST J_z SE OHRANJA

→ POSLEDICA: TRETJA KOMPONENTA VRTILNE KOLIČINE JE POVEZANA Z OPERATORJEM, KI ZAVRTI KOORDINATNI SISTEM OKOLI OSI Z

$$e^{i\frac{J_z}{\hbar}\varphi}$$

DN: IZPELJITE TO, ZAČNITE Z INFINITEZIMALNO ROTACIJO $\varphi \ll 1$

$$\left(1 + i\frac{J_z}{\hbar}\varphi\right)\psi(x,y,z)$$

OHRANITEV BARIONSKEGA IN LEPTONSKEGA ŠTEVILA

B: BARIONSKO ŠTEVILO. VSI BARIONI IMAJO $B=1$ → qqq

ANTI-BARIONI $B=-1$ → $\bar{q}\bar{q}\bar{q}$

p	uud	$B=1$
n	udd	1
lambda Λ	uds	1
$\bar{p}$	$\bar{u}\bar{u}\bar{d}$	-1
$\bar{\Lambda}$	$\bar{u}\bar{d}\bar{s}$	-1
π^+	$u\bar{d}$	0

$$p\,p \rightarrow p\,p\,p\,\bar{p}$$

B: $+1\ +1 \quad +1\ +1\ +1\ -1$

BARIONSKO ŠTEVILO SE OHRANJA

OK, DOVOLJEN

$$p\,p \not\rightarrow p\,\bar{p}\,\pi^+\pi^+$$

B: $+1\ +1 \quad +1\ -1\ 0\ 0$

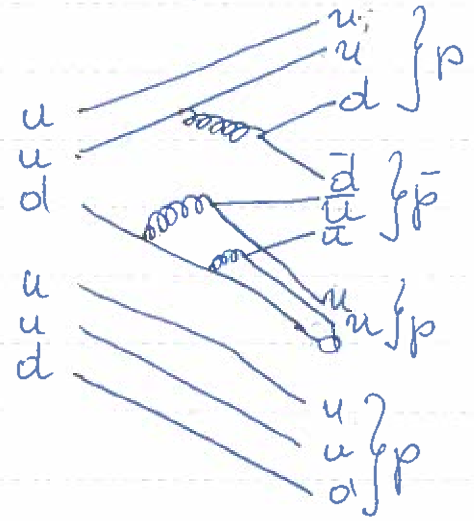
NE GRE, B SE NE OHRANJA

OHRANITEV B NA NIVOJU KVARKOV

$pp \to ppp\bar{p}$

KVARKI $B = +\frac{1}{3}$

ANTIKVARKI $= -\frac{1}{3}$



OHRANITEV LEPTONSKEGA ŠTEVILA

LEPTONI $L = +1$ ANTIDELCI $L = -1$ HADRONI $L = 0$

$e^+e^- \to \tau^+\tau^-$ DOVOLJEN PROCES

L: $\cancel{\mp}1 + 1 = \cancel{\mp}1 + 1$ ✓

$pp \to e^+e^+$ NE DOVOLJEN

L: $0 + 0 \neq -1 - 1$

B: $+1 + 1 \neq 0 + 0$

DN: $\pi^+ \to \mu^+ \nu_\mu$ DOVOLJEN ?

$\nu_\mu n \to p \mu^-$

L: $+1 + 0 \to 0 + 1$ ✓

B: $0 + 1 \to 1 + 0$ ✓

$L_\mu = +1 + 0 \to 0 + 1$ ✓

$\nu_\mu n \to p e^-$ NE GRE EKSPERIMENT!

L: $1 + 0 \to 0 + 1$

L_μ: $1 + 0 \to 0 + 0$

L_e: $0 + 0 \to 0 + 1$

L_e, L_μ, L_τ : VSA TRI SE LOČENO OHRANJAJO

$\mu^+ \to e^+ \gamma$ NI MOŽEN, L_μ, L_e SE NE OHRANJATA

OHRANITEV LEPTONSKEGA OKUSA.

DN: $\pi^0 \to e^+e^-$, $p \to n e^+ \nu_e$, $K^+ n \to \Sigma^+ \pi^0$, $K^- p \to \Sigma^- \pi^0$

SIMETRIJA VALOVNE FUNKCIJE

VAL. F. DVEH DELCEV (IDENTIČNIH)

$$|\psi(1,2)|^2 = |\psi(2,1)|^2 \Rightarrow \psi(1,2) = \pm\psi(2,1)$$

VALOV. FUNKCIJA POSAMEZNEGA DELCA $\phi(1)$, $\phi(2)$

MOŽNI STANJI: a, b

$$\psi_S = \frac{1}{\sqrt{2}}\left[\phi_a(1)\,\phi_b(2) + \phi_b(1)\,\phi_a(2)\right] \quad \text{SIMET.}$$

$$\psi_A = \frac{1}{\sqrt{2}}\left[\phi_a(1)\,\phi_b(2) - \phi_b(1)\,\phi_a(2)\right] \quad \text{ANTISIMET.}$$

ČE $a=b \Rightarrow \psi_A(1,2) = 0$, $\psi_S(1,2) \neq 0$

HADRONI V KVARKOVSKEM MODELU

BARIONI: SESTAVLJENI IZ TREH KVARKOV

q: $+\frac{2}{3}e_0$, $-\frac{1}{3}e_0$

IZ TREH KVARKOV u, d, s : $3^3 = 27$ RAZLIČNIH KOMBINACIJ

ZAČNIMO S SIMETRIČNIMA KOMBINACIJAMA u, d

$$\psi_{S1} = |u\,u\,u\rangle \quad \psi_{S2} = |d\,d\,d\rangle$$

IZOSPIN: HEISENBERG: p IN n

ISTI DELEC Z RAZLIČNO VREDNOSTJA TRETJE KOMPONENTE IZOSPINA I_3, $I_3 = +\frac{1}{2}$ ZA p

$I_3 = -\frac{1}{2}$ ZA n $\quad I_3|p\rangle = \frac{1}{2}|p\rangle \quad I_3|n\rangle = -\frac{1}{2}|n\rangle$

MOČNA INTERAKCIJA pp, pn, nn JE ENAKA

PO ANALOGIJI Z NAVADNIM SPINOM

$$\hat{I}_+|p\rangle = 0, \quad \hat{I}_-|p\rangle = |n\rangle$$
$$\hat{I}_+|n\rangle = |p\rangle, \quad \hat{I}_-|n\rangle = 0$$

KAKO JE TO PRI KVARKIH $u\ (+\frac{2}{3}e_0)$, $d\ (-\frac{1}{3}e_0)$

p: uud $\qquad n$: udd

$$I_+|u\rangle = 0, \quad I_-|u\rangle = |d\rangle$$
$$I_+|d\rangle = |u\rangle, \quad I_-|d\rangle = 0$$

$$\hat{I}_-|uuu\rangle = \sum_{i=1}^{3}\hat{I}_{i-}|uuu\rangle = |duu\rangle + |udu\rangle + |uud\rangle$$
$$\rightarrow \frac{1}{\sqrt{3}}\left(|duu\rangle + |udu\rangle + |uud\rangle\right) = \psi_{S3}$$

$$\hat{I}_+|ddd\rangle \rightarrow \frac{1}{\sqrt{3}}\left(|udd\rangle + |dud\rangle + |ddu\rangle\right) = \psi_{S4}$$

ČUDNOST

$\pi^- p \rightarrow K^0 \Lambda^0$ $\qquad$ ČUDNI DELCI NASTAJAJO V PARIH (MOČNA INTERAKCIJA)

$\Lambda^0 \rightarrow \pi^- p$ $\qquad$ V KONČNEM STANJU NOBENEGA ČUD. DELCA

$\tau \sim 10^{-10}\,s$

$\Delta^0 \rightarrow \pi^0 n \qquad \tau \sim 10^{-23}\,s$

ČUDNOST - STRANGENESS S : p, n $S=0$, Λ^0: $S=-1$

KVARKI u, d: $S=0$, s: $S=-1$.

OHRANITEV ČUDNOSTI : S SE OHRANJA PRI MOČNI IN ELEKTROMAGNETNI INTER., NE OHRANJA PA SE PRI ŠIBKI.

NADALJUJEMO S TVORBO VAL. FUNKCIJ (SIMETRIČNIH)

$u \to s$:

$$|uuu\rangle \to \frac{1}{\sqrt{3}}\left(|suu\rangle + |usu\rangle + |uus\rangle\right) = \psi_{S5}$$

$$\frac{1}{\sqrt{3}}\left(|udd\rangle + |dud\rangle + |ddu\rangle\right) \to \frac{1}{\sqrt{3}}\left(|sdd\rangle + |dsd\rangle + |dds\rangle\right) = \psi_{S6}$$

$$\frac{1}{\sqrt{3}}\left(|duu\rangle + |udu\rangle + |uud\rangle\right) \to$$
$$\frac{1}{\sqrt{6}}\left(|dsu\rangle + |dus\rangle + |sdu\rangle + |uds\rangle + |sud\rangle + |usd\rangle\right) = \psi_{S7}$$

$$\frac{1}{\sqrt{3}}\left(|suu\rangle + |usu\rangle + |uus\rangle\right) \to \frac{1}{\sqrt{3}}\left(|ssu\rangle + |sus\rangle + |uss\rangle\right) = \psi_{S8}$$

$$\psi_{S7} \to \frac{1}{\sqrt{3}}\left(|dss\rangle \cancel{+ |dss\rangle} + |sds\rangle + |ssd\rangle\right) = \psi_{S9}$$

$$\psi_{S8} \to |sss\rangle = \psi_{S10}$$

$\Rightarrow$ 10 SIMETRIČNIH VALOVNIH FUNKCIJ

ANTISIMETRIČNA VALOVNA F. 3 KVARKOV

ZAČNEM Z $|ud\rangle - |du\rangle$ ANTISIM. V.F. DVEH KVARKOV

$$\psi_{A1} = \frac{1}{\sqrt{6}}\left[|uds\rangle - |dus\rangle + |usd\rangle - |dsu\rangle + |sud\rangle - |sdu\rangle\right]$$

OSTANE 16 VALOVNIH FUNKCIJ ! NITI SIMETRIČNE, NITI ANTISIMETRIČNE – Z MEŠANO SIMETRIJO

PRIMER $\psi_{MA1} = \frac{1}{\sqrt{2}}\left[|udu\rangle - |duu\rangle\right]$

MEŠANA SIMETRIČNA (SIMETRIČNA NA ZAMENJAVO PRVIH DVEH DELCEV)

ψ_{MS1}

$$\langle \psi_{MS1} | \psi_{MA1} \rangle = 0 \qquad \langle \psi_{MS1} | \psi_{S3} \rangle = 0 \qquad \langle \psi_{MS1} | \psi_{MS1} \rangle =$$

$$|\psi_{MS1}\rangle = a\,|uud\rangle + b\,|udu\rangle + c\,|duu\rangle$$

$$\frac{1}{\sqrt{2}}\left(a\langle uud| + b\langle udu| + c\langle duu|\right)\left(|udu\rangle - |duu\rangle\right) = 0$$

$$\frac{1}{\sqrt{2}}(b-c) = 0 \Rightarrow b = c$$

$$0 = \frac{1}{\sqrt{3}}\left(a\langle uud| + b\langle udu| + c\langle duu|\right)\left(|uud\rangle + |udu\rangle + |duu\rangle\right)$$

$$\frac{1}{\sqrt{3}}(a + b + c) = 0 \Rightarrow a = -b - c = -2b$$

$$a^2 + b^2 + c^2 = 1 \Rightarrow 4b^2 + b^2 + b^2 = 6b^2 = 1$$

$$|\psi_{MS1}\rangle = \frac{1}{\sqrt{6}}\left(-2\,|uud\rangle + |udu\rangle + |duu\rangle\right)$$

$\Rightarrow$ 8 MEŠANIH ANTISIMETRIČNIH F.
8 — " — SIMETRIČNIH F.
TO JE <u>OKUSNI</u> DEL VALOVNE FUNKCIJE

SPINSKI DEL VALOVNE FUNKCIJE
KVARKI: SPIN $\frac{1}{2}$, 3 PROJEKCIJA $\pm\frac{1}{2}$
TRIJE KVARKI $2^3 = 8$ MOŽNOSTI, OD $|\uparrow\uparrow\uparrow\rangle$
DO $|\downarrow\downarrow\downarrow\rangle$
$|\uparrow\uparrow\uparrow\rangle$ SIMETRIČNA KOMBINACIJA, $J = \frac{3}{2}$

$$|uuu\rangle \to |\uparrow\uparrow\uparrow\rangle \qquad |ddd\rangle \to |\downarrow\downarrow\downarrow\rangle$$

$$\psi_{S3} \to \frac{1}{\sqrt{3}}\left(|\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\uparrow\uparrow\downarrow\rangle\right)$$

$$\psi_{S4} \to \frac{1}{\sqrt{3}}\left(|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle\right)$$

MEŠANE VALOVNE FUNKCIJE

$$\psi_{MA1} = \frac{1}{\sqrt{2}}\left[|udu\rangle - |duu\rangle\right] \longrightarrow \frac{1}{\sqrt{2}}\underbrace{\left[|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\right]}$$

$$\frac{1}{\sqrt{2}}\left(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle\right) \to J=0 \Rightarrow J=\frac{1}{2}$$

CELOTNA VALOV. FUNKCIJA IN DELEC Δ^{++}

Δ^{++}: BARION, $|uuu\rangle$ OKUS VAL. F. = SIM.

SPIN: $J=\frac{3}{2}$ SPINSKA V. F. = SIM.

$$\Psi_{\Delta^{++}} = \psi_S(\text{okus})\,\psi_S(\text{spin})\,\psi_S(\vec{r})\,\psi_A(\text{BARVA})$$

$\rightarrow$ POTREBUJEM PROSTOSTNO STOPNJO <u>BARVA</u>

R RDEČA, B MODRA, G ZELENA

$$\cancel{\psi_A(\text{BARVA}) = \frac{1}{\sqrt{6}}\left[|RGB\rangle - |GRB\rangle + |RBG\rangle - |GBR\rangle + |BGR\rangle - |BGR\rangle\right]}$$

$$= \frac{1}{\sqrt{6}}\left[|RGB\rangle + |GBR\rangle + |BRG\rangle - |GRB\rangle + |BGR\rangle + |RBG\rangle\right]$$

VSI BARIONI: ENAKA BARVNA VAL. FUNKCIJA

BARIONI SPIN $\frac{1}{2}$: VALOVNA F. V OKUSNEM IN SPINSKEM PROSTORU

$$\left(\psi_{MS1}(\text{okus})\,\psi_{S1}(\text{spin}) + \psi_{MA1}(\text{okus})\,\psi_{MA1}(\text{spin})\right)$$

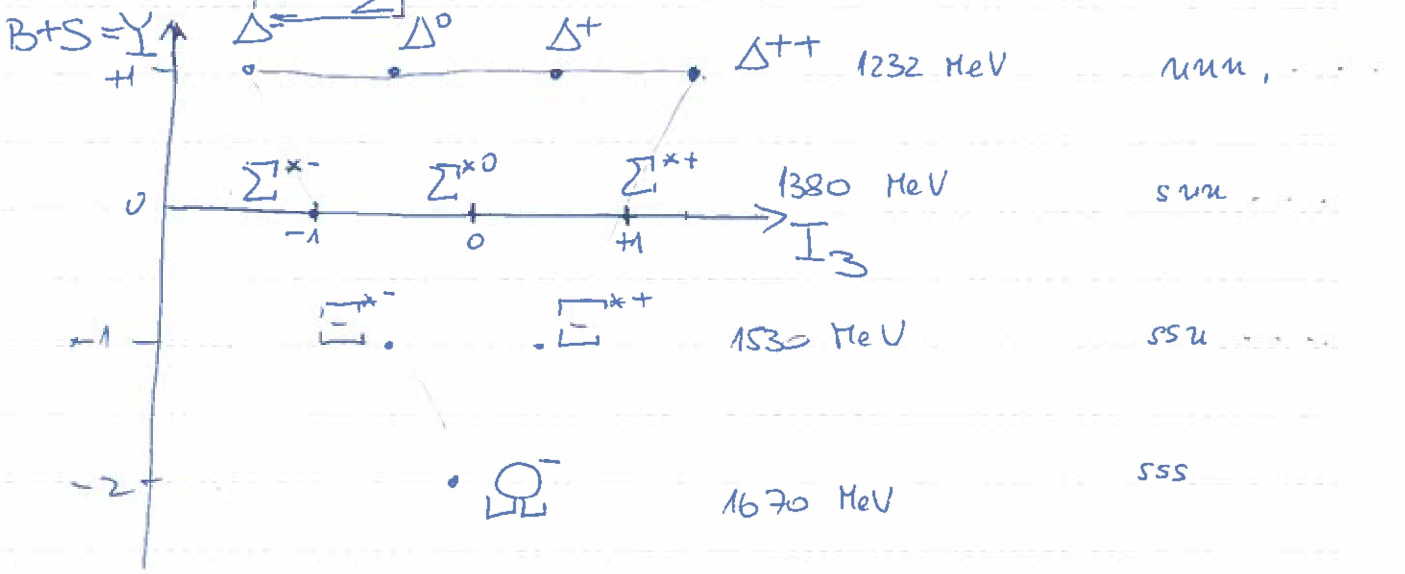
8 VALOVNIH FUNKCIJ, SPIN $\frac{1}{2}$, SIMETRIČNE V OKUSNO-SPINSKEM PROSTORU.

KLASIFIKACIJA BARIONOV I_3, $B+S=Y$

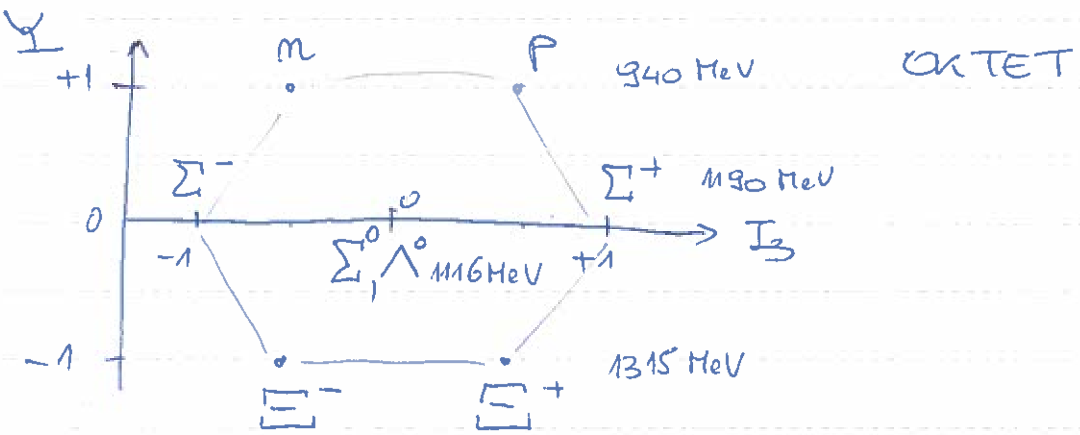
HIPERNABOJ

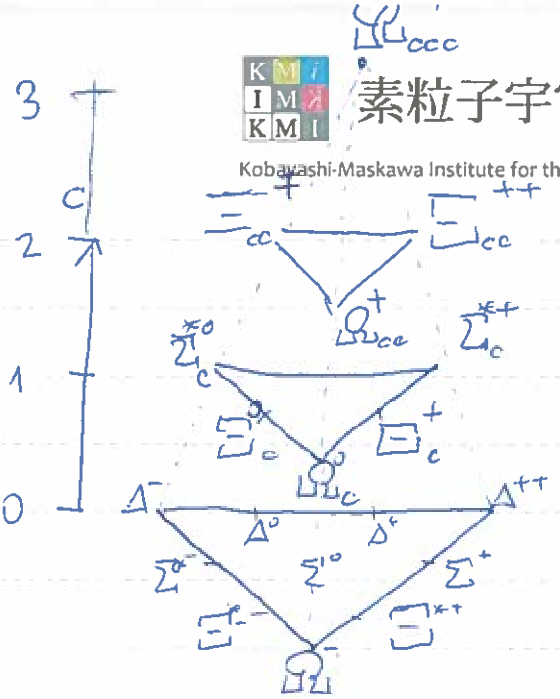
SIMETRIČNE VAL. F. V OKUSNEM IN SPINSKEM PROSTORU

$\boxed{J=\frac{3}{2}}$



DEKUPLET BARIONOV





MAGNETNI MOMENT p IN n

$J = \frac{1}{2}$

$$\Psi = \left[\psi_{MS1}(\text{okus})\,\psi_{MS1}(\text{spin}) + \psi_{MA1}(\text{okus})\,\psi_{MA1}(\text{spin})\right] \psi(\vec{r}) \cdot \psi(\text{barve})$$

OPERATOR i-ti kvark $\quad \vec{\mu}_i = g_s \dfrac{e_0 Q_i \vec{s}_i}{2 m_i}$

$$\mu_{3i} = g_s \frac{e_0 Q_i s_{3i}}{2 m_i} \quad \Rightarrow \quad \mu_p = \langle p | \sum_{i=1}^{3} \mu_{3i} | p \rangle$$

$\uparrow$ ψ_p

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}} \big[2|u\uparrow u\uparrow d\downarrow\rangle - |u\uparrow u\downarrow d\uparrow\rangle - |u\downarrow u\uparrow d\uparrow\rangle +$$
$$2|d\downarrow u\uparrow u\uparrow\rangle - |d\uparrow u\downarrow u\uparrow\rangle - |d\downarrow u\uparrow u\uparrow\rangle +$$
$$2|u\uparrow d\downarrow u\uparrow\rangle - |u\downarrow d\uparrow u\uparrow\rangle - |u\uparrow d\uparrow u\downarrow\rangle \big]$$

$$\mu_p = \frac{e_0}{2m_q} \qquad \mu_n = -\frac{2}{3}\frac{e_0}{2m_q}$$

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3} \qquad \text{EKSPERIMENT: } \frac{\mu_n}{\mu_p} = -0{,}685$$

$\mu_p = ?$ $\quad m_q$ NAIVNO: $\frac{m_p}{3}$ $\quad \mu_p = \dfrac{3 e_0}{2 m_p}$

V NARAVI $\quad \mu_p = g_{s,p} \dfrac{e_0 s}{2 m_p} = \dfrac{5{,}6 \cdot e_0 \frac{1}{2}}{2 m_p} = \dfrac{2{,}8 e_0}{2 m_p}$

MEZONI $q_i \bar{q}_j$

$\hat{C}: \quad |q\rangle \to |\bar{q}\rangle$ KONJUGACIJA NABOJA

$\hat{C}|q\rangle = e^{i\varphi}|\bar{q}\rangle \qquad \hat{C}^2|q\rangle = \hat{C}(e^{i\varphi}|\bar{q}\rangle) = e^{i\varphi}e^{-i\varphi}|q\rangle = |q\rangle$

$\hat{C}|u\rangle = -|\bar{u}\rangle$
$\hat{C}|d\rangle = |\bar{d}\rangle$

$\hat{I}_-|u\rangle = I_- |I=\frac{1}{2}, I_3=+\frac{1}{2}\rangle = |I=\frac{1}{2}, I_3=-\frac{1}{2}\rangle = |d\rangle$
$\hat{I}_+|d\rangle = |u\rangle$

$I_3(u) = +\frac{1}{2} \quad I_3(\bar{u}) = -\frac{1}{2} \quad I_3(\bar{d}) = +\frac{1}{2}$

$\hat{I}_-|\bar{d}\rangle = -|\bar{u}\rangle$
$\hat{I}_+|\bar{u}\rangle = -|\bar{d}\rangle$

KONSTRUKCIJA VEZANIH STANJA

$|u\bar{d}\rangle \qquad I_3 = +1$

$\hat{I}_-|u\bar{d}\rangle = |d\bar{d}\rangle - |u\bar{u}\rangle \qquad I_3 = 0$

$\hat{I}_-(|d\bar{d}\rangle - |u\bar{u}\rangle) = -|d\bar{u}\rangle - |d\bar{u}\rangle \qquad I_3 = -1$

$|\pi^+\rangle = |u\bar{d}\rangle = |I=1, I_3=+1\rangle$
$|\pi^0\rangle = \frac{1}{\sqrt{2}}(|d\bar{d}\rangle - |u\bar{u}\rangle) = |I=1, I_3=0\rangle$
$|\pi^-\rangle = |d\bar{u}\rangle = |I=1, I_3=-1\rangle$

$u, d \to s, \quad \bar{u}, \bar{d} \to \bar{s} \qquad |\pi^+\rangle \xrightarrow[\bar{d}\to\bar{s}]{} |u\bar{s}\rangle = |K^+\rangle$

$|\pi^+\rangle \xrightarrow[u\to s]{} |s\bar{d}\rangle = |K^0\rangle$

$|\pi^-\rangle \xrightarrow[d\to s]{} |s\bar{u}\rangle = |K^-\rangle \qquad |\pi^-\rangle \xrightarrow[\bar{u}\to\bar{s}]{} |d\bar{s}\rangle = |\bar{K}^0\rangle$

NABOJ 0, $I_3=0$, $S=0$ $\quad |d\bar d\rangle, |u\bar u\rangle, |s\bar s\rangle$

$$\frac{1}{\sqrt3}\left(|u\bar u\rangle + |d\bar d\rangle + |s\bar s\rangle\right) = |\eta_0\rangle \qquad u \Leftrightarrow s,\ u \Leftrightarrow d,\ d \Leftrightarrow s$$

$SU(3)_{OKUS}$

SINGLET

$$|\eta_8\rangle = a|u\bar u\rangle + b|d\bar d\rangle + c|s\bar s\rangle$$

$\langle \eta_8|\eta_0\rangle = 0$

$\langle \eta_8|\pi^0\rangle = 0$

$\langle \eta_8|\eta_8\rangle = 1$

$$0 = \frac{1}{\sqrt2}\left(\langle u\bar u| a + \langle d\bar d| b + \langle s\bar s| c\right)\left(|d\bar d\rangle - |u\bar u\rangle\right) = \frac{1}{\sqrt2}(b-a) \Rightarrow b=a$$

$$0 = \frac{1}{\sqrt3}\left(a\langle u\bar u| + b\langle d\bar d| + c\langle s\bar s|\right)\left(|u\bar u\rangle + |d\bar d\rangle + |s\bar s\rangle\right) = \frac{1}{\sqrt3}(a+b+c) \Rightarrow 2a+c=0$$

$$|\eta_8\rangle = \frac{1}{\sqrt6}\left(|u\bar u\rangle + |d\bar d\rangle - 2|s\bar s\rangle\right)$$

SPINSKI DEL VALOVNE FUNKCIJE ZA MEZONE

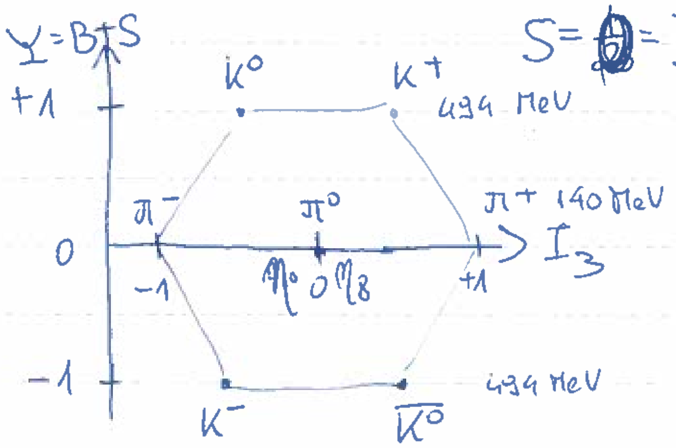
$$|\uparrow\uparrow\rangle = |S=1, S_3=+1\rangle$$

$$\hat S_-|\uparrow\uparrow\rangle \Rightarrow \frac{1}{\sqrt2}\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right) = |S=1, S_3=0\rangle \qquad 2s+1$$

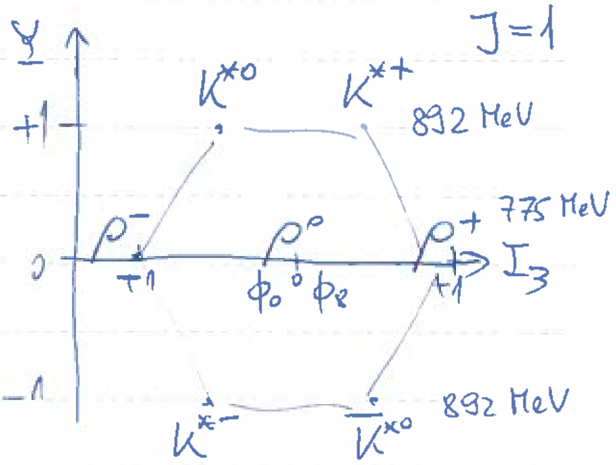
$$\hat S_- \frac{1}{\sqrt2}\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right) \Rightarrow |\downarrow\downarrow\rangle = |S=1, S_3=-1\rangle$$

$$\frac{1}{\sqrt2}\left(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle\right) = |S=0, S_3=0\rangle$$

$S=0=J$



$J=1$



$J=0$ $\quad |\eta_0\rangle, |\eta_8\rangle$

V NARAVI

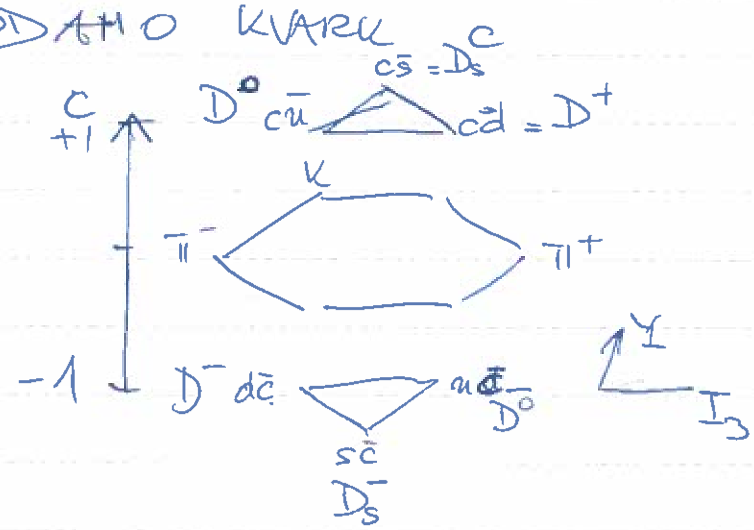
$$|\eta\rangle = \sin\Theta\, |\eta_0\rangle + \cos\Theta\, |\eta_8\rangle$$
$$|\eta'\rangle = \cos\Theta\, |\eta_0\rangle - \sin\Theta\, |\eta_8\rangle$$

$J=1$ $\quad |\phi_0\rangle, |\phi_8\rangle$

$$|\phi\rangle = \sin\Theta'\, |\phi_0\rangle + \cos\Theta'\, |\phi_8\rangle$$
$$|\omega\rangle = \cos\Theta'\, |\phi_0\rangle - \sin\Theta'\, |\phi_8\rangle$$

$$|\phi\rangle \sim |s\bar{s}\rangle, \quad \Theta' = -0.615$$

DODATNO O KVARKU C



$J=0$

VERJETNOSTNA GOSTOTA, TOK DELCEV, ANTI DELCI

$$E = \frac{p^2}{2m} \qquad E \to \hat{E} = -i\hbar\frac{\partial}{\partial t} \quad \vec{p} \to \hat{\vec{p}} = -i\hbar\vec{\nabla}$$

$$\hat{E}\psi = \frac{\hat{p}^2}{2m}\psi \quad \to \quad i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi = 0$$

$|\psi|^2 = \psi^*\psi$ = VERJETNOSTNA GOSTOTA, $|\psi|^2 dV$

TOK DELCEV $\vec{j}$

KONTINUITETNA ENAČBA $\quad \frac{\partial\rho}{\partial t} + \vec{\nabla}\vec{j} = 0$

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi=0 \qquad \Big|\cdot -i\psi^*$$

$$-i\hbar\frac{\partial\psi^*}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi^*=0 \qquad \Big|\cdot i\psi$$

$$\frac{\partial\psi}{\partial t}\psi^*-\frac{i\hbar}{2m}(\nabla^2\psi)\psi^*=0$$

$$\frac{\partial\psi^*}{\partial t}\psi+\frac{i\hbar}{2m}(\nabla^2\psi^*)\psi=0$$

$$\underbrace{\frac{\partial\psi}{\partial t}\psi^*+\frac{\partial\psi^*}{\partial t}\psi}+\frac{i\hbar}{2m}\left[(\nabla^2\psi^*)\psi-(\nabla^2\psi)\psi^*\right]=0$$

$$\frac{\partial}{\partial t}(\psi^*\psi)+\frac{i\hbar}{2m}\left[\psi(\nabla^2\psi^*)-\psi^*(\nabla^2\psi)\right]=0$$

$$0=\frac{\partial}{\partial t}(\psi^*\psi)+\frac{i\hbar}{2m}\vec\nabla\left[\psi\vec\nabla\psi^*-\cancel{(\nabla\psi)(\nabla\psi^*)}-\psi^*\vec\nabla\psi+\cancel{(\nabla\psi^*)(\nabla\psi)}\right]$$

$$0=\frac{\partial}{\partial t}(\psi^*\psi)+\frac{i\hbar}{2m}\vec\nabla\left[\psi\vec\nabla\psi^*-\psi^*\vec\nabla\psi\right]$$

$$\frac{\partial\rho}{\partial t}+\vec\nabla\cdot\vec j=0 \Rightarrow \boxed{\vec j=\frac{i\hbar}{2m}\left[\psi\vec\nabla\psi^*-\psi^*\nabla\psi\right]}$$

NARAVNE ENOTE $\hbar=1,\ c=1$

$\hbar c=197\ \mathrm{MeV\,fm}$, $c=3\cdot10^8\ \frac{m}{s}$

$$\psi=\frac{1}{\sqrt V}e^{i\vec p\cdot\vec x-iEt} \qquad \Rightarrow p=k,\ E=\omega$$

$$\rho=\psi^*\psi=\frac{1}{V} \qquad \vec j=\frac{i}{2m}\frac{1}{V}\left[-i\vec p-i\vec p\right]=\frac{2\vec p}{2m}\frac{1}{V}=$$

$$=\frac{1}{V}\cdot\vec v \qquad (\vec j=\rho\vec v)$$

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Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

KLEIN GORDONOVA E.
$$-\frac{\partial^2\phi}{\partial t^2} + \nabla^2\phi = m^2\phi \;\Big|\cdot(-i\phi^*)$$
$$-\frac{\partial^2\phi^*}{\partial t^2} + \nabla^2\phi^* = m^2\phi^* \;\Big|\cdot(-i\phi)$$

$$i\left[\frac{\partial^2\phi}{\partial t^2}\phi^* - \frac{\partial^2\phi^*}{\partial t^2}\phi\right] + i\left[(\nabla^2\phi)\phi^* - (\nabla^2\phi^*)\phi\right] = 0$$

$$\frac{\partial}{\partial t}\left[i\left(\phi^*\frac{\partial\phi}{\partial t} - \phi\frac{\partial\phi^*}{\partial t}\right)\right] + i\vec\nabla\left[\phi\vec\nabla\phi^* - \phi^*\vec\nabla\phi\right] = 0$$

$$\frac{\partial\rho}{\partial t} + \vec\nabla\cdot\vec j = 0$$

$$\vec j = i\left(\phi\vec\nabla\phi^* - \phi^*\vec\nabla\phi\right) \qquad \rho = i\left(\phi^*\frac{\partial\phi}{\partial t} - \phi\frac{\partial\phi^*}{\partial t}\right)$$

RAVNI VAL $\phi = \frac{1}{\sqrt{V}} e^{i\vec p\cdot\vec r - iEt}$ $\quad j = \frac{2\vec p}{V} \quad \rho = \frac{2E}{V}$

NORMALIZACIJA : 1 DELEC NA V SCHRÖD.
2E DELCEV NA V KLEIN-GORDON

$$d^3x \xrightarrow[\text{LORENTZOVA T.}]{} dx^3\sqrt{1-(v/c)^2}$$
$$\rho \to \frac{\rho}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\rho\, d^3x \to \rho\, d^3x$$

ČETVEREC ODVODOV
$$\partial^\mu = \left(\frac{\partial}{\partial t}, -\vec\nabla\right) \qquad \partial_\mu = \left(\frac{\partial}{\partial t}, \vec\nabla\right)$$

$$\partial^\mu\partial_\mu = \sum_{\mu=0}^{3}\partial^\mu\partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

KLEIN-GORDONOVA E. $\quad (\partial^\mu\partial_\mu + m^2)\phi = 0$

KONTINUITETNA E. $\quad \partial^\mu j_\mu = 0 \qquad j^\mu = (\rho, \vec j),\ j_\mu = (\rho, -\vec j)$

ANTIDELCI

KLEIN GORDONOVA E.: ZA PROST DELEC

$$\phi = \frac{1}{\sqrt{V}} e^{i p^\mu x_\mu}$$

$x_\mu = (t, -\vec{r})$
$p_\mu = (E, -\vec{p})$

$$E^2 = p^2 + m^2$$

$$\Rightarrow E = \pm\sqrt{p^2 + m^2}$$

KAJ SO REŠITVE Z $-$ = NEGATIVNE ENERGIJE

$$j^\mu = \frac{2}{V}(E, \vec{p})$$

OBVESTILO: 30.4. VAJE → PREDAVANJA

$$j^\mu = -e_0 \frac{2}{V}(E, \vec{p})$$ ELEKTROM. TOK Z e^-

$$j^\mu = e_0 \frac{2}{V}(E, \vec{p}) = -e_0 \frac{2}{V}(-E, -\vec{p})$$ POZITRON e^+

FEYNMAN-STÜCKELBERGOVA INTERPRETACIJA

DIRACOVA ENAČBA

ENAČBA S PRVIMI ODVODI, IN $H^2\psi = (p^2 + m^2)\psi$

$$\hat{H}\psi = i\hbar \frac{\partial \psi}{\partial t}$$

POSKUSIMO

$$\hat{H}\psi = [\vec{\alpha}\cdot\vec{p} + \beta m]\psi$$

$$[\vec{\alpha}\vec{p} + \beta m]^2 = (\vec{\alpha}\cdot\vec{p})^2 + \vec{\alpha}\cdot\vec{p}\,\beta m + \beta m\,\vec{\alpha}\cdot\vec{p} + \beta^2 m^2 =$$

$$(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3)(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p) + \alpha_1 p_1 \beta m +$$
$$+ \alpha_2 p_2 \beta m + \alpha_3 p_3 \beta m + \beta m \alpha_1 p_1 + \beta m \alpha_2 p_2 + \beta m \alpha_3 p_3 +$$
$$+ \beta^2 m^2 = \alpha_1^2 p_1^2 + \alpha_2^2 p_2^2 + \alpha_3^2 p_3^2 + (\alpha_1\alpha_2 + \alpha_2\alpha_1) p_1 p_2 +$$
$$+ (\alpha_2\alpha_3 + \alpha_3\alpha_2) p_2 p_3 + (\alpha_1\alpha_3 + \alpha_3\alpha_1) p_1 p_3 +$$
$$+ (\alpha_1\beta + \beta\alpha_1) p_1 m + (\alpha_2\beta + \beta\alpha_2) p_2 m + (\alpha_3\beta + \beta\alpha_3) p_3 m + \beta^2 m^2$$

$$\Rightarrow \quad \alpha_i^2 = 1, \ \beta^2 = 1, \ \alpha_i \alpha_j + \alpha_j \alpha_i = 0 \quad i \neq j \qquad \alpha_i \beta + \beta \alpha_i = 0, \ i = 1,2,3$$

$$\vec{\alpha} = \begin{bmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix} \qquad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \ \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \ \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\beta = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\vec{\alpha}, \beta$ 4x4 MATRIKE $\rightarrow$ ψ IMA 4 KOMPONENTE

$$\hat{H}\psi = [\vec{\alpha}\cdot\vec{p} + \beta m]\psi \Rightarrow i\frac{\partial}{\partial t}\psi = [-i\vec{\alpha}\cdot\vec{\nabla} + \beta m]\psi$$

$$i\beta\frac{\partial}{\partial t}\psi = [-i\beta\vec{\alpha}\cdot\vec{\nabla} + \underset{1}{\beta^2} m]\psi$$

DEFINIRAMO

$$\gamma^\mu = (\beta, \beta\vec{\alpha}) \qquad \Rightarrow \boxed{[i\gamma^\mu\partial_\mu - m]\psi = 0}$$

KOVARIANTNA OBLIKA DIRACOVE E.

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \qquad g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

REŠITVE
IŠČEMO V OBLIKI $\quad \psi = u(\vec{p})\, e^{-ip^\mu x_\mu}$

$u(\vec{p})$ BISPINOR

$$\Rightarrow \quad [\gamma^\mu p_\mu - m]\, u(\vec{p}) = 0$$

$$\hat{H} u(\vec{p}) = (\vec{\alpha}\cdot\vec{p} + \beta m]\, u(\vec{p}) = E\, u(\vec{p})$$

$$\begin{bmatrix} 0 & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & 0 \end{bmatrix} u(\vec{p}) + \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix} u(\vec{p}) = E\, u(\vec{p})$$

$$u(\vec{p}) = \begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

$$\begin{bmatrix} 0 & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & 0 \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} + \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} = E\begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

$$\vec{\sigma}\cdot\vec{p}\, u_B + m\, u_A = E\, u_A \qquad \vec{\sigma}\cdot\vec{p}\, u_B = (E-m)u_A$$

$$\vec{\sigma}\cdot\vec{p}\, u_A + (-m)u_B = E\, u_B \qquad \vec{\sigma}\cdot\vec{p}\, u_A = (E+m)u_B$$

$$u_A^{(1)} = \chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{SPINOR} \qquad u_A^{(2)} = \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\boxed{\text{ZA } E>0}$

$$u_B^{(s)} = \frac{1}{E+m}\vec{\sigma}\cdot\vec{p}\, u_A^{(s)} \qquad s = 1, 2$$

$$u^{(s)} = N\begin{bmatrix} \chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \end{bmatrix} \qquad s=1,2 \qquad \chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\boxed{E<0}$

$$u_B^{(s)} = \chi^{(s)} \qquad u_A^{(s)} = \frac{\vec{\sigma}\cdot\vec{p}}{E-m}\underbrace{u_B^{(s)}}_{=\chi^{(s)}}$$

$$u^{(s+2)} = N\begin{bmatrix} -\frac{\vec{\sigma}\cdot\vec{p}}{|E|+m}\chi^{(s)} \\ \chi^{(s)} \end{bmatrix}$$

ODKOD DVOJNA DEGENERACIJA? 2 REŠ. Z $E>0$
2 REŠ. Z $E<0$

KOMUTATOR H IN $\hat{L}$

$$\hat{H} = \vec{\alpha}\cdot\hat{\vec{p}} + \beta m, \quad \hat{\vec{L}} = \hat{\vec{r}} \times \vec{p} \qquad p_i = -i\frac{\partial}{\partial x_i}$$

$$[\hat{H}, \hat{L}_1] = [\vec{\alpha}\cdot\vec{p} + \beta m, x_2 p_3 - x_3 p_2] = [\vec{\alpha}\cdot\vec{p}, x_2p_3 - x_3p_2] +$$
$$+ \underbrace{[\beta m, x_2 p_3 - x_3 p_2]}_{=0}$$

$$[x_i, p_j] = i\delta_{ij}$$

$$[\vec{\alpha}\cdot\vec{p},\, x_2p_3 - x_3p_2] = [\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3,\, x_2p_3 - x_3p_2] = [\alpha_2 p_2, x_2 p_3] - [\alpha_3 p_3, x_3 p_2]$$
$$= -i\alpha_2 p_3 + i\alpha_3 p_2$$

$$[\hat{H}, \hat{L}_1] = -i(\vec{\alpha}\times\vec{p})_1 \quad \Rightarrow \quad [\vec{H}, \vec{L}] = -i(\vec{\alpha}\times\vec{p})$$

$\Rightarrow$ $\vec{L}$ NE KOMUTIRA S $\hat{H}$, $\vec{L}$ NI VEČ DOBRO KV. ŠT.

UPELJEMO

$$\vec{\Sigma} = \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix} \qquad [\vec{\alpha}\cdot\vec{p}, \Sigma_1] = \begin{bmatrix} 0 & [\sigma, \sigma_1 \\ & \end{bmatrix}$$

$$[\alpha_i, \Sigma_1] = \begin{bmatrix} 0 & [\sigma_i, \sigma_1] \\ [\sigma_i, \sigma_1] & 0 \end{bmatrix}$$

$$\sigma_i^2 = 1 \qquad [\sigma_1, \sigma_2] = 2i\sigma_3 \qquad [\sigma_2, \sigma_3] = 2i\sigma_1 \qquad [\sigma_3, \sigma_1] = 2i\sigma_2$$

$$[\vec{\alpha}\cdot\vec{p}, \Sigma_1] = p_2\begin{bmatrix} 0 & -2i\sigma_3 \\ -2i\sigma_3 & \end{bmatrix} + p_3\begin{bmatrix} 0 & 2i\sigma_2 \\ 2i\sigma_2 & 0 \end{bmatrix} =$$
$$= 2i\left[-p_2\alpha_3 + p_3\alpha_2\right] = 2i(\vec{\alpha}\times\vec{p})_1$$

$$[\beta_m, \Sigma_i] = 0$$

$$\Rightarrow [\hat{H}, \vec{\Sigma}] = 2i(\vec{\alpha}\times\vec{p})$$

$$\hat{\vec{J}} = \underset{\substack{\uparrow \\ \text{TIRNA} \\ \text{VRTILNA KOLIČINA}}}{\hat{\vec{L}}} + \underbrace{\frac{1}{2}\vec{\Sigma}}_{\text{SPIN DELCA}} \quad \Rightarrow \quad [\hat{H}, \hat{\vec{J}}] = 0$$

VIJAČNOST $\qquad \vec{\Sigma}\cdot\frac{\vec{p}}{p}$

$$\left(\vec{\Sigma}\cdot\frac{\vec{p}}{p}\right)\psi^{(1)} = +\psi^{(1)}$$
$$\left(\vec{\Sigma}\cdot\frac{\vec{p}}{p}\right)\psi^{(2)} = -\psi^{(1)}$$

VERJETNOSTNA GOSTOTA IN TOK ZA REŠITVE D.E.

$$[i\gamma^\mu \partial_\mu - m]\psi = 0 \qquad i\gamma^0 \frac{\partial\psi}{\partial t} + i\gamma^k \frac{\partial\psi}{\partial x^k} - m\psi = 0, \quad k=1,2,3$$

$$-i\frac{\partial\psi^+}{\partial t}\gamma^0 + (-i)(-1)\frac{\partial\psi^+}{\partial x^k}\gamma^k - m\psi^+ = 0 \quad /\cdot\gamma^0$$

$$(\gamma^0)^+ = \gamma^0; \quad (\gamma^k)^+ = -\gamma^k$$

$$-i\frac{\partial\psi^+}{\partial t}\gamma^0\gamma^0 + i\frac{\partial\psi^+}{\partial x^k}\gamma^k\gamma^0 - m\psi^+\gamma^0 = 0$$

$$i\frac{\partial\psi^+}{\partial t}\gamma^0\gamma^0 + i\frac{\partial\psi^+}{\partial x^k}\gamma^0\gamma^k + m\psi^+\gamma^0 = 0$$

ADJUNGIRAN BISPINOR $\quad \psi^+\gamma^0 = \bar\psi$

$$i\frac{\partial\bar\psi}{\partial t}\gamma^0 + i\frac{\partial\bar\psi}{\partial x^k}\gamma^k + m\bar\psi = 0$$

$$i\partial_\mu\bar\psi\gamma^\mu + m\bar\psi = 0 \qquad \text{DIRAK. E. ZA } \bar\psi$$

$$i(\partial_\mu\bar\psi)\gamma^\mu\psi + m\bar\psi\psi = 0$$

$$i\bar\psi\gamma^\mu(\partial_\mu\psi) - m\bar\psi\psi = 0$$

SEŠTEJEM, DELIM Z i

$$(\partial_\mu\bar\psi)\gamma^\mu\psi + \bar\psi\gamma^\mu(\partial_\mu\psi) = 0$$

$$\partial_\mu(\bar\psi\gamma^\mu\psi) = 0 \qquad \text{KONTINUITETNA ENAČBA}$$

$$\Rightarrow \quad j^\mu = \bar\psi\gamma^\mu\psi$$

ELEKTROMAGNETNI TOK $\quad j^\mu = -e_0\bar\psi\gamma^\mu\psi$

INTERAKCIJA Z E.M. POLJEM

$$\partial_\mu \longrightarrow \partial_\mu - ieA_\mu = D_\mu$$ KOVARIANTNI ODVOD

$$A_\mu = (A_0, \vec{A}) \qquad E = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} A_0 , \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

DROBNI TISK: INVARIANTNOST $\psi(x) \to e^{i\alpha(x)}\psi(x)$
UMERITVENA TRANSFORMACIJA

$$[i\gamma^\mu(\partial_\mu - ieA_\mu) - m]\psi = 0$$

$$[i\gamma^\mu \partial_\mu - m]\psi = -e\gamma^\mu A_\mu \psi \equiv \gamma^0 V \psi$$

$$\gamma^0 \cancel{A} V = -e\gamma^\mu A_\mu$$

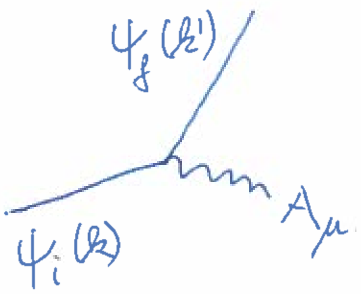
$$\underline{V = -e\gamma^0\gamma^\mu A_\mu}$$

V POVZROČI PREHOD IZ $\psi_i \to \psi_f$

$$T_{fi} = -i\int \psi_f^\dagger(k',x)\, V\, \psi_i(k,x)\, d^4x =$$

$$= ie\int \psi_f^\dagger \gamma^0 \gamma^\mu A_\mu \psi_i\, d^4x = ie\int \bar{\psi}_f \gamma^\mu A_\mu \psi_i\, d^4x =$$

$$= -i\int \underbrace{(-e\bar{\psi}_f \gamma^\mu \psi_i)}_{j^\mu_{fi}} A_\mu\, d^4x = \underline{\underline{-i\int j^\mu_{fi} A_\mu\, d^4x}}$$



A_μ : USTVARJA GA DRUG DELEC (ČE GLEDAM SIPANJE ENEGA DELCA NA DRUGEM)

ZA A_μ VELJA MAX. ENAČBA

$$\cancel{\partial^\nu \partial_\mu A} \qquad \partial^\nu \partial_\nu A_\mu = j^\mu$$

POTENCIAL, KI GA ČUTI DELEC 1

TOK, KI GA USTVARI DELEC 2

DELEC $\psi(p) \to \psi(p')$

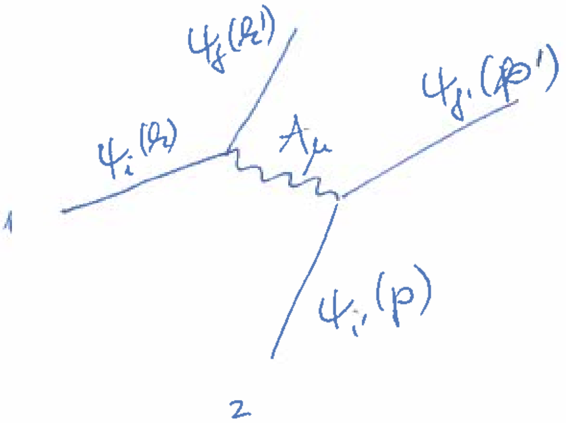
TOK $j_\mu = -e\,\bar{\psi}(p')\,\gamma_\mu\,\psi(p) = -e\,\bar{u}(p')\,e^{ip'x}\,\gamma_\mu u(p)\,e^{-i}$

$$= -e\,\bar{u}(p')\,\gamma_\mu\,u(p)\,e^{iqx} \qquad q = p' - p$$

$$\Rightarrow A_\mu = -q^{-2}\,j^\mu$$

$$\boxed{A_\mu = -\frac{j^\mu}{q^2}}$$

$$T_{fi} = -i \int j^\mu_{fi} \left(-\frac{1}{q^2}\right) j_\mu^{f'i'}\, d^4x$$



E.M. INTERAKCIJA V NERELATIVISTIČNI LIMITI

$$[\gamma^\mu p_\mu + e\gamma^\mu A_\mu - m]\psi = 0 \qquad \gamma^\mu = (\beta, \beta\vec{\alpha}) \quad A^\mu = (A_0, \vec{A})$$

$$[\beta E - \beta\vec{\alpha}\vec{p} + e\beta A_0^\circ - e\beta\vec{\alpha}\vec{A} - m]u = 0 \qquad /\cdot\beta \text{ Z LEVE}$$

$$[E - \vec{\alpha}\vec{p} + eA_0 - e\vec{\alpha}\cdot\vec{A} - \beta m]u = 0$$

$$[\vec{\alpha}\cdot(\vec{p} + e\vec{A}) - eA_0 + \beta m]u = Eu$$

$$\begin{bmatrix} m - eA_0 & \vec{\sigma}\cdot(\vec{p}+e\vec{A}) \\ \vec{\sigma}(\vec{p}+e\vec{A}) & -m - eA^0 \end{bmatrix} \begin{bmatrix} u_A \\ u_B \end{bmatrix} = E \begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

↓ SKICIRAM IZPELJAVO, DETAJLI V GOLOB, ZAPISKI

$$(E + m + eA^0)^{-1}\vec{\sigma}(\vec{p}+e\vec{A})u_A = u_B$$

$$\vec{\sigma}(\vec{p}+e\vec{A})(E+m+eA^0)^{-1}\vec{\sigma}(\vec{p}+e\vec{A})u_A = (E - m + eA^0)u_A$$

$$m \gg p\ ; \quad m \gg eA^0 \qquad E - m + eA^0 = E_{NR} + eA^0$$

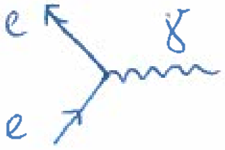
⋮

$$\left(\frac{(\vec{p}+e\vec{A})^2}{2m} + \frac{e}{2m}\vec{\sigma}\cdot\vec{B} - eA^0\right)u_A = E_{NR}u_A$$

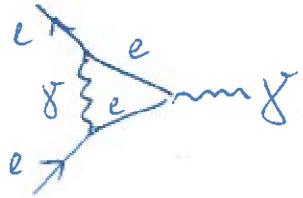
↑ $-\vec{\mu}\cdot\vec{B}$ ↑ ELEKTROST. INT.

$$-\frac{e}{2m}\vec{\sigma} = \vec{\mu} = -\frac{e}{2m}g_s\frac{\vec{\sigma}}{2} \Rightarrow g_s = 2$$

$$\vec{\mu} = -\frac{e}{2m}g_s\vec{s}$$



1. RED PERTURBACIJSKEGA RAZVOJA



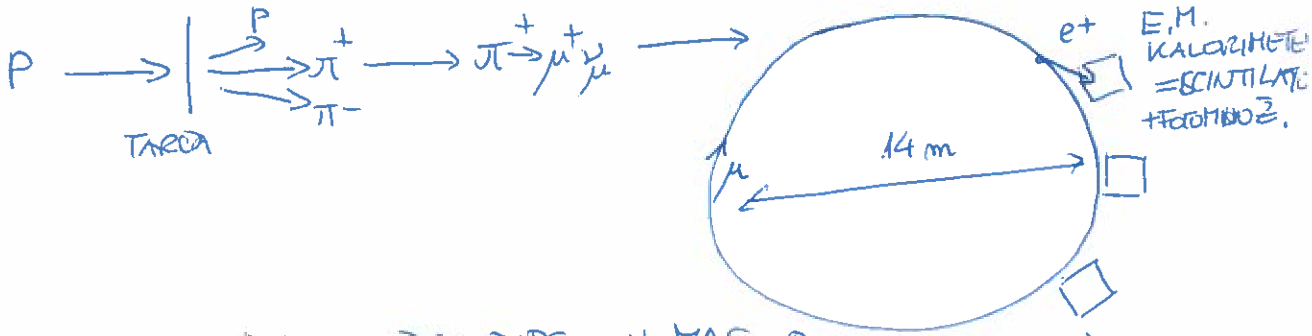
$\Rightarrow g_s \neq 2$

TUDI ZA MION VELJA PODOBNO

$$\frac{g_s(\mu)-2}{2}$$

NAJBOLJ NATANČEN POSKUS IN RAČUN V FIZIKI DELCEV

$$(g_s-2)_\mu = 11{,}659214 \cdot 10^{-4}$$



GIBALNA ENAČBA ZA ~~ELEC~~DIPOL V MAG. POLJU

→ $g_s = 2$ ⇒ M. DIPOL VRTI SKUPAJ S $\vec{p}$

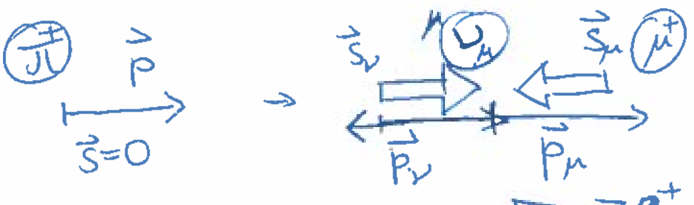
NA ZAČETKU VZPOREDNA → VES ČAS VZPOREDNA

IDEJA EKSPERIMENTA! MERIMO FREKVENCO VRTENJA $\vec{\mu}$ GLEDE NA $\vec{p}$

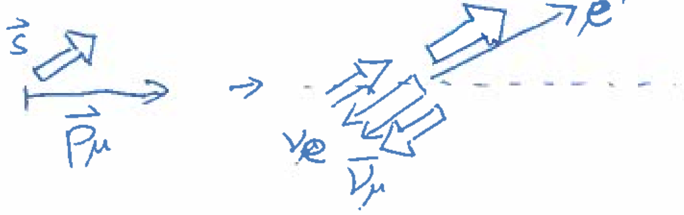
(1.) ZAČETNO STANJE (KO μ MION VSTOPI V OBROČ)

$\vec{p} \parallel \vec{\mu} \parallel \vec{s}$

SPIN ν_μ JE VEDNO NASPROTEN $\vec{p}_\nu$

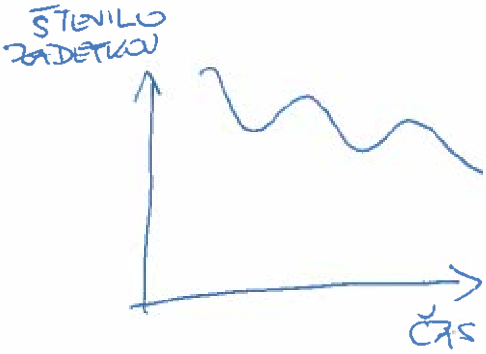


NASTANEK



RAZPAD $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$

ČE IZBEREM e^+ MAX. MOŽNA



MERIMO TIPIČNO ~~[illegible]~~ $t > 700\,\mu s$

$$\tau_\mu = 2{,}1\,\mu s$$

VROČA TEMA: EKSPERIMENT IN TEORIJA SE NE UJEMATA DO NATANČNOSTI.

NORMALIZACIJA REŠITEV DIRACOVE ENAČBE ZA PROST DELEC

KLEIN-GORDONOVA E. → REŠITVE NORMIRAMO NA VOLUMEN NA 2E DELCEV V VOLUMNU.

ISTA NORMALIZACIJA TUDI PRI DIR. ENAČBI

$$\rho = \bar{\psi}\gamma^0\psi = \psi^+\psi \qquad \bar{\psi}\gamma^0 = \psi^+\gamma^0\gamma^0 = \psi^+$$

$$\int \psi^+\psi \, dV = u^+u = 2E \qquad \psi = \frac{1}{\sqrt{V}} e^{ipx} u$$

$$u^{(s)} = N \begin{bmatrix} \chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \end{bmatrix}$$

$$u^+u = |N|^2 \left[\chi^{(s)T} \quad \left[\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)}\right]^T \right] \begin{bmatrix} \chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \end{bmatrix}$$

$$(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{p}) = \vec{p}^{\,2}$$

$$= |N|^2 \left(1 + \chi^{(s)T} \frac{(\vec{\sigma}\cdot\vec{p})^T}{E+m} \frac{(\vec{\sigma}\cdot\vec{p})}{E+m} \chi^{(s)} \right) =$$

$$= |N|^2 \left(1 + \frac{p^2}{(E+m)^2} \right) = |N|^2 \left(\frac{(E+m)^2 + p^2}{(E+m)^2} \right)$$

$$= |N|^2 \left(\frac{2E}{E+m} \right) \underset{\text{ZAHTEVA}}{=} 2E \qquad E^2 + 2mE + m^2 + p^2 = 2E(E+m)$$

$$\Rightarrow |N| = \sqrt{E+m}$$

$$[\gamma^\mu p_\mu - m]\psi = 0 \quad \Rightarrow \quad [\not p - m]\psi = 0$$

DEFINICIJA ZA POLJUBEN ČETVEREC a_μ

$$\not a = \gamma^\mu a_\mu$$

LASTNOST BISPINORJEV

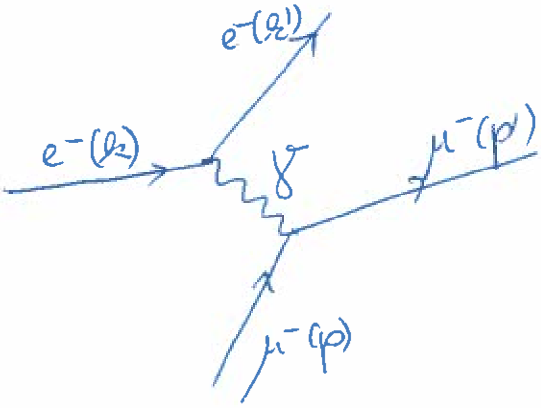
$$\boxed{\sum_{s=1,2} u^{(s)}(p)\,\bar{u}^{(s)}(p) = \not p + m}$$

← 4x4 MATRIKA

POLNOSTNA RELACIJA

ELEKTROMAGNETNO SIPANJE DIRACOVIH DELCEV

1. PRIMER $e^-\mu^- \to e^-\mu^-$



$$T_{fi} = -i\int j^e_\nu \left(-\frac{1}{q^2}\right) j^\nu_\mu \, d^4x$$

$$j^e_\nu = -e\,\bar{u}(k')\gamma_\nu u(k)\, e^{i(k-k')x}$$

$$j^\mu_\nu = -e\,\bar{u}(p')\gamma_\nu u(p)\, e^{i(p-p')x}$$

$$T_{fi} = -i\left[-e\,\bar{u}(k')\gamma_\nu u(k)\right]\left(\frac{-1}{q^2}\right)\left[-e\bar{u}(p')\gamma^\nu u(p)\right]\underbrace{\int e^{i(k'+p'-k-p)x} d^4x}_{(2\pi)^4\delta^4(k'+p'-k-p)}$$

$\Rightarrow$ OHRANITEV $E, \vec{p}$

$$T_{fi} = -(2\pi)^4\delta^4(k'+p'-k-p)\,\mathcal{M}$$

$$-i\mathcal{M} = \left[-e\bar{u}(k')\gamma^\kappa u(k)\right]\left(\frac{-g_{\kappa\nu}}{q^2}\right)\left[-e\bar{u}(p')\gamma^\nu u(p)\right]$$

$\mathcal{M}$ INVARIANTNA AMPLITUDA

$$g_{\kappa\nu}\gamma^\nu = \gamma_\kappa$$

NEPOLARIZIRANI PRESEK

$$e^- \mu^- \to e^- \mu^-$$
$s_1 \quad s_2 \qquad s_3 \quad s_4$

s_1, s_2 : POLARIZIRANI e, μ
s_3, s_4 : MERIM POLARIZACIJO (SMER SPINA)

NEPOLARIZIRAN PRESEK: SIPALNI PRESEK ZA 'NEPOLARIZIRANE' VPADNE DELCE IN ZA PRIMER, KO NE MERIM SPINOV V KONČNEM STANJU.

POLARIZIRAN PRESEK

$$\sigma \propto |\mathcal{M}|^2 \qquad \sigma \propto |\mathcal{M}_{s_1 s_2 s_3 s_4}|^2$$

NEPOLARIZIRAN PRESEK

$$\sigma \propto \frac{1}{(2s_a+1)(2s_b+1)} \sum_{s_1,s_2,s_3,s_4} |\mathcal{M}_{s_1,s_2,s_3,s_4}|^2$$

s_a, s_b ZA NAŠ PRIMER: $s_e = \frac{1}{2}$, $s_\mu = \frac{1}{2}$

$$\frac{1}{(2s_a+1)(2s_b+1)} = \frac{1}{4}$$

ZAKAJ $\sum |\mathcal{M}_{s_1,s_2,s_3,s_4}|^2$ IN NE $\cancel{\left|\sum \mathcal{M}_{s_1,s_2,s_3,s_4}\right|^2}$

ZATO, KER LAHKO SPINE V ZAČETKU IN NA KONCU V PRINCIPU IZMERIMO. SEŠTEVANJE AMPLITUD PRIDE V POŠTEV, KADAR PROCESI NISO LOČLJIVI.

$$|\mathcal{M}|^2 \propto [\bar{u}(k')\gamma^k u(k)][\overset{u^+(k')\gamma^0}{\bar{u}(k')}\gamma^3 u(k)]^+ = [\bar{u}(k')\gamma^k u(k)][\bar{u}(k')\gamma^3 u(k)]^*$$

$$= [\bar{u}(k')\gamma^k u(k)][u(k)^+ \underbrace{\gamma^{3+}\gamma^{0+}}_{\gamma^{3+}\gamma^0 = \gamma^0\gamma^3} u(k')] =$$

$$= [\bar{u}(k')\gamma^k u(k)][u(k)^+ \gamma^0 \gamma^3 u(k')] = [\bar{u}(k')\gamma^k u(k)][\bar{u}(k)\gamma^3 u(k')]$$

$$\sum_{s,s'} [\bar{u}^{(s')}(k')\gamma^k u^{(s)}(k)][\bar{u}^{(s)}(k)\gamma^3 u^{(s')}(k')] =$$

IMPLICITNA VSOTA PO PARIH INDEKSOV

$$= \sum_{s,s'} \left[\bar{u}_\alpha^{(s')}(k')\gamma^\kappa_{\alpha\beta} u_\beta^{(s)}(k)\right]\left[\bar{u}_\delta^{(s)}(k)\gamma^\tau_{\delta\varepsilon} u_\varepsilon^{(s')}(k')\right] =$$

$$= \sum_{s,s'} \underbrace{u_\varepsilon^{(s')}(k')\bar{u}_\alpha^{(s')}(k')}_{[\not k'+m_e]_{\varepsilon\alpha}}\gamma^\kappa_{\alpha\beta}\underbrace{u_\beta^{(s)}(k)\bar{u}_\delta^{(s)}(k)}_{[\not k+m_e]_{\beta\delta}}\gamma^\tau_{\delta\varepsilon}$$

$$= [\not k'+m_e]_{\varepsilon\alpha}\gamma^\kappa_{\alpha\beta}[\not k+m_e]_{\beta\delta}\gamma^\tau_{\delta\varepsilon} =$$

$$= \left[[\not k'+m_e]\gamma^\kappa[\not k+m_e]\gamma^\tau\right]_{\varepsilon\varepsilon} = \mathrm{Tr}\left[(\not k'+m_e)\gamma^\kappa(\not k+m_e)\gamma^\tau\right]$$

ENAK RAZMISLEK ZA MIONE

$$\overline{|\mathcal{M}|^2} = \frac{1}{2}\cdot\frac{1}{2}\frac{e^2e^2}{q^4}\cdot\mathrm{Tr}\left[(\not k'+m_e)\gamma^\kappa(\not k+m_e)\gamma^\tau\right]\mathrm{Tr}\left[(\not p'+m_\mu)\gamma_\kappa(\not p+m_\mu)\gamma_\tau\right]$$

TEOREMI O SLEDEH

EDEN OD NJIH

$$\mathrm{Tr}\left[(\not k'+m_e)\gamma^\kappa(\not k+m_e)\gamma^\tau\right] = \cancel{4[k'^\kappa k^\tau + k'^\tau k^\kappa - (k'k)}$$

$$= 4\left[k'^\kappa k^\tau + k'^\tau k^\kappa - (k'\cdot k - m_e^2)g^{\kappa\tau}\right]$$

$$\overline{|\mathcal{M}|^2} = 4\frac{e^4}{q^4}\left[(k'p')(kp) + (k'p)(kp') - m_e^2 p'p - m_\mu^2 k'k + 2m_e^2 m_\mu^2\right]$$

— LORENTZOVA INVARIANTA (KER SKALARNI PRODUKTI)

SIPALNI PRESEK $\quad \dfrac{d\sigma}{d\Omega} = \dfrac{dW_{fi}/d\Omega}{\rho_i v_i}$

$$\frac{dW_{fi}}{d\Omega} = \frac{2\pi}{\hbar}|T_{fi}|^2\frac{d\rho_f}{d\Omega}$$

$$d^3N = V\frac{d^3p}{(2\pi\hbar)^3} = \frac{d^3p}{\rho(2\pi\hbar)^3}$$

$$\rho = \frac{1}{V}$$ NERELATIVISTIČNI PRIMER

RELATIVISTIČNI DELCI

$$d^3N = \frac{V}{2E}\frac{d^3p}{(2\pi\hbar)^3}$$

$$d^3N = \frac{V}{2E}\frac{d^3p}{(2\pi\hbar)^3}$$

$$\rho_2 \propto \frac{d^3p}{E}$$

LORENTZOVA TRANSF. V SMERI x

$$dp_x' = \gamma(dp_x - \beta dE) \quad dp_y' = dp_y \quad dp_z' = dp_z$$

$$dE' = \gamma(dE - \beta dp_x)$$

$$\frac{d^3p'}{E'} = \frac{\gamma(dp_x - \beta dE)\,dp_y\,dp_z}{\gamma(E - \beta p_x)} = \frac{dp_x\left(1-\beta\frac{dE}{dp_x}\right)dp_y\,dp_z}{E-\beta p_x}$$

$$= \frac{dp_x\left(1-\beta\frac{p_x}{E}\right)dp_y\,dp_z}{E\left(1-\beta\frac{p_x}{E}\right)} =$$

$$E^2 = p_x^2 + p_y^2 + p_z^2 + m^2$$

$$E\,dE = p_x\,dp_x$$

$$= \frac{d^3p}{E}$$

LORENTZOVA INVARIANTA

$$\rho_i v_i = \underbrace{\frac{2E_a}{V} v_a}_{\text{UPADNI DELEC}} \underbrace{\frac{2E_b}{V}}_{\text{TARČA}}$$

PRIMER, KO DELEC b MIRUJE, a PA SE NA NJEM SIPLJE

ČE SE OBADVA GIBLJETA

$$\rho_i v_i = F = \frac{2E_a}{V}\frac{2E_b}{V}|\vec{v}_a - \vec{v}_b|$$

$$\frac{v}{c} = \beta = \frac{\gamma m v \cdot c}{\gamma m c\, c} = \frac{cp}{E}$$

$$\vec{v}_a - \vec{v}_b = \frac{|\vec{p}_a E_b - \vec{p}_b E_a|}{E_a E_b}$$

$$|\vec{p}_a E_b - \vec{p}_b E_a| = \sqrt{(p_a p_b)^2 - m_a^2 m_b^2}$$

$\Rightarrow$ TUDI TO JE RELATIVISTIČNA INVARIANTA

DIFERENCIALNI PRESEK

$$d\sigma = \frac{|\mathcal{M}|^2}{F}\,dQ$$

$$F = 4\sqrt{(p_a p_b)^2 - m_a^2 m_b^2}$$

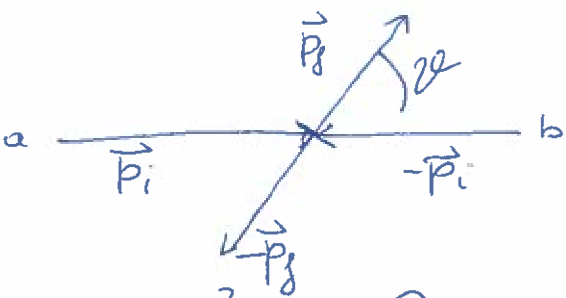
$$dQ = (2\pi)^4 \delta^4(p' + k' - p - k)\cdot$$

$$\cdot \frac{d^3p_c}{(2\pi)^3\, 2E_c} \cdot \frac{d^3p_d}{(2\pi)^3\, 2E_d}$$

$$ab \to cd$$

$$-i\mathcal{M} = [J^\mu_{ca}]\left(\frac{g_{\mu\nu}}{q^2}\right)[J^\nu_{db}]$$

V TEŽIŠČNEM SISTEMU (CENTER-OF-MASS, C.M.S.)



$$\vec{p}_a = \vec{p}_i = -\vec{p}_b$$
$$\vec{p}_c = \vec{p}_f = -\vec{p}_d$$

$$d^3p_c = p_f^2\, dp_f\, d\Omega \qquad d\Omega = 2\pi \sin\vartheta\, d\vartheta$$

$$\int \frac{d^3p_d}{2E_d} \underbrace{\delta^4(p_c+p_d-p_a-p_b)}_{\delta(E_c+E_d-E_a-E_b)\delta^3(\vec{p}_c+\vec{p}_d-\vec{p}_a-\vec{p}_b)} = \frac{1}{2E_d}\delta(E_c+E_d-E_a-E_b)$$

ENERGIJA V TEŽIŠČNEM SISTEMU $E = E_a + E_b$

$$dQ = \frac{1}{4\pi^2}\frac{p_f^2\, dp_f\, d\Omega}{4E_c E_d}\delta(E_c+E_d-E)$$

$$E = E_c + E_d = \sqrt{p_f^2+m_c^2} + \sqrt{p_f^2+m_d^2}$$

$$\frac{dE}{dp_f} = \frac{p_f}{E_c} + \frac{p_f}{E_d}$$

$$dp_f = dE\left(\frac{p_f}{E_c}+\frac{p_f}{E_d}\right)^{-1} = dE\, \frac{E_c E_d}{p_f \underbrace{(E_c+E_d)}_{E}}$$

$$dQ = \frac{1}{4\pi^2}\frac{p_f\, dE}{4(E_c+E_d)}\, p_f\, d\Omega\, \delta(E_c+E_d-E)$$

$$\int dE \rightarrow \quad dQ = \frac{1}{4\pi^2}\frac{p_f}{4E}d\Omega$$

$$F = 4p_i E$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2 p_f}{64\pi^2 p_i E^2}}$$

V ULTRARELATIVISTIČNI LIMITI $m_x \ll p_x$ $\quad p_i = p_f$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 E^2}}$$

UPORABIMO TA REZULTAT ZA $e^-\mu^- \to e^-\mu^-$,
NEPOLARIZIRANO SIPANJE. V ULTRARELAT. LIMITI

$$k = \left(\tfrac{E}{2}, \vec{p}_i\right) \quad k' = \left(\tfrac{E}{2}, \vec{p}_f\right) \quad p = \left(\tfrac{E}{2}, -\vec{p}_i\right) \quad p' = \left(\tfrac{E}{2}, -\vec{p}_f\right)$$

$$k'p = \frac{E^2}{4} + \vec{p}_f \cdot \vec{p}_i = \frac{E^2}{4}(1+\cos\vartheta)$$

$$kp = \frac{E^2}{4} + \frac{E^2}{4} = \frac{E^2}{2} = k'p'$$

$$\vec{p}_i = (p_i, 0, 0) = (E/2, 0, 0)$$

$$\vec{p}_f = \left(\tfrac{E}{2}\cos\vartheta, \tfrac{E}{2}\sin\vartheta, 0\right)$$

$$q^2 = (k'-k)^2 = (0, \vec{p}_f - \vec{p}_i)^2 =$$

$$= -(\vec{p}_f - \vec{p}_i)^2 = -\frac{E^2}{4}\left((\cos\vartheta - 1)^2 + \sin^2\vartheta\right) =$$

$$= -\frac{E^2}{2}(1-\cos\vartheta)$$

$$\overline{|\mathcal{M}|^2} = 4\frac{e^4}{q^4}\left[(k'p')(kp) + (k'p)(kp')\right] = \cancel{(1-\cos\vartheta)^2}$$

$$= 4\frac{e^4}{q^4}\left[\frac{E^4}{4} + \frac{E^4}{16}(1+\cos\vartheta)^2\right] =$$

$$= \frac{1}{4}\frac{e^4}{q^4}\left[4 + (1+\cos\vartheta)^2\right]E^4$$

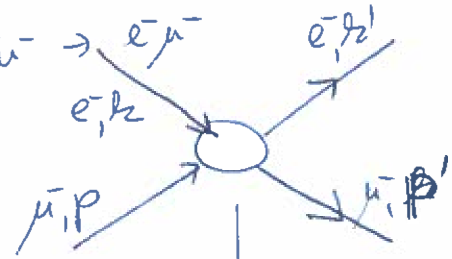
$$= \frac{e^4\left[4 + (1+\cos\vartheta)^2\right]}{(1-\cos\vartheta)^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 E^2} = \frac{e^4\left(4 + (1+\cos\vartheta)^2\right)}{32\pi^2 E^2 (1-\cos\vartheta)^2}$$

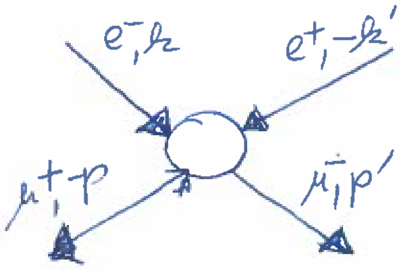
PREVERITI DOMA

$e^+e^- \to \mu^+\mu^-$ IN KRIŽANJE

$e^-\mu^- \to e^-\mu^-$



$e^-e^+ \to \mu^-\mu^+$



VHODNI		IZHODNI	
p_a	p_b	p_c	p_d
k	p	k'	p'
k	$-k'$	$-p$	p'

$$\overline{|\mathcal{M}|^2} = \frac{8e^4}{q^4}\left[(pp')(-kk') + (k'p)(kp') + m_e^2 p'k' + m_\mu^2 pk + 2m_e^2 m_\mu^2\right]$$

V ULTRARELATIVISTIČNI LIMITI

$$k = \left(\frac{E}{2}, \vec{p_i}\right),\quad k' = \left(\frac{E}{2}, \vec{p_f}\right),\quad p = \left(\frac{E}{2}, -\vec{p_i}\right),\quad p' = \left(\frac{E}{2}, -\vec{p_f}\right)$$

$$q^2 = (k'-k)^2 \xrightarrow[\text{PO KRIŽANJU}]{} (p+k)^2 = E^2$$

$$\frac{d\sigma}{d\Omega} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 E^2} = \frac{8e^4\left[(1-\cos\vartheta)^2 + (1+\cos\vartheta)^2\right]\frac{E^4}{4\cdot 4}}{64\pi^2 E^2\, E^4} =$$

$$\boxed{\frac{d\sigma}{d\Omega}} = \frac{e^4}{4\cdot 32\pi^2 E^2}\left(2 + 2\cos^2\vartheta\right) = \frac{e^4}{4\cdot 16\pi^2 E^2}\left(1+\cos^2\vartheta\right)$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = \frac{e^4}{4\cdot 16\pi^2 E^2}\int_{-1}^{+1}\left(1+\cos^2\vartheta\right) 2\pi \cdot d(\cos\vartheta) =$$

$$= \frac{e^4}{4\cdot 8\pi E^2}\left(\cos\vartheta + \frac{1}{3}\cos^3\vartheta\right)\Big|_{-1}^{+1} = \frac{e^4}{4\cdot 8\pi E^2}\left(2 + \frac{2}{3}\right) =$$

$$\sigma = \frac{e^4}{4\cdot 3\pi E^2} = \frac{e^4}{12\pi E^2} \qquad \sigma \propto \frac{1}{E^2}$$

$$\alpha = \frac{1}{137} = \frac{e^2}{4\pi\cancel{\varepsilon_0}\hbar c}$$

$$\Rightarrow \frac{e^2}{4\pi} = \alpha\,\hbar c$$

$$\sigma_{e^+e^- \to \mu^+\mu^-} = \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{3} 4\pi\, \frac{1}{E^2} =$$

$$= \alpha^2 (\hbar c)^2 \frac{4\pi}{3}\frac{1}{E^2}$$

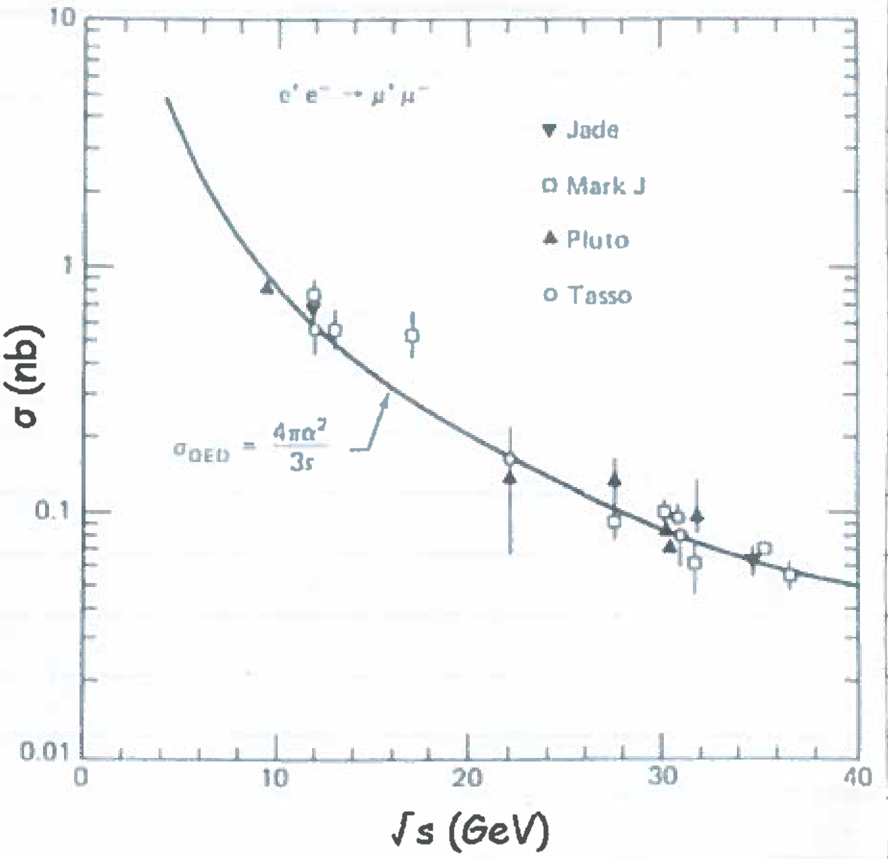
$$\boxed{\sigma_{e^+e^-\to\mu^+\mu^-} = \frac{4\pi}{3}\alpha^2(\hbar c)^2\frac{1}{E^2}}$$

σ pri $E = \boxed{10\ \text{GeV}}$

$$\sigma = \frac{4\pi\,(0{,}2)^2\,(\text{GeV})^2\,\text{fm}^2}{3\;10^2(\text{GeV})^2\,(137)^2} =$$

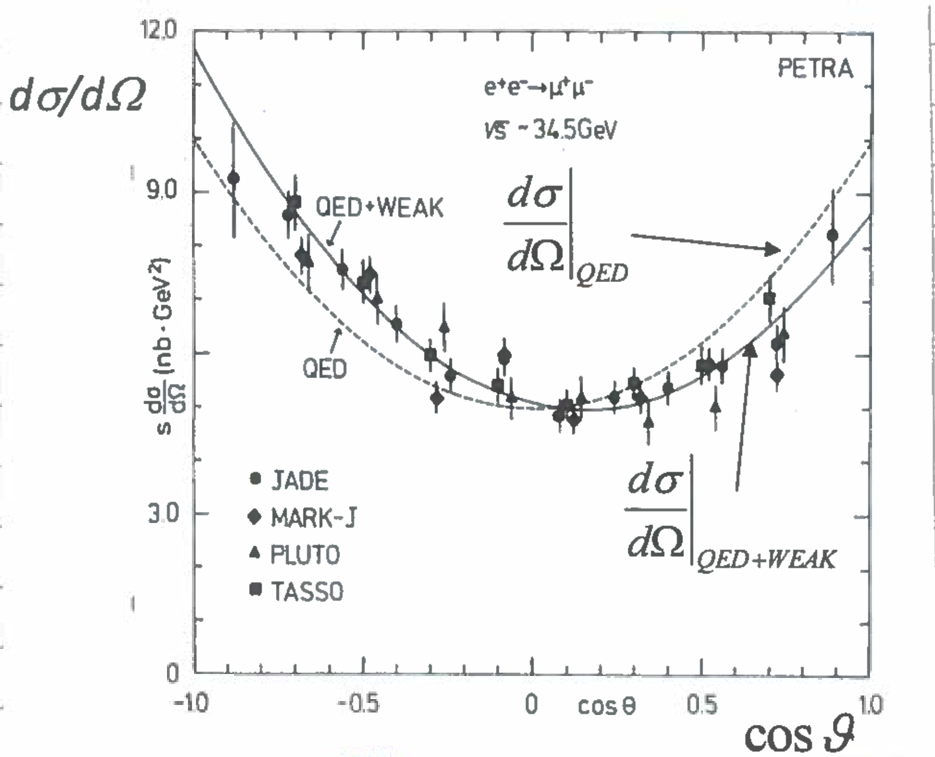
$$= \frac{4\pi\; 0{,}04}{3\;10^{-2}\,10^{2}\;137^2}\,\text{b} \qquad 1\,\text{b} = (10\,\text{fm})^2$$

$$\approx 9\;10^{-10}\,\text{b} = \boxed{0{,}9\ \text{nb}}$$

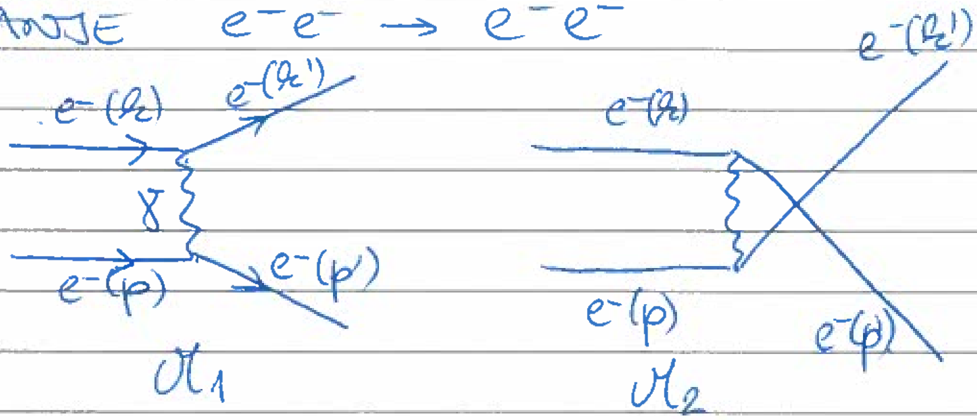


$E^2 = s$

$$\left(\frac{d\sigma}{d\Omega}\right)_{e^+e^-\to\mu^+\mu^-} = \frac{e^4}{4\cdot 16\pi^2E^2}\left(1+\cos^2\vartheta\right) = \frac{\alpha^2(\hbar c)^2}{4E^2}\left(1+\cos^2\vartheta\right)$$



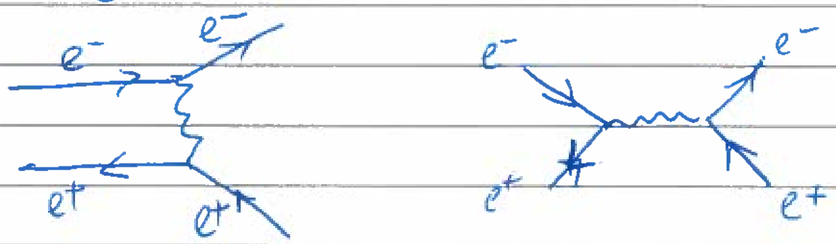
SIPANJE $e^-e^- \to e^-e^-$



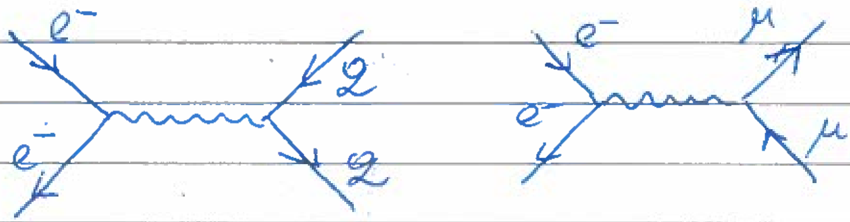
$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 \qquad \mathcal{M}_1 \propto \left[\bar{u}(k')\gamma^\mu u(k)\right]\left[\bar{u}(p')\gamma_\mu u(p)\right]$$

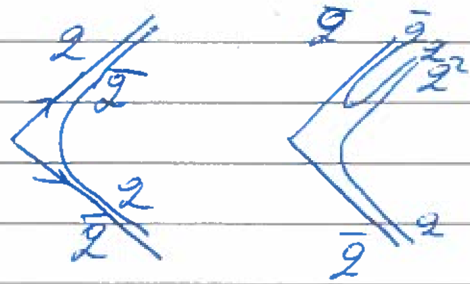
$$\mathcal{M}_2 \propto \left[\bar{u}(p')\gamma^\mu u(k)\right]\left[\bar{u}(k')\gamma_\mu u(p)\right]$$

$e^-e^- \to e^-e^-$ $\to$ s KRIŽANJEM $\to$ $e^-e^+ \to e^-e^+$



SIPANJE $e^+e^- \to q\bar{q}$





$$\sigma_{e^+e^-\to\mu^+\mu^-} \propto e^4 = \underset{\substack{\uparrow \\ e}}{e^2} \cdot \underset{\substack{\uparrow \\ \mu}}{e^2}$$

$$\sigma_{e^+e^-\to q\bar{q}} \propto \underset{\substack{\uparrow \\ e}}{e^2} \cdot \underset{\substack{\uparrow \\ q}}{e_q^2} \qquad e_q^2 = \frac{1}{9}, \frac{4}{9}$$

$$\frac{\sigma_{e^+e^-\to q\bar{q}_i}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = 3\,\frac{e_e^2 \cdot e_{q_i}^2}{e_e^2 \cdot e_\mu^2} = 3Q_{q_i}^2$$

3: 3 BARVE

$$Q_{q_i} = \frac{e_{q_i}}{e_0}$$

$$\sigma(e^+e^- \to \text{hadroni}) = \sum_{q=u,d,s,c\ldots} \sigma(e^+e^- \to q\bar{q})$$

LEP

e^-e^+ v TUNELU LHC

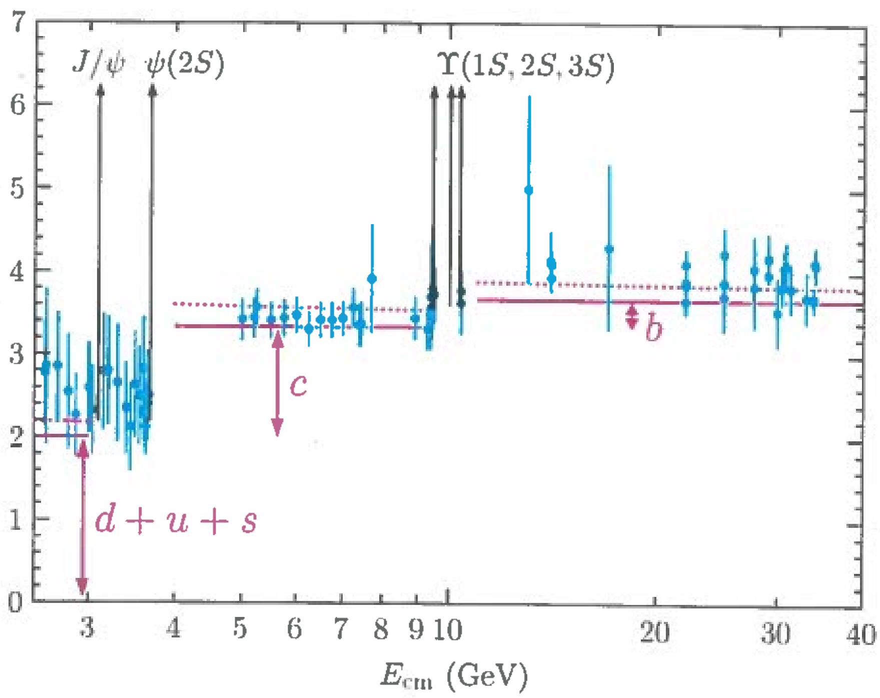
$E \sim 90$ GeV

$e^+e^- \to q\bar{q}$

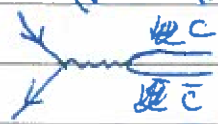
JET

$$R = \frac{\sigma(e^+e^- \to \text{hadroni})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_i Q_{q_i}^2$$

$$\begin{aligned}
E < 3\ \text{GeV}: &\quad R = 3\left(\overset{u}{\tfrac{4}{9}} + \overset{d}{\tfrac{1}{9}} + \overset{s}{\tfrac{1}{9}}\right) = 2 \\
4\ \text{GeV} < E < \sim 9\ \text{GeV}: &\quad R = 3\left(\tfrac{4}{9} + \tfrac{1}{9} + \tfrac{1}{9} + \tfrac{4}{9}\right) = \tfrac{10}{3} = 3\tfrac{1}{3} \\
E > 10\ \text{GeV}: &\quad R = 3\tfrac{2}{3}
\end{aligned}$$

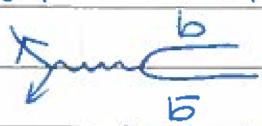


3 GeV < E < 4 GeV



$J/\psi, \psi(2S)$

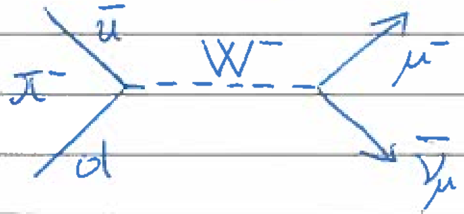
9 GeV < E < 10 GeV



$\Upsilon(1S), \Upsilon(2S), \Upsilon(3S), \Upsilon(4S)$

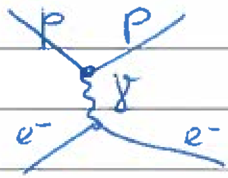
ŠIBKA INTERAKCIJA

$\tau(\pi^-) = 2{,}6 \cdot 10^{-8}\,s$ $\to$ ŠIBKA $\tau(\pi^0) = 8{,}4 \cdot 10^{-17}\,s$ $\to$ EM $\pi^0 \to \gamma\gamma$



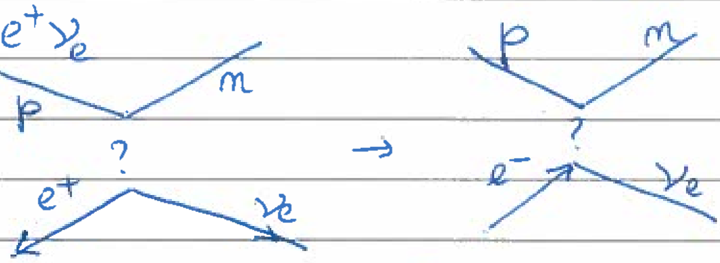
$\pi^- \to \mu^- \bar{\nu}_\mu$

FERMI 1932 – ANALOGIJA Z E.M. INTERAKCIJO



$$-i\mathcal{M} = e^2 [\bar{u}_p \gamma^\mu u_p] \left(-\frac{g_{\mu\nu}}{q^2}\right) [\bar{u}_e \gamma^\nu u_e]$$

$^{10}C \to {}^{10}Be\, e^+ \nu_e$



$$-i\mathcal{M} = \frac{G_F}{\sqrt{2}} [\bar{u}_n \gamma^\mu u_p][\bar{u}_\nu \gamma_\mu u_e]$$

G_F = FERMIJEVA SKLOPITVENA KONSTANTA

1950 LETA "Θ^+/τ^+" UGANKA: "Θ^+" $\to \pi^+\pi^0$, "τ^+" $\to \pi^+\pi^+\pi^-$

$\Theta^+ \equiv \tau^+ \equiv K^+$ IZ EKSPERIMENTOV

PARNOST PIONA: $P = -1$

K^+ RAZPADE ENKRAT V STANJE S $P = (-1)^2 = +1$

ENKRAT PA V $P = (-1)^3 = -1$

E.M. IN MOČNA INT. OHRANJATA PARNOST

$P_i = P_f$

1956 T.D. LEE CHEN NING YANG

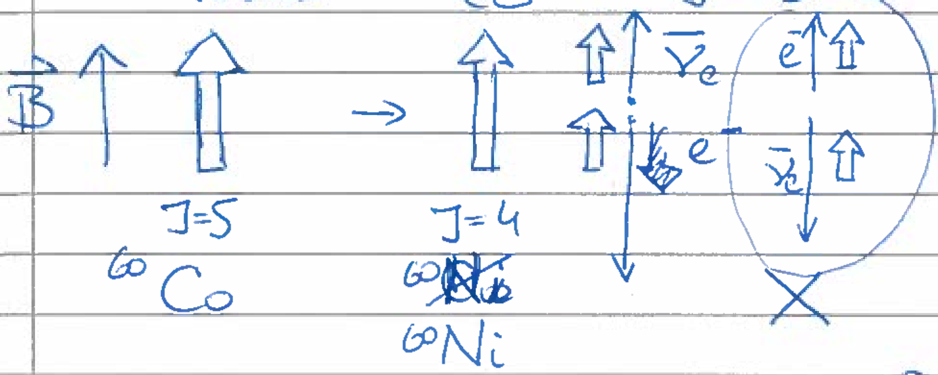
→ ŠIBKA INTERAKCIJA NE OHRANJA PARNOSTI

EKSPERIMENT S ^{60}Co

CHIEN SHIUNG WU

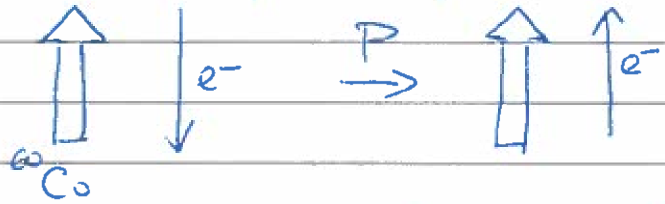
$$^{60}Co \to {}^{60}Ni\; e^- \; \bar{\nu}_e$$

JEDRO ^{60}Co : $J = 5$



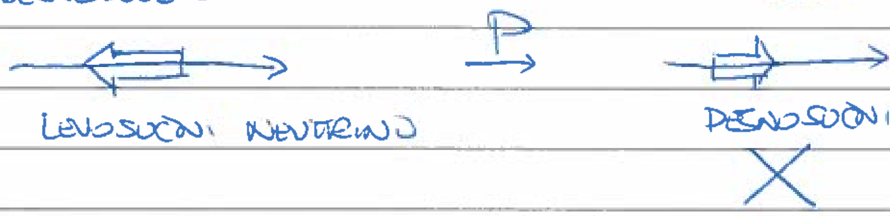
^{60}Co V MAGNETNEM POLJU B, MERIMO SMER ELEKTRONOV

P: $\vec{r} \to -\vec{r}$: $\vec{p} \to -\vec{p}$, $\vec{L} \to \vec{L}$, $\vec{J} \to \vec{J}$



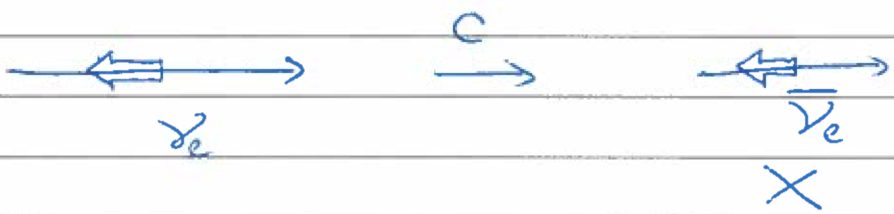
$\Rightarrow$ PARNOST JE KRŠENA PRI ŠIBKI INTERAKCIJI

DESNOSUČEN ANTINEVTRINO — LEVOSUČNI ANTINEVTRINO

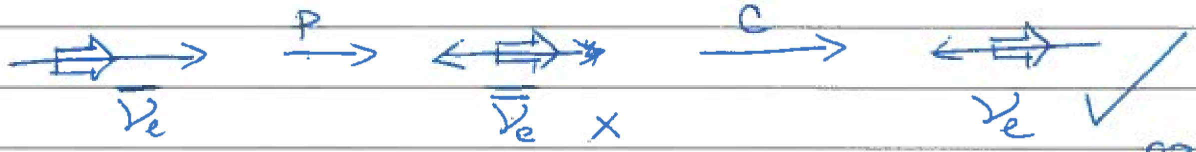


LEVOSUČNI NEVTRINO — DESNOSUČNI NEVTRINO

KRŠENA JE TUDI PARNOST C : DELEC $\to$ ANTIDELEC



P, C KRŠENI, OHRANJA SE PA CP



DN $\nu_e \xrightarrow{CP}$?

1964 FITCH, CRONIN RAZPADI K^0 : CP
SE NE OHRANJA

KAKO NAPISATI $\mathcal{M}$, DA BO AVTOMATSKO
ZA TO POSKRBLJENO? (UPOŠTEVANO)
NEOHRANITEV P

$$-i\mathcal{M} = \frac{G_F}{\sqrt{2}} \left[\bar{u}_p \gamma^\kappa (1-\gamma^5) u_p\right] \left[\bar{u}_\nu \gamma_\kappa \overset{(1-\gamma^5)}{} u_e\right]$$

$$\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \text{"}1\text{"} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\gamma^{5+} = \gamma^5 \,;\; (\gamma^5)^2 = 1 \,;\; \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0$$

$\frac{1}{2}(1-\gamma^5)u = u_L$ LEVOROČNI BISPINOR

$\frac{1}{2}(1+\gamma^5)u = u_R$ DESNOROČNI

$$u_L + u_R = u$$

$$\gamma^5 u_L = \gamma^5 \frac{1}{2}(1-\gamma^5) u = \frac{1}{2}(\gamma^5 - 1) u = -\frac{1}{2}(1-\gamma^5)u = -u_L$$

$$\gamma^5 u_R = +u_R$$

LEVOROČNI BISPINOR

$$u_L = \frac{1}{2}(1-\gamma^5)u = \frac{N}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \chi \\ \frac{\vec{\sigma}\vec{p}}{E+m}\chi \end{bmatrix} = \frac{N}{2}\begin{bmatrix} \chi - \frac{\vec{\sigma}\vec{p}}{E+m}\chi \\ -(\chi - \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi) \end{bmatrix}$$

za $E \gg m$ $\frac{\vec{p}}{E+m} \to \hat{p}$ ENOTSKI VEKTOR

$$\to = \frac{N}{2}\begin{bmatrix} \chi - \vec{\sigma}\hat{p}\chi \\ -(\chi - \vec{\sigma}\hat{p}\chi) \end{bmatrix}$$

SUČNOST u_L!

OPERATOR $\vec{\Sigma}\hat{p}$

$$\vec{\Sigma}\hat{p}\, u_L = \begin{bmatrix} \vec{\sigma}\hat{p} & 0 \\ 0 & \vec{\sigma}\hat{p} \end{bmatrix} u_L =$$

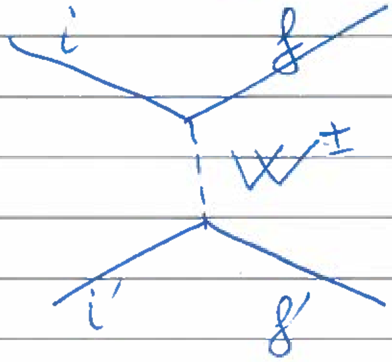
$$= \frac{N}{2}\begin{bmatrix} \vec{\sigma}\hat{p}\chi - (\vec{\sigma}\hat{p})^2\chi \\ -(\vec{\sigma}\hat{p}\chi - (\vec{\sigma}\hat{p})^2\chi) \end{bmatrix} = \frac{N}{2}\begin{bmatrix} \vec{\sigma}\hat{p}\chi - \chi \\ -\vec{\sigma}\hat{p}\chi + \chi \end{bmatrix} = -u_L$$

$\boxed{\text{LEVOSUČEN}}$

$\vec{\Sigma}\hat{p}\, u_R = + u_R$ ZA ULTRARELATIVISTIČNE DELCE

V ULTRAREL. LIMITI: ROČNOST $\equiv$ SUČNOST

KONONA OBLIKA MATRIČNEGA ELEMENTA ZA ŠIBEK PROCES



$$-i\mathcal{M} = \left[\frac{g_W}{\sqrt{2}}\bar{u}_f \gamma^\kappa (1-\gamma^5) u_i\right]\left(-\frac{g_{\kappa\nu}}{M_W^2 - q^2}\right)\left[\frac{g_W}{\sqrt{2}}\bar{u}_{f'} \gamma^\nu (1-\gamma^5) u_{i'}\right]$$

RAZPADI $\underset{(\mathrm{MeV})^2}{q^2} \ll \underset{(83\,\mathrm{GeV})^2}{M_W^2}$ $\qquad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{2M_W^2} \Rightarrow \boxed{G_F = \frac{g_W^2}{\sqrt{2}\, M_W^2}}$

A. SALAM, S. GLASHOW, S. WEINBERG

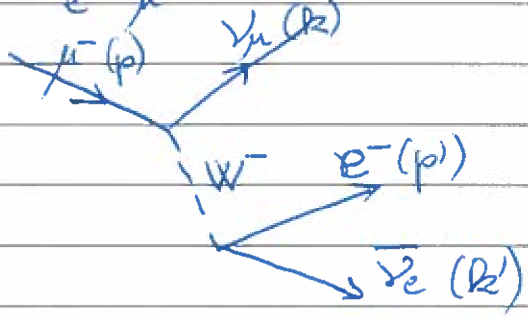
→ ELEKTROŠIBKA INTERAKCIJA

→ STANDARDNI MODEL

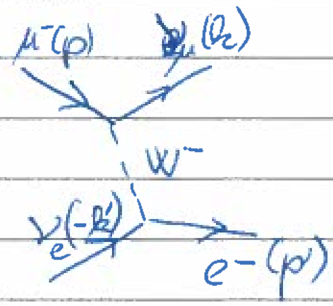
ENA OD NAPOVEDI: NEVTRALNI ŠIBKI TOKOVI Z^0

PRIMERI ŠIBKEGA PROCESA PRI OSNOVNIH DELCIH

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$



$\equiv$



RAZPAD → RAZPADNA ŠIRINA $\Gamma = \frac{\hbar}{\tau}$

$$d\Gamma = \frac{|\mathcal{M}|^2}{2E}\, dQ$$

FAZNI PROSTOR

$$dQ = \frac{d^3p'}{(2\pi)^3 2E'} \cdot \frac{d^3k}{(2\pi)^3 2\omega} \cdot \frac{d^3k'}{(2\pi)^3 2\omega'} (2\pi)^4 \delta^4(p-p'-k-k')$$

$\uparrow e$ $\quad \uparrow \nu_\mu$ $\quad \uparrow \bar{\nu}_e$

RAČUNAMO $\frac{d\Gamma}{dE'}$ – SPEKTER ELEKTRONOV

NAJPREJ INTEGRAL PO k, UPOŠTEVAMO IDENTITETO

$$\int \frac{d^3k}{2\omega} = \int d^4k\, \Theta(\omega)\, \delta(k^2) \qquad \Theta(\omega) = \begin{cases} 1, \omega>0 \\ 0, \omega<0 \end{cases}$$

$$dQ = \frac{1}{(2\pi)^5} \frac{d^3p'}{2E'} \frac{d^3k'}{2\omega'} \int d^4k\, \Theta(\omega)\, \delta(k^2)\, \delta^4(p-p'-k-k')$$

$$= \frac{1}{(2\pi)^5} \frac{d^3p'}{2E'} \frac{d^3k'}{2\omega'} \Theta(E-E'-\omega')\, \delta((p-p'-k')^2)$$

MATRIČNI ELEMENT

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} [\bar{u}(k)\gamma^\mu(1-\gamma^5)u(p)][\bar{u}(p')\gamma_\mu(1-\gamma^5)u(-k')]$$

$$= \frac{G_F}{\sqrt{2}} [\bar{u}(k)\gamma^\mu(1-\gamma^5)u(p)][\bar{u}(p')\gamma_\mu(1-\gamma^5)v(k')]$$

$v(k)$ BISPINOR ZA ANTIDELEC

$u^{(1,2)} e^{-ipx} \quad E>0$

$u^{(3,4)} e^{-(-ipx)} \equiv v^{(2,1)} e^{ipx} \quad E>0$

DIRACOVA ENAČBA ZA v: ZA u: $(\not{p}-m)u=0$

ZA v: $(\not{p}+m)v=0$ $\qquad \gamma^\mu p_\mu = \not{p}$

POLNOSTNA RELACIJA

ZA v: $\sum_{s=1,2} v^{(s)}(p)\bar{v}^{(s)}(p) = \not{p}-m$ $\qquad$ ZA u: $\sum_{s=1,2} u^{(s)}(p)\bar{u}^{(s)}(p) = \not{p}+m$

$$\overline{|\mathcal{M}|}^2 = \frac{1}{2}\frac{G_F^2}{2} \sum [\bar{u}(k)\gamma^\mu(1-\gamma^5)u(p)][\bar{u}(p')\gamma_\mu(1-\gamma^5)v(k')] [\bar{u}(k)\gamma^\lambda(1-\gamma^5)u(p)]^\dagger[\bar{u}(p')\gamma_\lambda(1-\gamma^5)v(k')]^\dagger$$

$a^* = a^\dagger$ (ČE a KOMPL. ŠTEVILO)

$$\left[\bar{u}(k)\gamma^3_\circ(1-\gamma^5)u(p)\right]^+ = \left[u^+(k)\gamma^0\gamma^3(1-\gamma^5)u(p)\right]^+ =$$

$$= u^+(p)(1-\gamma^5)^+\gamma^{3+}\gamma^{0+}u(k) = \underbrace{u^+(p)}_{u^+\gamma^0}(1-\gamma^5)\gamma^{3+}\gamma^0 u(k) =$$

$$= -\bar{u}(p)(1+\gamma^5)\gamma^{3+}u(k) = \bar{u}(p)\gamma^{3+}(1-\gamma^5)u(k) =$$

$$= \bar{u}(p)\gamma^3(1-\gamma^5)u(k)$$

PODOBNO:

$$\left[\bar{u}(p')\gamma_3(1-\gamma^5)v(k')\right]^+ = \bar{v}(k')\gamma_3(1-\gamma^5)u(p')$$

$$\overline{|\mathcal{M}|}^2 = \frac{G_F^2}{4}\sum_{\substack{\text{SPINA}\\ \mu,\nu_\mu}}\left[\bar{u}(k)\gamma^\mu(1-\gamma^5)u(p)\right]\left[\bar{u}(p)\gamma^6(1-\gamma^5)u(k)\right]\cdot$$

$$\cdot\sum_{\substack{\text{SPINA}\\ e,\bar{\nu}_e}}\left[\bar{u}(p')\gamma_\mu(1-\gamma^5)v(k')\right]\left[\bar{v}(k')\gamma_3(1-\gamma^5)u(p')\right]$$

POLNOSTNA RELACIJA →

TAKO KOT PRI E.M. INTERAKCIJI: VSOTA PO SPINIH → SLED. MATRIKE

$$\overline{|\mathcal{M}|}^2 = \frac{G_F}{4}\mathrm{Tr}\left[(\not{k}+m_{\nu_\mu})\gamma^\mu(1-\gamma^5)(\not{p}+m_\mu)\gamma^3(1-\gamma^5)\right]$$

$$\mathrm{Tr}\left[(\not{p}'+m_e)\gamma_\mu(1-\gamma^5)(\not{k}'-m_{\bar{\nu}_e})\gamma_6(1-\gamma^5)\right]$$

V NADALJEVANJU: UPOŠTEVAMO $m_{\nu_e}\simeq 0$, $m_{\nu_\mu}\simeq 0$

$m_e \ll m_\mu$

PRODUKTA

SLED LIHEGA ŠTEVILA MATRIK γ $=0$, $\gamma^5=\gamma^0\gamma^1\gamma^2\gamma^3$

$$\overline{|\mathcal{M}|}^2 = \frac{G_F^2}{4}\mathrm{Tr}\left[\not{k}\gamma^\mu(1-\gamma^5)\not{p}\gamma^3(1-\gamma^5)\right]$$

$$\mathrm{Tr}\left[\not{p}'\gamma_\mu(1-\gamma^5)\not{k}'\gamma_3(1-\gamma^5)\right]$$

+ IZREKI O SLEDI $\quad \overline{|\mathcal{M}|}^2 = \frac{G_F^2}{4}256\,(k\cdot p')(p\cdot k')$

V MIROVNEM SISTEMU MIONA : $p=(m_\mu,0)$

$$(p-k')^2=(p'+k)^2=\underbrace{p'^2}_{m_e^2\sim 0}+\underbrace{k^2}_{m_\mu^2\sim 0}+2p'k$$

$$\Rightarrow p'k \doteq \tfrac{1}{2}(p-k')^2$$

$$\overline{|\mathcal{M}|}^2=32G_F^2(p-k')^2(pk')=32G_F^2\underbrace{(m_\mu-\omega',-\vec{k}')^2}\,m_\mu\omega'=$$

$$=32G_F^2m_\mu^2(m_\mu-2\omega')$$

$$(m_\mu-\omega')^2-\vec{k}'^2=$$
$$=(m_\mu^2-2m_\mu\omega'+\underbrace{\omega'^2-\vec{k}'^2}_{\doteq 0}$$

$$d\Gamma=\frac{\overline{|\mathcal{M}|}^2}{2E}\,dQ=$$

$\leftarrow \Theta(\omega)\doteq 1$

$$=\frac{1}{2m_\mu}32G_F^2m_\mu^2(m_\mu-2\omega')\frac{1}{(2\pi)^5}\frac{d^3p'}{2E'}\cdot\frac{d^3k'}{2\omega'}\delta\left((p-p'-k')^2\right)$$

$$d^3p'=4\pi E'^2dE'$$
$$d^3k'=2\pi\,\omega'^2d\omega'\,d(\cos\vartheta)$$

ϑ: kot med elektronom in $\overline{\nu}_e$, os z v smeri e^-

$$\delta\left((p-p'-k')^2\right)=\dots=\delta\left(m_\mu^2-2m_\mu E'-2m_\mu\omega'+2E'\omega'(1-\cos\vartheta)\right)$$

$$=\delta\left(\dots+2E'\omega'\cos\vartheta\right)=\frac{1}{2E'\omega'}\delta\left(\dots+\cos\vartheta\right)$$

$$d\Gamma=\frac{G_F^2}{2\pi^3}m_\mu\,\omega'(m_\mu-2\omega')\,dE'\,d\omega'$$

$$\cos\vartheta=\frac{m_\mu^2-2m_\mu E'-2m_\mu\omega'}{2E'\omega'}+1$$

$$-1\leq\cos\vartheta\leq 1$$

$$-2\leq\frac{m_\mu^2-2m_\mu E'-2m_\mu\omega'}{2E'\omega'}\leq 0$$

$$\omega'(2E'-m_\mu)\geq\frac{m_\mu}{2}(2E'-m_\mu)\qquad\boxed{\omega'\geq\frac{m_\mu}{2}-E'}$$

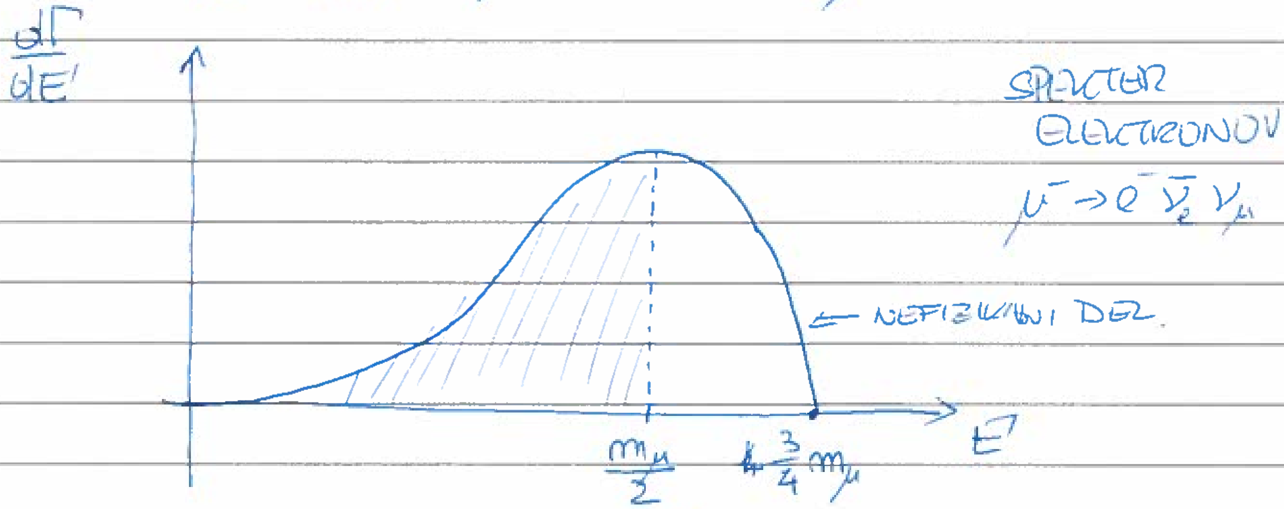
ČE BI BILA PRI RAZPADU DVA DELCA V KONČNEM STANJU (BREZMASNA) $E' = \frac{m_\mu}{2}$

3 DELCI → $E' \leq \frac{m_\mu}{2}$

⇒ LEVA MEJA → $\boxed{\omega' \leq \frac{m_\mu}{2}}$

$$d\Gamma = \frac{G_F^2}{2\pi^3} m_\mu dE' \int_{\frac{m_\mu}{2}-E'}^{\frac{m_\mu}{2}} \omega'(m_\mu - 2\omega') d\omega' =$$

$$\frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)$$



$$\Gamma = \int \frac{d\Gamma}{dE'} dE' = \frac{G_F^2}{12\pi^2} \int^{\frac{m_\mu}{2}} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right) dE' =$$

$$= \cancel{G_F^2 m_\mu^5}$$

$$\boxed{\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}}$$

$$\boxed{\tau_\mu = \frac{\hbar}{\Gamma} = 2{,}1\,\mu s}$$

RAZPADNI ČAS MIONA → G_F

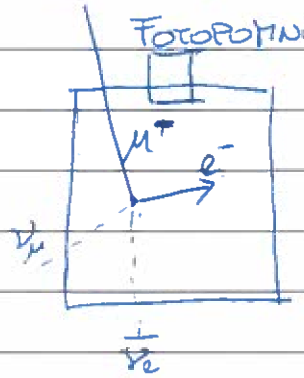
POSKUS

POSKUS (FIZIKALNI EKSPERIMENTI 1)

FOTOPOMNOŽEVALKA

200 l

TEKOČI SCINTILATOR

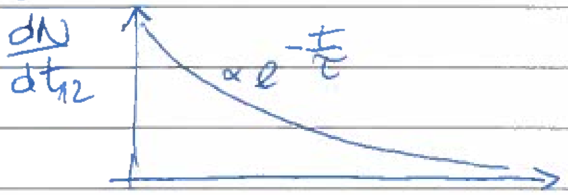


$pp \to pp\pi^+\pi^-$

$\pi^+ \to \mu^+ \nu_\mu$

μ^- SVETLOBNI SUNEK (1)

e^- — " — (2)



RAZVEJITVENO RAZMERJE
(BRANCHING FRACTION, BR)

$\mu^- \to e^- \bar{\nu}_e \nu_\mu \qquad \Gamma_1(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$

$$\Gamma_\mu = \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) + \underbrace{\Gamma_2 + \Gamma_3 + \dots}_{\text{OSTALI RAZPADNI KANALI}}$$

$$BR(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma_\mu}$$

RAZVEJITVENO RAZMERJE ZA DOLOČEN RAZPAD

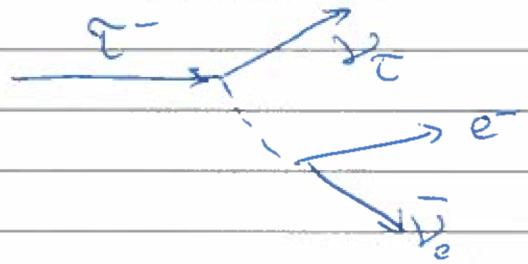
PRI TEM KONKRETNEM PRIMERU (MION):

$$\Gamma_2 + \Gamma_3 + \dots = 0$$

$$\Gamma_\mu = \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$$

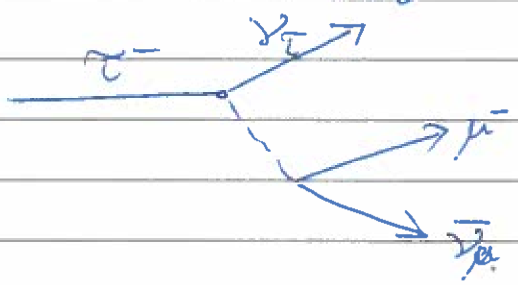
RAZPADNA ŠIRINA ZA LEPTON τ

EDEN OD RAZPADOV TAK KOT PRI MIONU

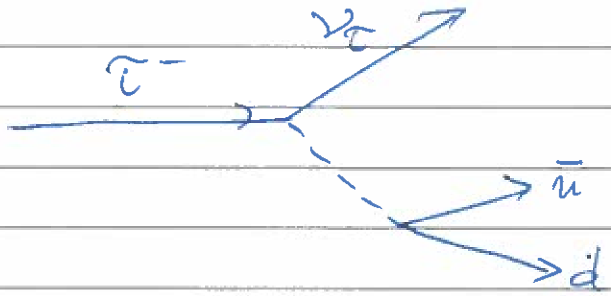


$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \frac{G_F^2 m_\tau^5}{192\pi^3}$$

$$m_\tau \doteq 1{,}8 \text{ GeV}$$



$$\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = \frac{G_F^2 m_\tau^5}{192\pi^3}$$



$$\Gamma(\tau^- \to \nu_\tau \bar{u} d) \sim 3 \frac{G_F m_\tau^5}{192\pi^3}$$

$$\tau^- \to \nu_\tau \pi^-, \; \nu_\tau \pi^- \pi^0, \; \ldots$$

$$\Gamma_\tau = \sum \Gamma_i \doteq 5\, \Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$$

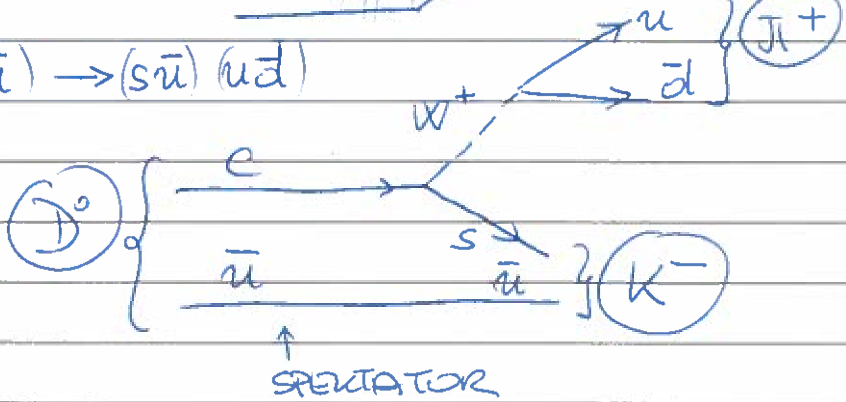
$$\boxed{\frac{\tau_\tau}{\tau_\mu} = \frac{1}{5}\left(\frac{m_\mu}{m_\tau}\right)^5}$$

DN τ_τ

ŠIBKI RAZPADI MEZONOV

$D^0 \to K^- \pi^+$

$(c\bar{u}) \to (s\bar{u})(u\bar{d})$



NAIVNO: $\Gamma(D^0 \to K^-\pi^+) \propto m_D^5$

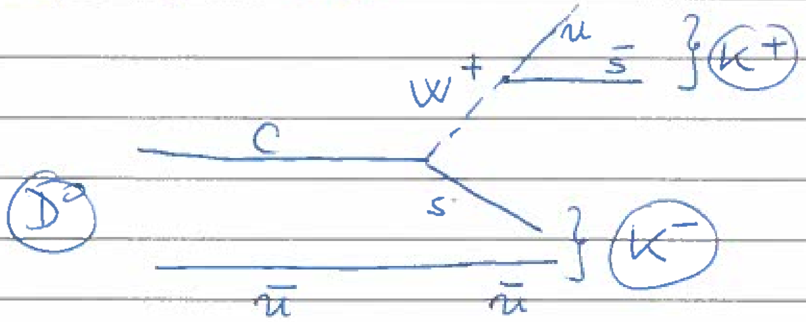
(ČE KONČNI DELCI Z ZANEMARLJIVIMI MASAMI)

$m_{D^0} \sim 1{,}86\ \mathrm{GeV}$ $\quad m_K \sim 0{,}5\ \mathrm{GeV}$

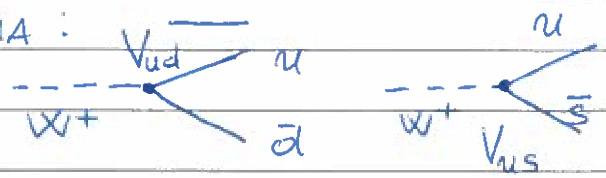
$m_\pi \sim 0{,}14\ \mathrm{GeV}$

BOLJŠE: $\Gamma(D^0 \to K^-\pi^+) \propto (m_{D^0} - m_{K^-})^5$

KAJ PA RAZPAD $D^0 \to K^- K^+$?



RAZLIKA MED RAZPADOMA:

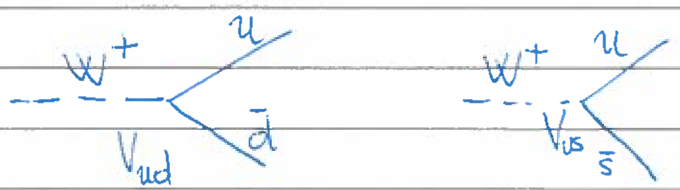


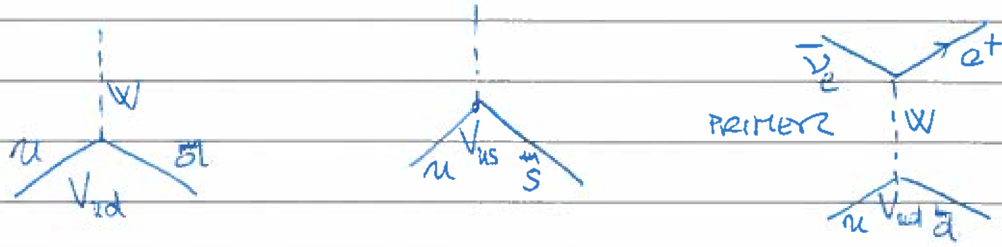
V_{ud}, V_{us} ELEMENTA MATRIKE

MATRIKA CABIBBA-KOBAYASHIJA IN MASKAWE (CKM)

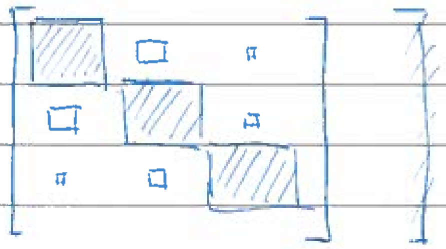
3x3 MATRIKA

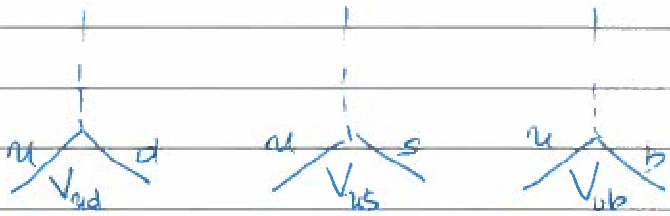
$V_{us} \sim 0{,}2\, V_{ud}$





$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} =$$





$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

ORTOGONALNE VRSTICE IN STOLPCI $\Rightarrow$ $\boxed{\text{UNITARNA MATRIKA}}$

NA PRIMER $\quad V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0$

ZA 4 KVARKE : $\begin{bmatrix} \cos\vartheta_c & \sin\vartheta_c \\ -\sin\vartheta_c & \cos\vartheta_c \end{bmatrix}$ $\quad \vartheta_c$ = CABIBBOV KOT

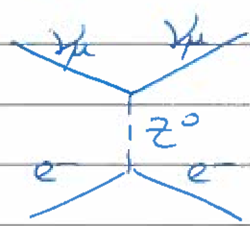
u d s c

ORTOGONALNA MATRIKA $\quad \sin\vartheta_c \sim 0{,}22$

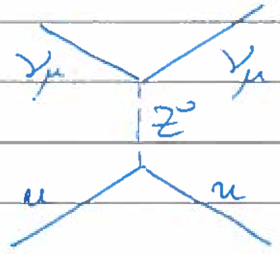
NEVTRALNI ŠIBKI TOK

NOSILEC ŠIBKE SILE Z^0

PRIMER



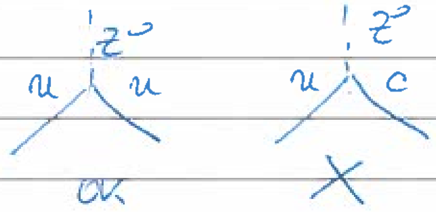
$\nu_\mu e^- \to \nu_\mu e^-$



$$J_\mu^{NC} = \bar{u}_f \gamma_\mu (C_V - C_A \gamma^5) u_i$$

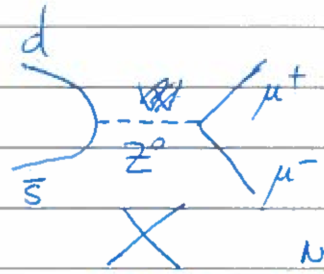
	ν	e^-, μ^-, τ^-	u, c, t	d s b
C_V	1	-0,03	0,19	0,34
C_A	1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

NEVTRALNI TOKOVI NE SPREMINJAJO OKUSA DELCA



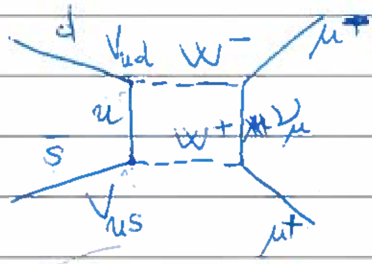
$K^0 \rightarrow \mu^+ \mu^-$
$d\bar{s}$

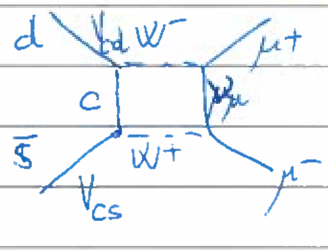
$BR(K^0 \to \mu^+\mu^-) \doteq 9 \cdot 10^{-9}$



NE GRE

ZARADI





$V_{ud} = \cos\vartheta_c \quad V_{us} = \sin\vartheta_c \quad V_{cd} = -\sin\vartheta \quad V_{cs} = \cos\vartheta_c$

$\mathcal{M}_1 \propto V_{ud} V_{us} = \cos\vartheta_c \sin\vartheta$

$\mathcal{M}_2 \propto V_{cs} V_{cd} = \cos\vartheta_c (-\sin\vartheta_c)$

$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 \sim 0 \Rightarrow BR(K^0 \to \mu^+\mu^-)$ ZELO MAJHEN

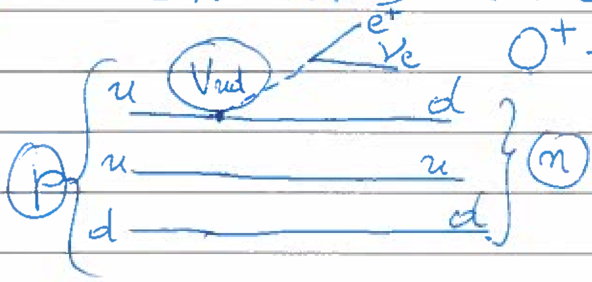
MEHANIZEM GIM

3x3 MATRIKA CKM

$$V_{CKM} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix},$$

POT DO MATRIČNIH ELEMENTOV

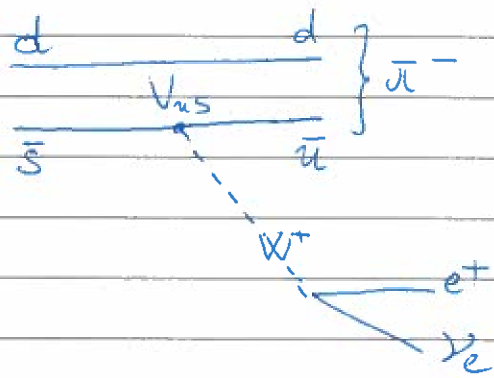
V_{ud} BETA RAZPAD (SUPERDOVOLJENI RAZPADI) $0^+ \to 0^+$



$$\frac{1}{\tau} = \frac{\cancel{\hbar}\Gamma}{\hbar} \propto |V_{ud}|^2$$

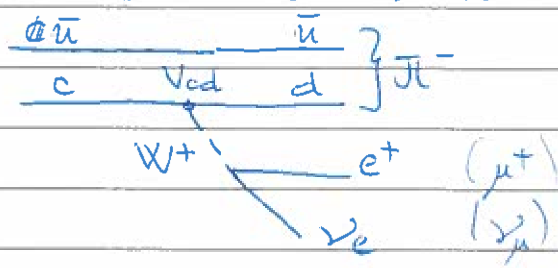
↳ IZRAČUNAM $(G_F^2)_\beta = (G_{F_\mu}^2) \cdot |V_{ud}|^2$

V_{us} RAZPAD $K^0 \to \pi^- e^+ \nu_e$



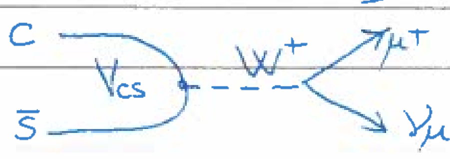
BR $\propto |V_{us}|^2$

V_{cd} RAZPAD $D^0 \to \pi e \nu_e$



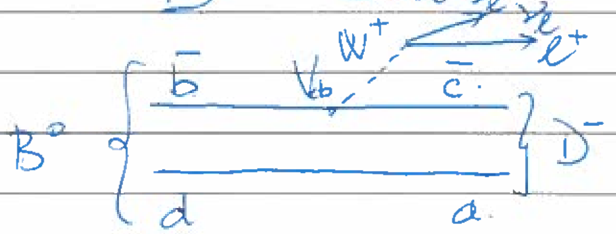
BR $\propto |V_{cd}|^2$

V_{cs} LEPTONSKI RAZPAD D_s^+ $\quad D_s^+ \to \mu^+ \nu_\mu$



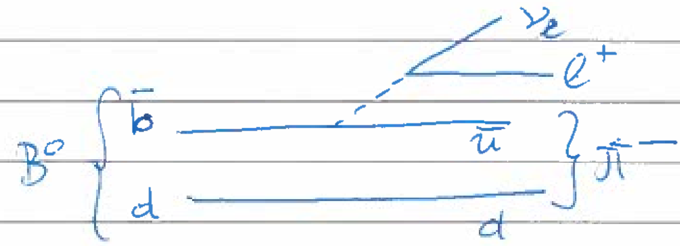
BR $\propto |V_{cs}|^2$

V_{cb} $\qquad B^0 \to D^- \ell^+ \nu_\ell \qquad \ell = e, \mu$



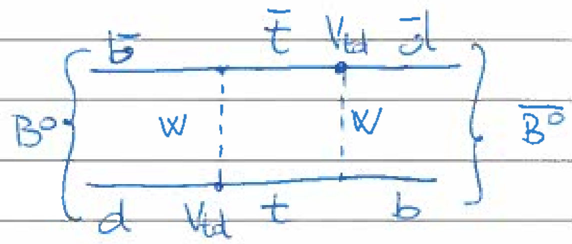
$BR \propto |V_{cb}|^2$

V_{ub} $\qquad B^0 \to \pi^- \ell^+ \nu_\ell$



$BR \propto |V_{ub}|^2$

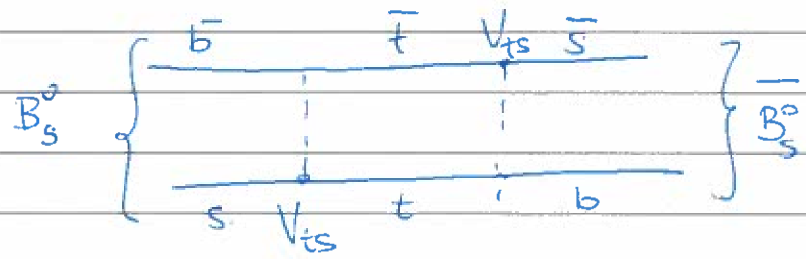
V_{td} IN V_{ts} IZ MEŠANJA MEZONOV B IN B_s



$B^0 \leftrightarrow \bar{B}^0$

MEŠANJE

FREKVENCA MEŠANJA $\propto |V_{td}|^2$



$B_s^0 \leftrightarrow \bar{B}_s^0$

FREKVENCA MEŠANJA $\propto |V_{ts}|^2$

WOLFENSTEINOVA PARAMETRIZACIJA

$$
\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)
$$

$$\lambda = \sin\vartheta_c = 0{,}22$$

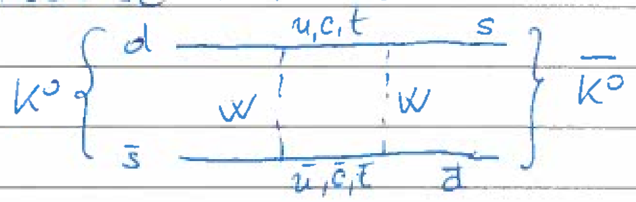
$$\lambda = 0.22453 \pm 0.00044\,, \qquad A = 0.836 \pm 0.015\,,$$
$$\bar{\rho} = 0.122^{+0.018}_{-0.017}\,, \qquad \bar{\eta} = 0.355^{+0.012}_{-0.011}\,.$$

MEŠANJE PRI NEUTRALNIH MEZONIH

K^0, D^0, B^0, B_s^0 ← 2005 CDF FERMILAB

K^0: 50-ta; D^0: BELLE, BABAR 2007; B^0: 1987 ARGUS

MEŠANJE PRI KAONIH



$$K^0 \Leftrightarrow \overline{K^0}$$

$$\psi(K^0) \neq A\, e^{\frac{iEt}{\hbar}} \underbrace{e^{-\frac{\Gamma t}{2\hbar}}}_{=e^{-t/\tau}}$$

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K^0}\rangle\right) \qquad |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K^0}\rangle\right)$$

CP PARNOST STANJ K_1 IN K_2

$$CP|K^0\rangle = |\overline{K^0}\rangle \qquad CP|\overline{K^0}\rangle = |K^0\rangle$$

$$CP|K_1\rangle = \frac{1}{\sqrt{2}}\left(|\overline{K^0}\rangle + |K^0\rangle\right) = +|K_1\rangle$$

$$CP|K_2\rangle = \frac{1}{\sqrt{2}}\left(|\overline{K^0}\rangle - |K^0\rangle\right) = -|K_2\rangle$$

RAZPADI NEVTRALNI KAONOV V PIONE

$\rightarrow 2\pi$ $\qquad\qquad$ $\rightarrow 3\pi$

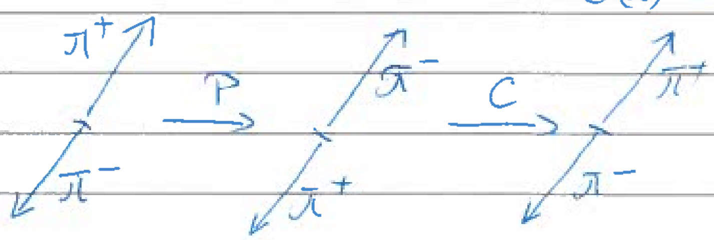
$\pi^+\pi^-$ $\qquad\qquad$ $\pi^+\pi^-\pi^0$

KAKŠNA JE CP PARNOST TEH KONČNIH STANJ?

$P(\pi) = -1$ $\qquad\qquad$ $P(\pi^+\pi^-) = P_\pi P_\pi (-1)^l = +1$

<u>$\pi^+\pi^-$</u> $\qquad\qquad$ ker $l=0$

$C(\pi) = +1$ *



$CP(2\pi) = (+1)\cdot(+1) = +1$

* $C(\pi) = +1$ se vidi iz tega, ker pion π^0 razpada v 2γ $\qquad$ $\pi^0 \rightarrow \gamma\gamma$

$$CP|\pi^+\pi^-\rangle = (+1)|\pi^+\pi^-\rangle$$

$\pi^+\pi^-\pi^0$ $\qquad\qquad$ $P(\pi^+\pi^-\pi^0) = (-1)^3 = -1$

$C(\pi^+\pi^-\pi^0) = +1$

$CP(\pi^+\pi^-\pi^0) = -1$

$CP(K_1) = +1 \qquad CP(K_2) = -1$

$CP(2\pi) = +1 \qquad CP(3\pi) = -1$

$\Rightarrow$ ČE SE OHRANJA PARNOST CP $\rightarrow$

$K_1 \rightarrow 2\pi \quad , \quad K_2 \rightarrow 3\pi$

ŽIVLJENJSKA DOBA K_1 IN K_2

$$d\Gamma = \frac{|M|^2}{2M_K} \, dQ \qquad \frac{d^3p}{(2\pi)^3}$$

$M_{K^0} \sim 0.5 \text{ GeV} \qquad M_\pi \sim 0.14 \text{ GeV}$

$2M_\pi = 0.28 \text{ GeV}$

$3M_\pi = 0.42 \text{ GeV}$

$\Rightarrow$ FAZNI PROSTOR ZA 2π RAZPAD $\gg$ 3π RAZPAD

$\tau_{K_1} \ll \tau_{K_2} \qquad \tau_{K_2} \sim 600\, \tau_{K_1}$

$0.893 \cdot 10^{-10} \qquad 0.517 \cdot 10^{-7}\,s$

$K_1 \sim K_S^0 \qquad K_2 \sim K_L^0$

POSLEDICA :

$\pi^+ p \rightarrow K^0 \Lambda$

MOČNA INT.

$|K_1\rangle = \frac{1}{\sqrt{2}} \left(|K^0\rangle + |\bar{K}^0\rangle \right) \qquad |K_2\rangle = \frac{1}{\sqrt{2}} \left(|K_0\rangle - |\bar{K}_0\rangle \right)$

$$|K^0\rangle = \frac{1}{\sqrt{2}} \left(|K_1\rangle + |K_2\rangle \right)$$

K^0 RAZPADA, K_1 Z RAZ. Č. τ_{K_1} , K_2 S τ_{K_2}

$K_S^0 \qquad \tau_S \qquad K_L^0$ S τ_L

ČE POČAKAMO DOVOLJ DOLGO (≡ ČE SMO DOVOLJ DALEČ OD TARČE, KJER JE K^0 NASTAL)

$t \gg \tau_S \Rightarrow$ VSI K_S^0 RAZPADEJO, OSTANEJO SAMO K_L^0 (K_2) $= \frac{1}{\sqrt{2}} \left(|K^0\rangle - |\bar{K}^0\rangle \right)$

1964 FITCH, CRONIN (BNL)

$K^0 \longrightarrow$ DOVOLJ DALEČ OD TARČE SAMO K_L^0

RAZPADAJO $K_L^0 \to \pi^+\pi^-\pi^0$ $(CP = -1)$

VIDELI PA SO TUDI RAZPADE $K_L^0 \to \pi^+\pi^-$ $(CP = +1)$!

$\boxed{\Rightarrow \text{ CP SE NE OHRANJA PRI ŠIBKI INTERAKCIJI!}}$

SAHAROV: RAZVOJ VESOLJA

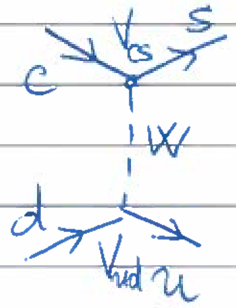
- ZGODNJE VESOLJE — DELCI + ANTIDELCI
- DANES — DELCI

POGOJI SAHAROVA

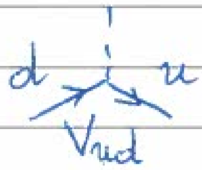
1. ANTIDELCI SE RAZLIKUJEJO OD DELCEV
   $\equiv$ KRŠENA SIMETRIJA CP
   (PARNOST CP SE NE OHRANJA)
2. BARIONSKO ŠTEVILO SE NE OHRANJA
3. RAZVOJ DALEČ OD RAVNOVESNEM STANJU

$K_L^0 \to \pi^+\pi^-$ KRŠITEV OHRANITVE PARNOSTI CP

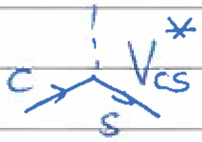
KRŠITEV CP V STANDARDNEM MODELU



$$\mathcal{M} \propto \left[\bar{u}_u \gamma^\mu (1-\gamma^5) u_d\right] \cdot V_{ud}\, V_{cs}^* \left[\bar{u}_s \gamma_\mu (1-\gamma^5) u_c\right]$$



$-\frac{1}{3} \to +\frac{2}{3}$



$+\frac{2}{3} \to -\frac{1}{3}$

ČE ZAMENJAM DELCE Z ANTIDELCI

$$\mathcal{M}' \propto \left[\bar{u}_d \gamma^\mu (1-\gamma^5) u_u\right] \cdot V_{ud}^* \, V_{cs} \cdot \left[\bar{u}_c \gamma_\mu (1-\gamma^5) u_s\right]$$

$\mathcal{M} \propto V_{ud} V_{cs}^*$ $\qquad$ $\mathcal{M}' \propto V_{ud}^* V_{cs}$

ČE CKM REALNA MATRIKA – AMPLITUDA ZA PROCES Z DELCI = AMPL. ZA PROCES Z ANTIDELCI $\Rightarrow$ CP NI KRŠENA

EKSPERIMENT: CP JE KRŠENA $\to$ MATRIKA IMA KOMPLEKSNE MAT. ELEMENTE

PREJŠNJA PREDAVANJA: GOVORILI SMO O MERITVAH VELIKOSTI MATRIČNIH ELEMENTOV

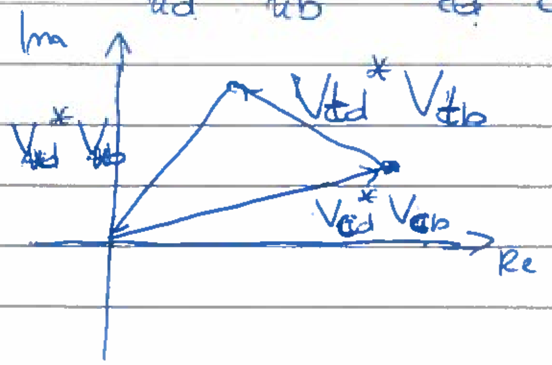
VERJETNOST $\propto |V_{ij}|^2$ ZA PROCES $j \to i$

MATRIKA CKM UNITARNA $V^\dagger V = I$

$$\begin{bmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{bmatrix} \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

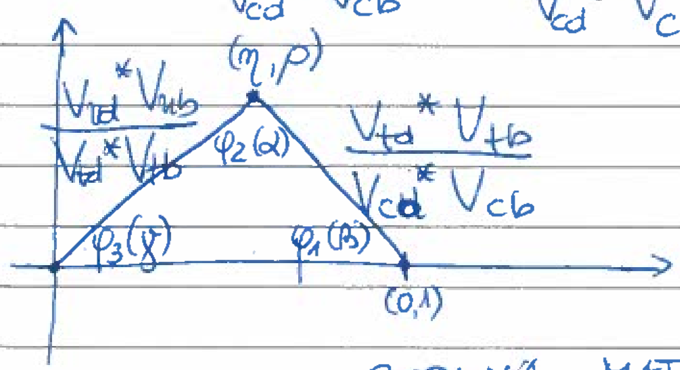
$$V_{ud}^* V_{ud} + V_{cd}^* V_{cd} + V_{td}^* V_{td} = 1$$

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$



UNITARNI TRIKOTNIK

$$1 + \frac{V_{td}^* V_{tb}}{V_{cd}^* V_{cb}} + \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}} = 0 \qquad \Big| : V_{cd}^* V_{cb}$$



REALNA MATRIKA → IZROJEN TRIKOTNIK

$\varphi_1, \varphi_2, \varphi_3$ (α, β, γ) ! ALI $0°$ ALI $180°$

MERITEV KOMPLEKSNIH ELEMENTOV V_{CKM}

→ MERITEV KOTOV UNITARNEGA TRIKOTNIKA

NAJBOLJŠI PRIMER: MERITEV KRŠITVE SIMETRIJE CP PRI RAZPADIH $B^0 \to J/\psi K_S^0$

$$B^0 \to J/\psi\, K_S^0$$

$$K_S^0 \to \pi^+\pi^-$$

$$J/\psi \to e^+e^-,\ \mu^+\mu^-$$

PRIMERJAMO Z RAZPADOM $\bar{B}^0 \to \underbrace{J/\psi\, K_S^0}_{\text{LASTNO STANJE CP}}$

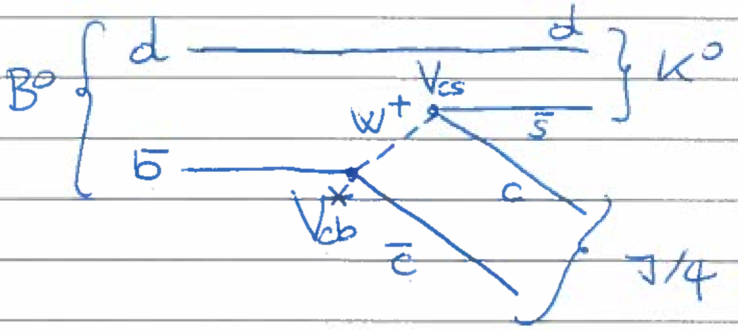
$B^0 \to$ } ISTO KONČNO STANJE

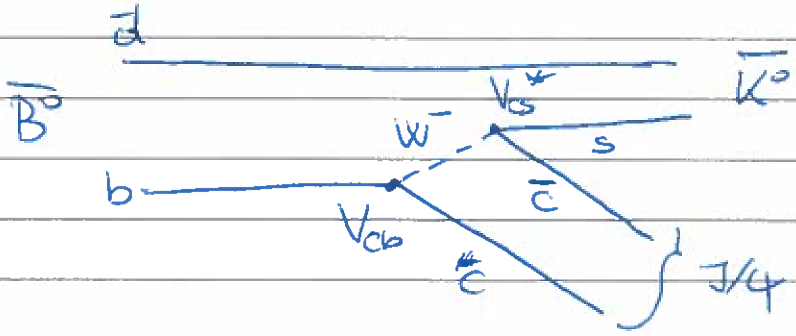
$\bar{B}^0 \to$ } LASTNO STANJE CP

RAZLIKA MED RAZPADI B^0 IN $\bar{B}^0$ →

→ SIMETRIJA CP JE KRŠENA

MERITEV RAZLIKE → KOT UNITARNEGA TRIKOTNIKA.





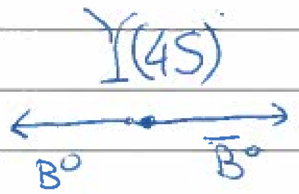
MERITEV RAZLIKE RAZPADOV $B^0 \to J/\psi K^0_S$
IN $\bar{B}^0$ V ISTI KANAL
$\Rightarrow$ KOT φ_1 (β)

KAKO IZVEDEMO TO MERITEV?
1.) NAREDIMO B^0 OZ. $\bar{B}^0$
2.) IZMERIMO RAZPADNE PRODUKTE
$J/\psi \to \mu^+\mu^-$, $K^0_S \to \pi^+\pi^-$

3.) UGOTOVIMO, ALI JE RAZPADEL B^0
ALI $\bar{B}^0$
4.) IZMERIMO ČAS RAZPADA (= RAZLIKA ČASOV)

$e^+e^- \longrightarrow \Upsilon(4S)$ VEZANO STANJE $b\bar{b}$
$\hookrightarrow B^+B^-$, $B^0\bar{B}^0$

$\Upsilon(4S)$: VRTILNA KOLIČINA $J=1$, B^0, $\bar{B}^0$: $J=0$
OHRANITEV VRT. KOL. $\Rightarrow$ $\ell=1$

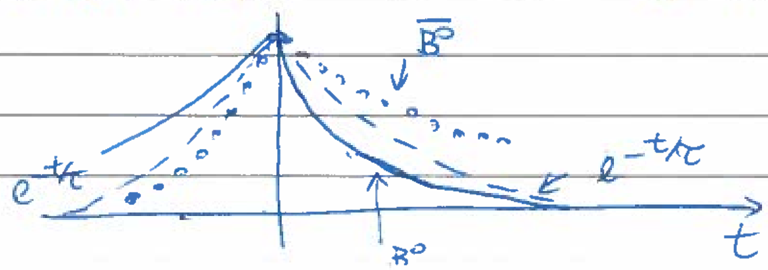


KVANTNO PREPLETENO STANJE

$B^0 \leftrightarrow \bar{B}^0$ ČE PROSTA
ČE BI SE EDEN OD OBEH SPREMENIL, RECIMO
$B^0 \to \bar{B}^0$: $(B^0\bar{B}^0) \to (\bar{B}^0\bar{B}^0)$

$(\bar{B}^0\bar{B}^0) \xrightarrow{P} (\bar{B}^0\bar{B}^0)$, KER PA JE $\ell=1$
$\Rightarrow \psi(\bar{B}^0, \bar{B}^0) \to -\psi(\bar{B}^0, \bar{B}^0)$
TO PA JE PREPOVEDANO, KER STA BOZONA

ČASOVNI POTEK RAZPADOV B^0 $(\bar{B}^0)$

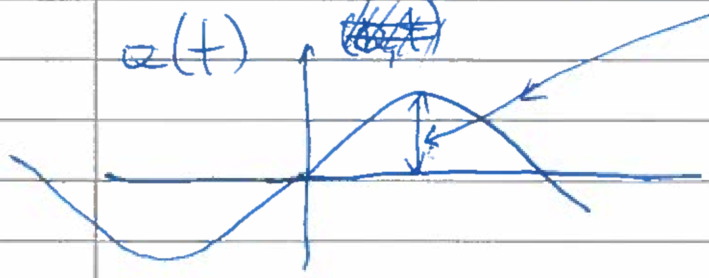


$$\frac{N(\bar{B}^0_{,t}) - N(B^0_{,t})}{N(\bar{B}^0_{,t}) + N(B^0_{,t})} = a$$

$$a = \frac{N(\bar{B}^0_1,t) - N(B^0_1,t)}{N(\bar{B}^0_1,t) + N(B^0_1,t)} \propto \sin 2\varphi \cdot \sin(\omega t)$$

$\downarrow$

$B^0 \Leftrightarrow \bar{B}^0$

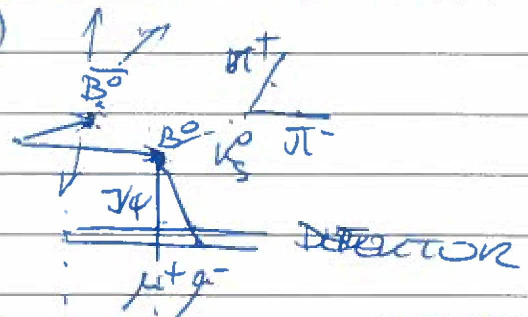


KDAJ STA RAZPADLA? RAZLIKA ~ 1ps

POSPEŠENI DELCI: IZMERIMO KOORDINATO, KJER SO RAZPADLI → POZNAM HITROST → DOLOČIM ČAS

→ e^-e^+ Z RAZLIČNIMI ENERGIJAMI ($E_{e^-} \sim 8$ GeV, $E_{e^+} \sim 3{,}5$ GeV)

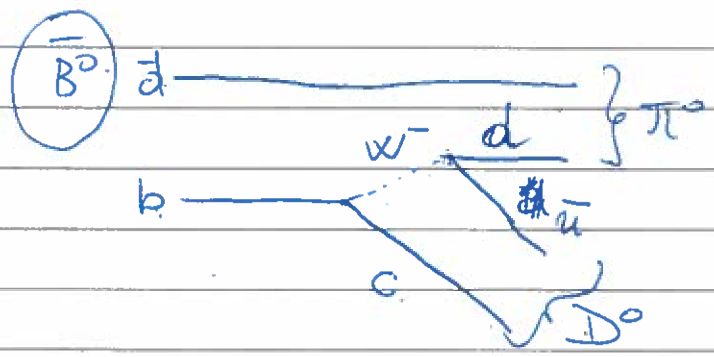
$\xrightarrow{e^-} \; \xleftarrow{e^+}$ $\quad \xrightarrow{\Upsilon(4S)}$



KOORDINATO RAZPADA DOLOČIM Z EKSTRAPOLACIJO SLEDI.

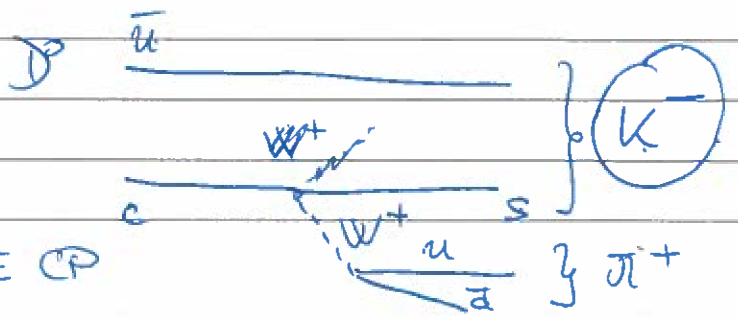
$\Delta z \to \Delta t$

$\bar{B}^0$ ALI B^0?



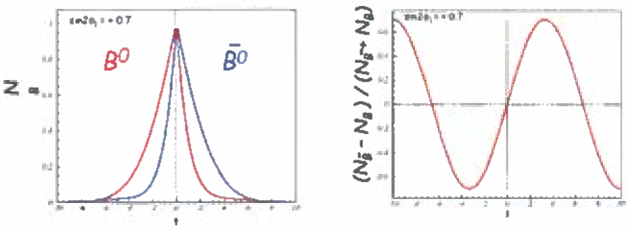
$\bar{B}^0 \to K^- \ldots$

$B^0 \to K^+ \ldots$



→ SLIDES BELLE CP

Kršitev CP: asimetrija v razpadni verjetnosti



$$a(t)=\frac{P(\bar B^0(t)\to f_{CP})-P(B^0(t)\to f_{CP})}{P(\bar B^0(t)\to f_{CP})+P(B^0(t)\to f_{CP})}=-\xi_f \sin 2\phi_1 \sin \Delta m_d t$$

$\boxed{\xi_f=\pm 1 \text{ for } CP=\pm 1}$

Meritev kršitve CP pri mezonih B

Kako izmeriti kršitev CP pri mezonih B?

Najprej jih moramo ustvariti: uporabimo reakcijo pri trku elektrona in pozitrona z dovolj veliko energijo: $e^- e^+ \to \Upsilon(4s) \to B^0 \bar B^0$

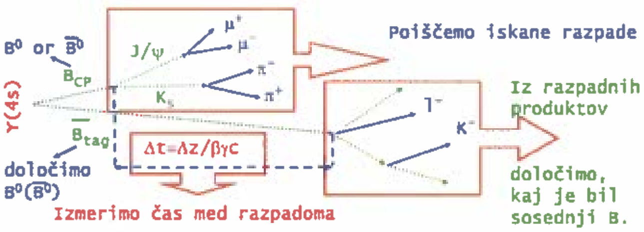
Nato počakamo, da eden od obeh B^0 razpade v stanje, za katero vemo, kakšna je njegova CP parnost (torej kako se obnaša pri simetrijski operaciji CP). Primer takega stanja je razpad

$B^0 \to J/\psi\, K_S$. Razpadna produkta naprej razpadeta:

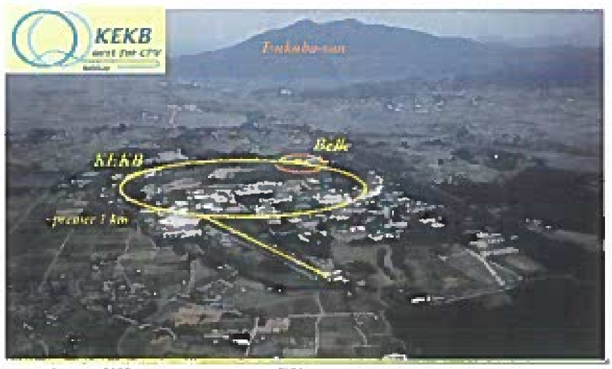
$J/\psi \to \mu^- \mu^+$ in $K_S \to \pi^- \pi^+$

Izmeriti moramo, kje se je to zgodilo, in določiti ali je v $J/\psi\, K_S$ razpadel B^0 ali njegov anti-delec $\bar B^0$ (=meritev okusa B).

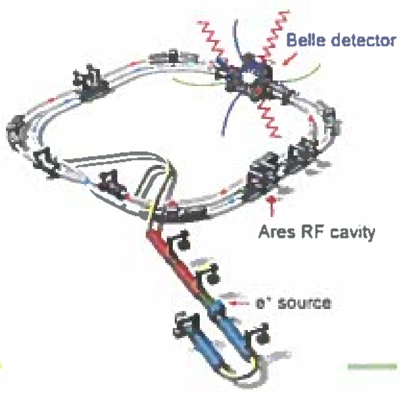
Kako merimo kršitev CP



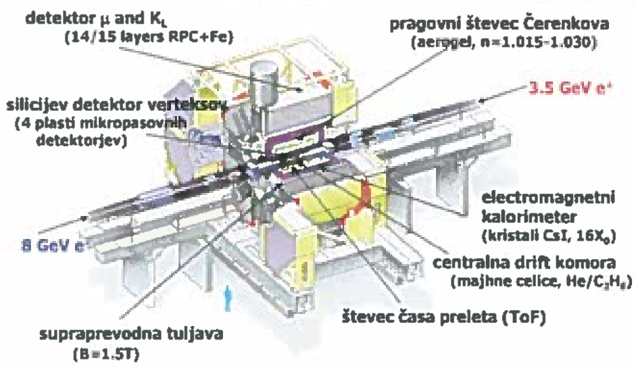
Trkalnik KEK-B in detektor Belle v Tsukubi



Trkalnik KEK-B
pospešuje elektrone in pozitrone do trka



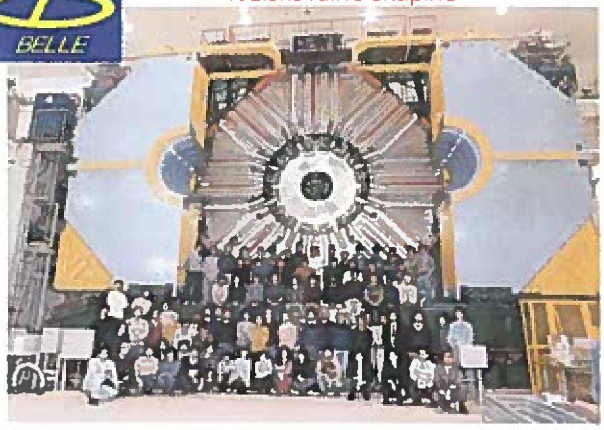
Spektrometer Belle



Detektor verteksov

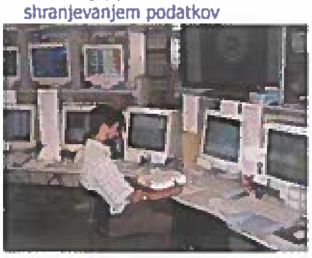
- Eden bistvenih elementov detektorja je detektor verteksa, točke, kjer je mezon B razpadel.
- Zelo občutljiv kos aparature iz 300μm debelih silicijevih plošč z gosto nanešenimi elektrodami: natančnost meritve mesta preleta nabitega delca: 10 μm!

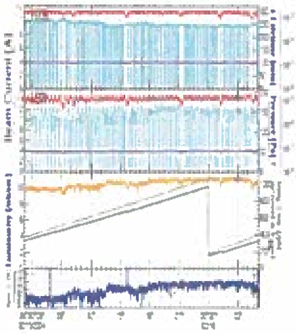
Spektrometer Belle in del raziskovalne skupine



S potrpežljivim merjenjem, dan in noč, nekaj let...

Kontrolna soba eksperimenta Belle: nadzor na vsemi komponentami detektorja, prenosom in shranjevanjem podatkov





V enem dnevu naberemo ~trikrat toliko podatkov kot v celotnem času obratovanja eksperimenta ARGUS...

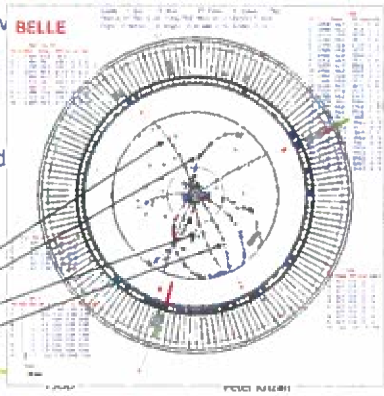
Kaj izmerimo z detektorjem?

- sledi nabitih delcev v magnetnem polju (polmer kroga je odvisen od gibalne količine delca)
- koordinate točke, od koder sledi izhajajo
- dodatne podatke o identiteti delca

$B^0 \to K_S J/\psi$

$K_S \to \pi^- \pi^+$

$J/\psi \to \mu^- \mu^+$

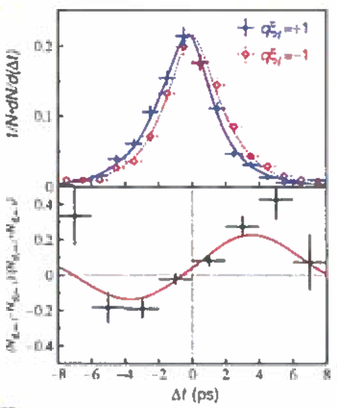


2001, rezultat meritve: CP je kršena!

Razlika med delci in antidelci:

Modra: časovni potek razpada anti-B

Rdeča: isto za B



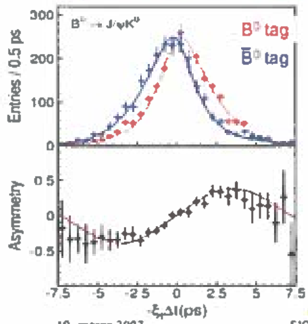
Razlika med obema porazdelitvama

→objavi v PRL in PRD imata več kot 500 citatov!

2005: $B^0 \to J/\psi K^0$

$\sin 2\phi_1 = 0.652 \pm 0.039$ (stat) ± 0.020 (syst)

C = 0.010 ± 0.026 (stat) ± 0.036 (syst)



$$a_f = S\sin(\Delta m t) + C\cos(\Delta m t)$$

Sestavljen vzorec, razpadi

- $J/\psi K_S$: N(Δt),
- $J/\psi K_L$: N(-Δt)

2001: odkritje →

2005: precizijska meritev!

ŠTEVILO PARAMETROV V_{CKM}

KOMPL. MATRIKA $N \times N$: $2N^2$ PARAMETROV

UNITARNOST: N^2 ENAČB $\rightarrow$ N^2 PARAMETROV

$$-\frac{1}{3} \rightarrow +\frac{2}{3} \qquad V_{ji}\,\bar{u}_j\,\gamma^\mu(1-\gamma^5)\,u_i$$

$$N \qquad N$$

$(2N-1)$ RELATIVNIH FAZ: POLJUBNE

ŠTEVILO PARAMETROV V_{CKM} $= N^2 - (2N-1) =$
$= (N-1)^2$

$N=2$: 1 $\begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix}$

$N=3$: 4

ZA $N=2$ V_{CKM} REALNA! $\rightarrow$ NI KRŠITVE CP

ŠELE PRI $N=3$: 4 PARAMETRI =
= 3 KOTI (RECIMO EULERJEVI)
+ 1 FAZA!

WOLFENSTEINOVA PARAMETRIZACIJA

λ, A, ρ, η $\qquad V_{ub} = A\lambda^3(\rho - i\eta)$

1964: KRŠITEV CP PRI KAONIH
(TAKRAT SO POZNALI 3 KVARKE!)

1973 KOBAYASHI, MASKAWA
V_{CKM} MORA BITI 3×3 $\rightarrow$ OBSTAJAJO
ŠE 3 VRSTE KVARKOV

1974 KVARK c, 77 b, 95 t

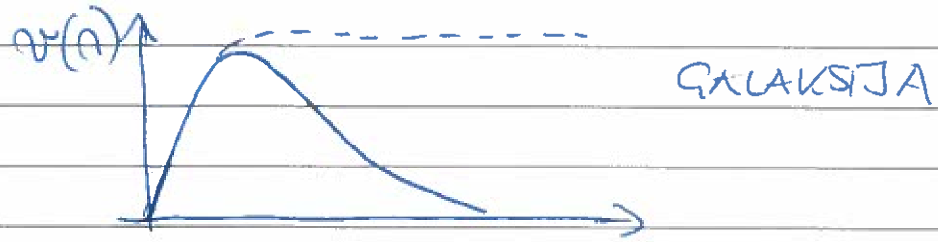
2001 KRŠITEV CP V RAZPADIH B $\rightarrow$
KOMPLEKSNA FAZA V_{CKM}

2008 NN ZA K+M.

SM IZREDNO DOBRO OPISUJE INTERAKCIJE MED OSNOVNIMI DELCI

TODA

- SM NE VSEBUJE TEMNE SNOVI



- RAZLIKA MED KOLIČINO SNOVI IN ANTI-SNOVI V VESOLJU

FD: IŠČEMO SIGNALE „NOVE FIZIKE"

→ NOVI DELCI
→ NOVE VRSTE INTERAKCIJ

1) VISOKE ENERGIJE (LHC) $pp \to X$

2) ZELO NATANČNE MERITVE PRI NIŽJIH ENERGIJAH! REDKI RAZPADI MEZONOV B^0 → ODSTOPANJE OD NAPOVEDI STANDARDNEGA MODELA

$$\left.\begin{array}{l} B^0 \to D^+ \mu^- \bar{\nu}_\mu \\ B^0 \to D^+ \tau^- \bar{\nu}_\tau \end{array}\right\} \text{KRŠITEV LEPTONSKE UNIVERZALNOSTI?}$$

ENA OD RAZLAG: LEPTOKVARKI