

Particle identification

Peter Kriz̃an

Introduction: Why Particle ID?

Efficiency and purity in particle identification

Time-of-flight

Basics: dE/dx , Čerenkov, Trans. Rad.

dE/dx measurement

Čerenkov counters

Transition radiation counters

Calorimeters

Muon and K_L detection

Introduction: Why Particle ID?

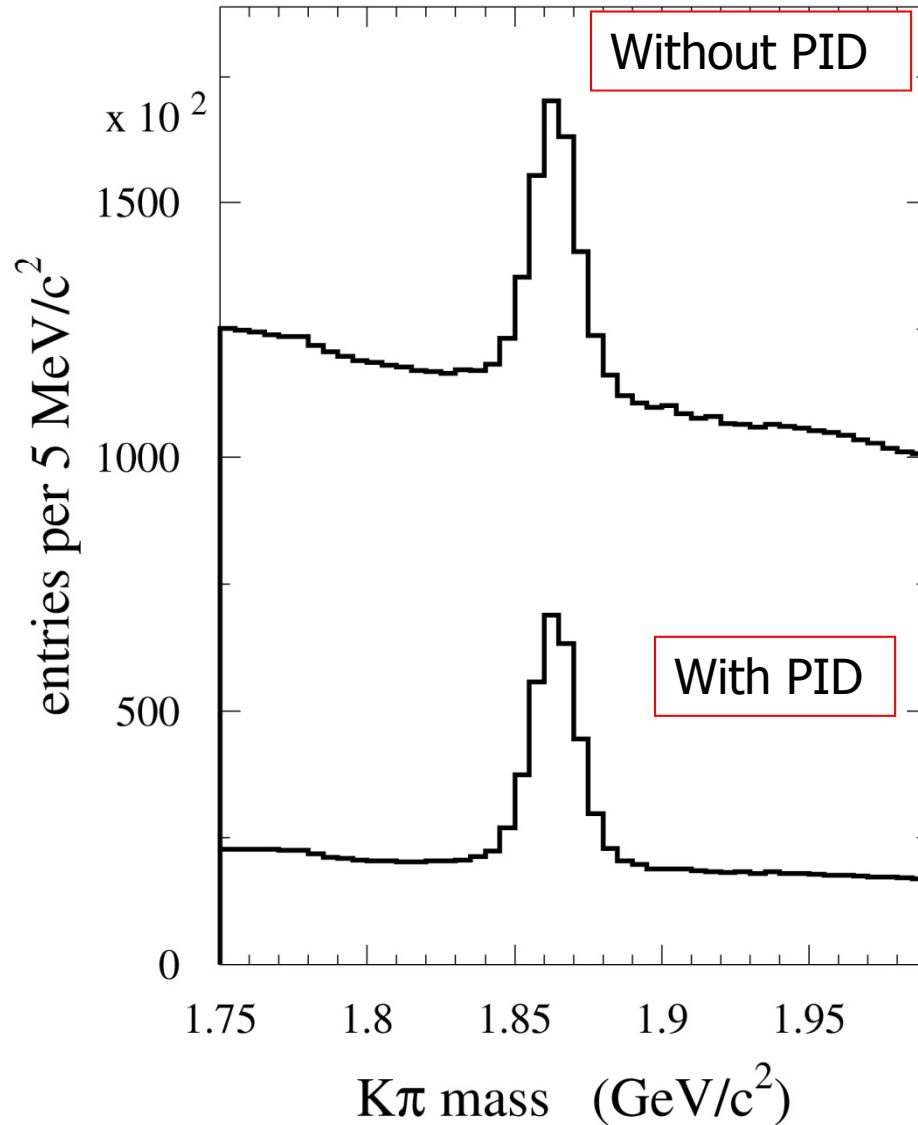
Particle identification is an important aspect of particle, nuclear and astroparticle physics experiments.

Some physical quantities in particle physics are only accessible with sophisticated particle identification (B-physics, CP violation, rare decays, search for exotic hadronic states).

Nuclear physics: final state identification in quark-gluon plasma searches, separation between isotopes

Astrophysics/astroparticle physics: identification of cosmic rays – separation between nuclei (isotopes), charged particles vs high energy photons

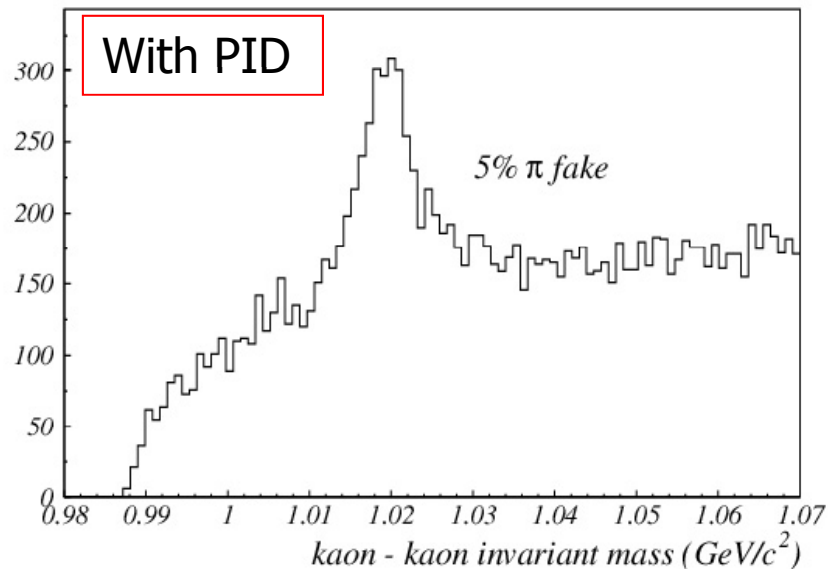
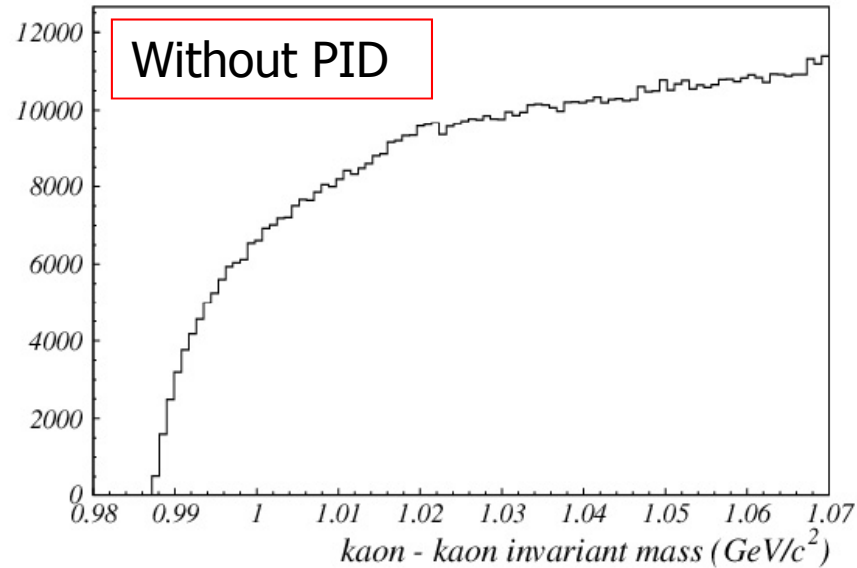
Introduction: Why Particle ID?



Example 1: BaBar

Particle identification
reduces
combinatorial
background by $\sim 5x$

Introduction: Why Particle ID?

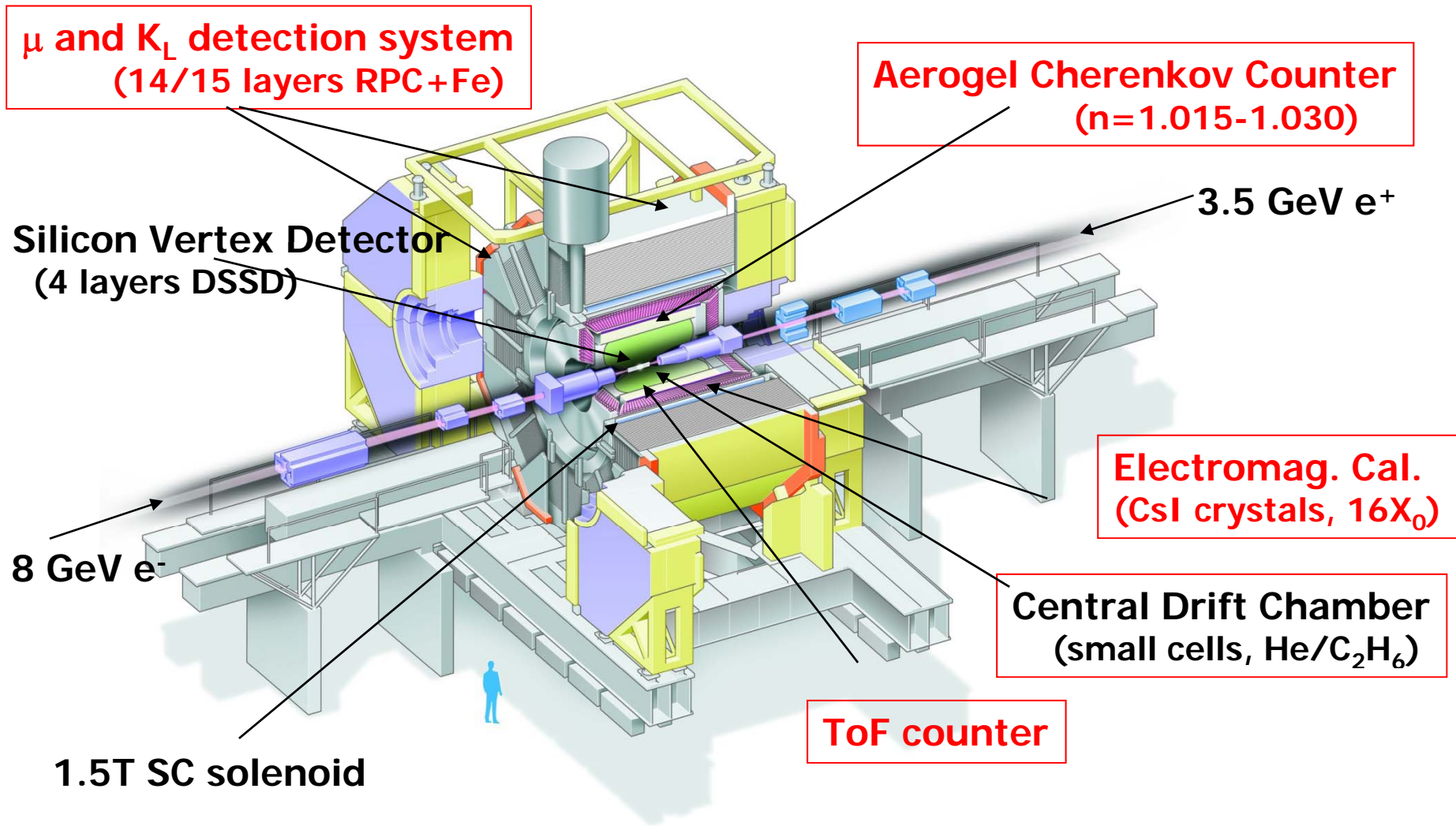


Example 2: HERA-B

K^+K^- invariant mass.

The $\phi \rightarrow K^+K^-$ decay only becomes visible after particle identification is taken into account.

Particle identification systems in Belle

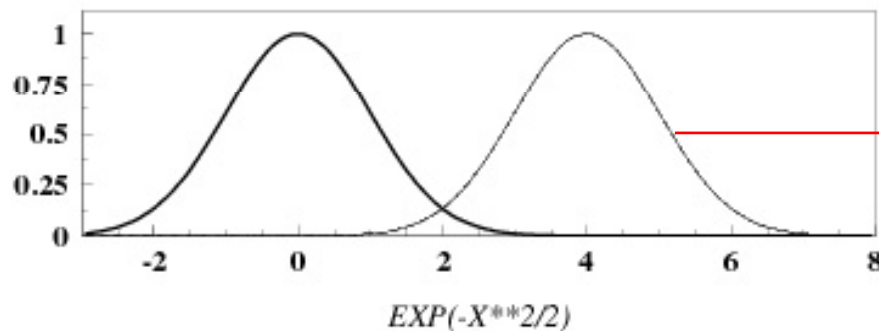
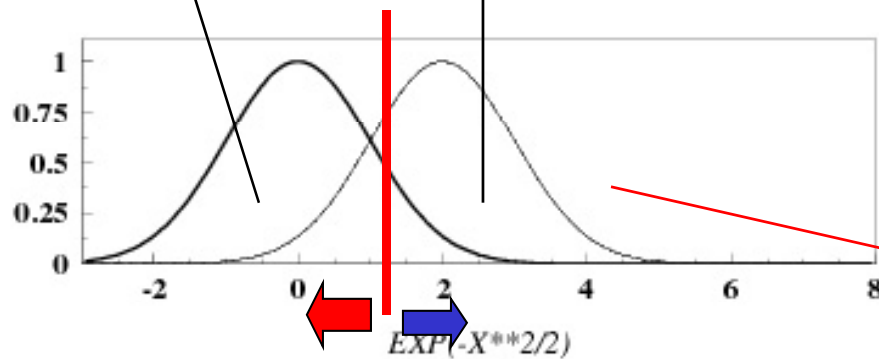


Efficiency and purity in particle identification

Efficiency and purity are tightly coupled!

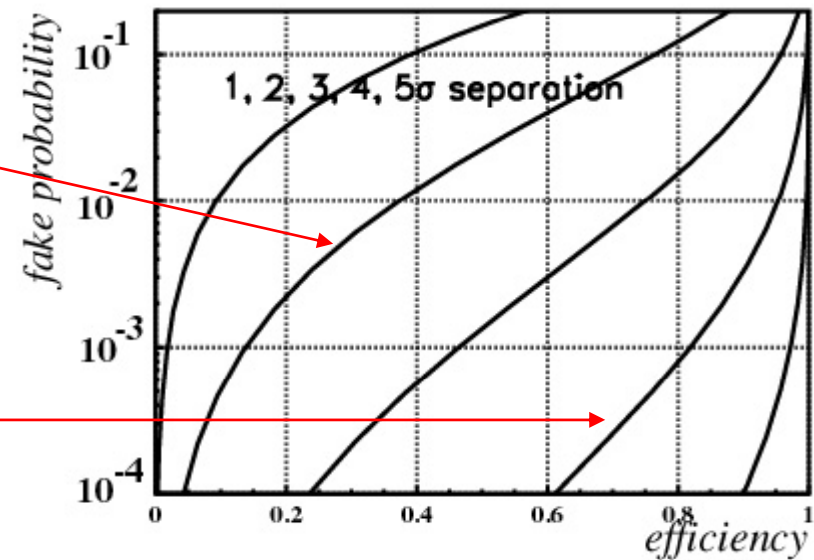
Two examples:

particle type 1 type 2



some discriminating variable

eff. vs fake probability



Identification of charged particles

Particles are identified by their **mass** or by the **way they interact**.

Determination of mass: from the relation between momentum and velocity, $p = \gamma m v$.

Momentum known (radius of curvature in magnetic field)

→ Measure velocity:

time of flight

ionisation losses dE/dx

Cherenkov angle

transition radiation

Mainly used for the identification of hadrons.

Identification through interaction: electrons and muons

Time-of-flight measurement (TOF)

Measure time difference over a known distance, determine velocity

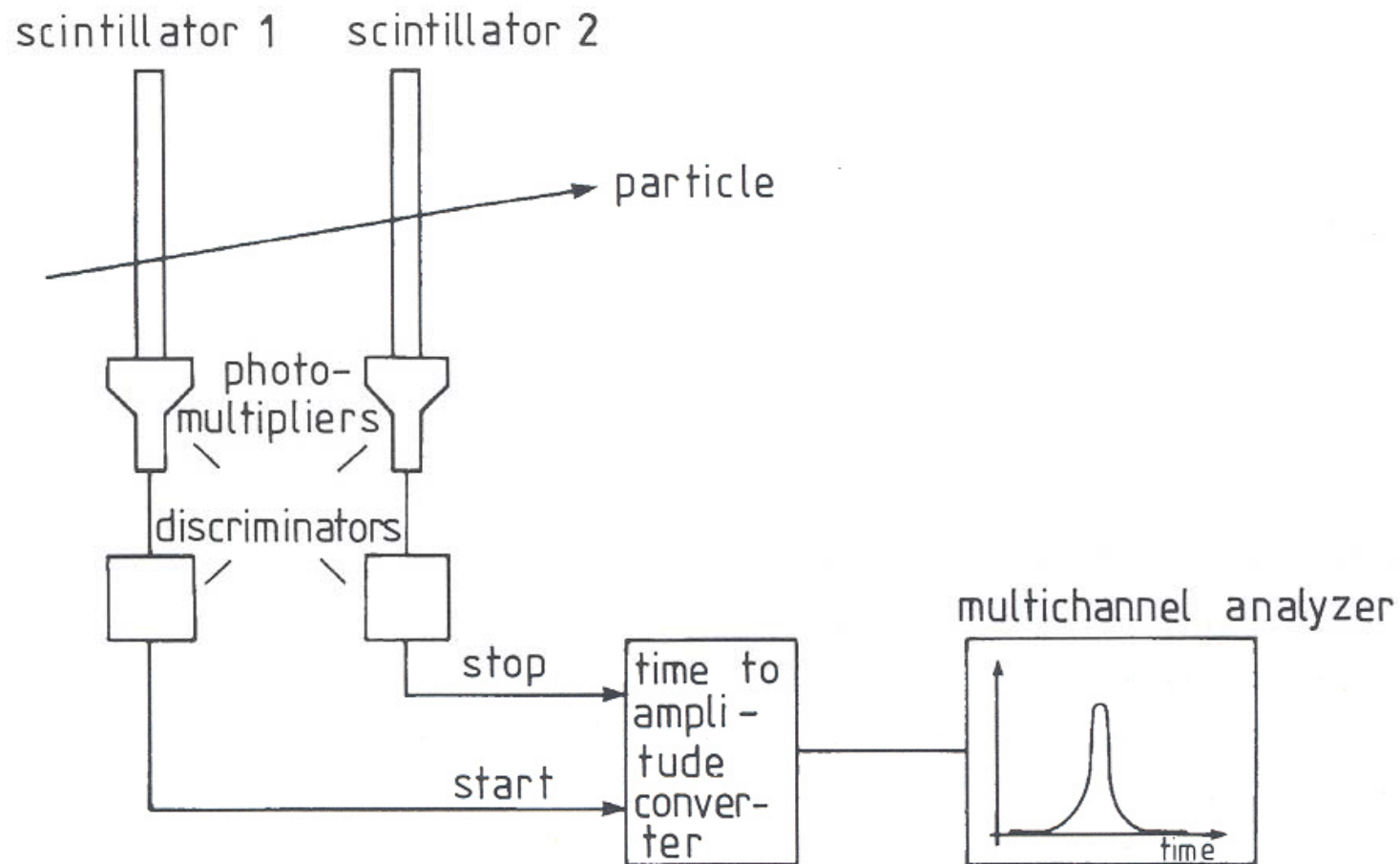
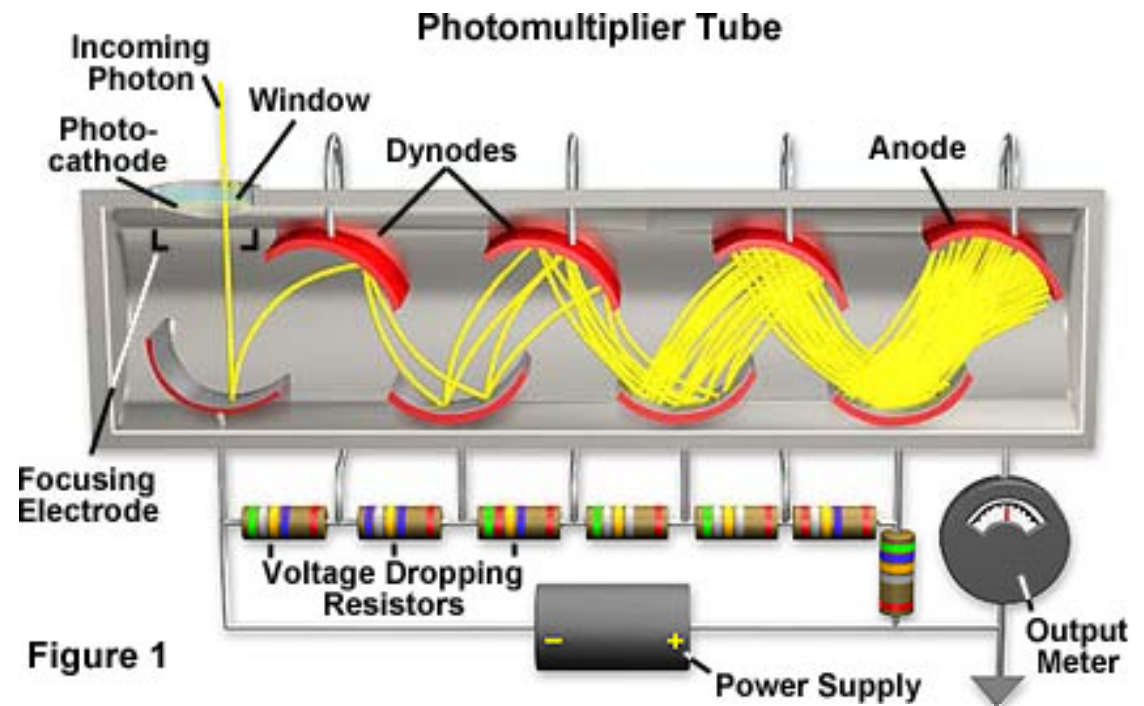


Fig. 6.5. Working principle of time-of-flight measurement.

Photomultiplier tube



Time-of-flight measurement 2

Required resolution, example:

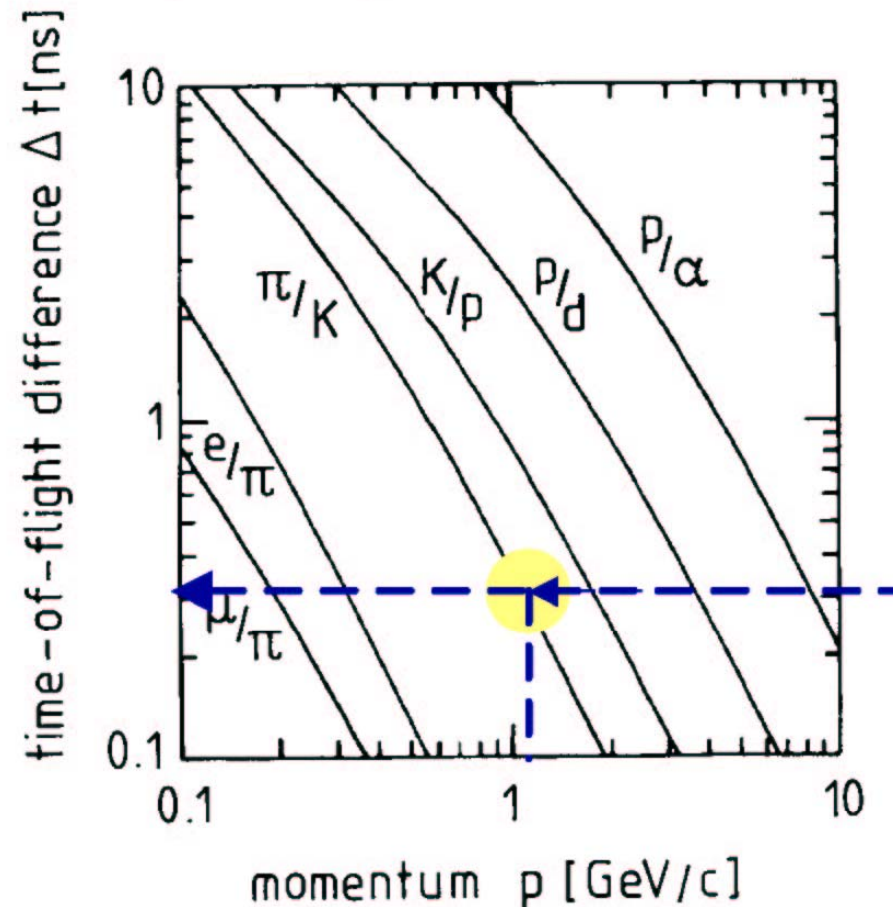
π/K difference at 1GeV/c: 300ps

For a 3σ separation need
 $\sigma(\text{TOF})=100\text{ps}$

Resolution contributions:

- PMT: transient time spread (TTS)
- Path length variation
- Momentum uncertainty

Time difference between two particle species for path length=1m



Time-of-flight measurement 3

Resolution of a PMT: transient time spread (TTS), time variation for single photons

Tubes for TOF have to be optimized for small TTS.

Main contribution after the optimisation: photoelectron time spread before it hits the first dynode.

Estimate: take two cases, one with $T=1\text{eV}$ and the other with $T=0$ after the photoelectron leaves the photocathode; take $U=200\text{V}$ and $d=10\text{mm}$

$T=1\text{eV}$: $v_0 = \sqrt{2T/m} = 0.002 c$, $a=F/m=200\text{eV}/(10\text{mm} \cdot 0.5 \cdot 10^6\text{eV}/c^2)$

$$d = v_0 t + at^2/2 \rightarrow t = \sqrt{(2d/a + (v_0/a)^2)} - v_0/a$$

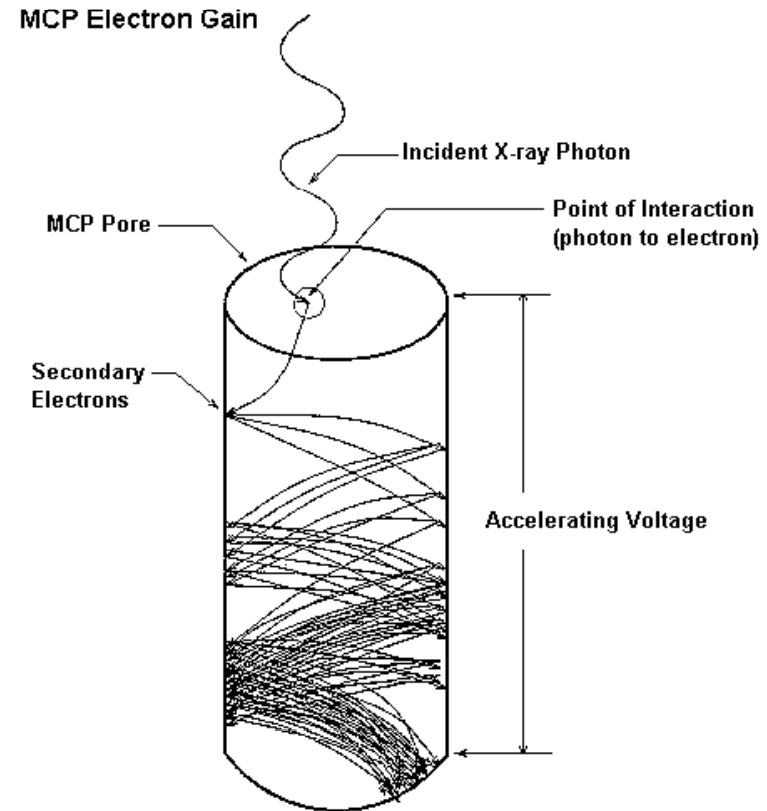
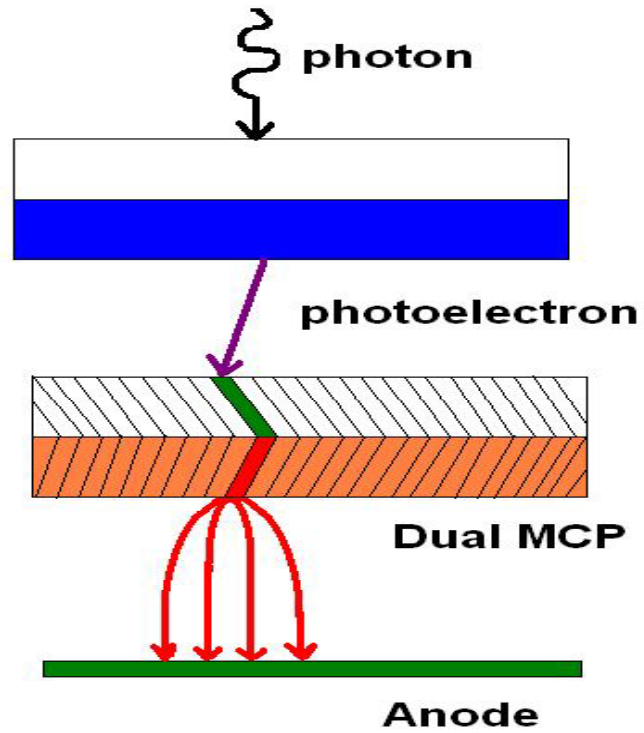
$T=0\text{eV}$: $v_0 = 0 \rightarrow t = \sqrt{2d/a} = 2.3\text{ns}$

Time difference: 170ps is a typical value.

Good tubes: $\sigma(\text{TTS}) = 100\text{ps}$

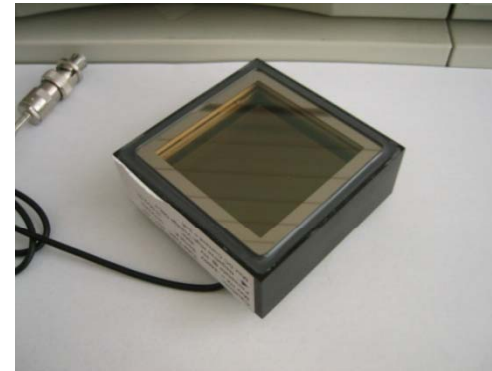
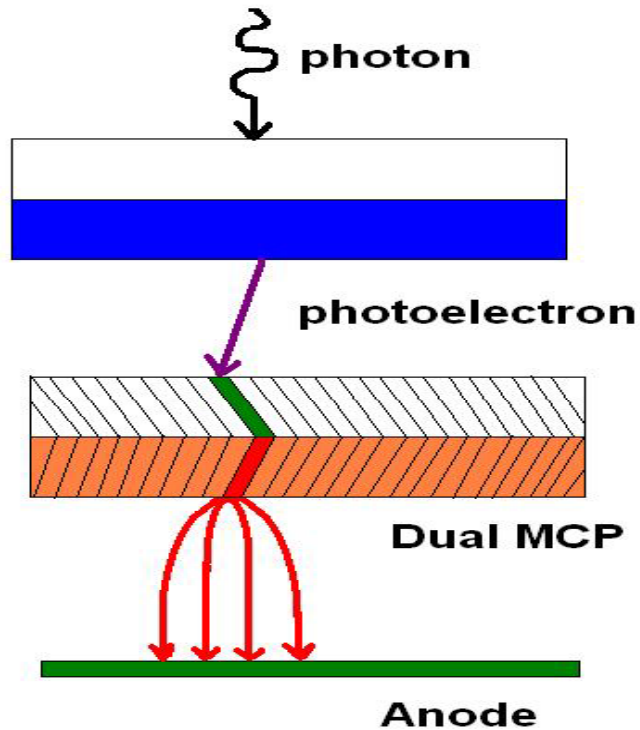
For N photons: $\sigma \sim \sigma(\text{TTS}) / \sqrt{N}$

Very fast: MCP-PMT

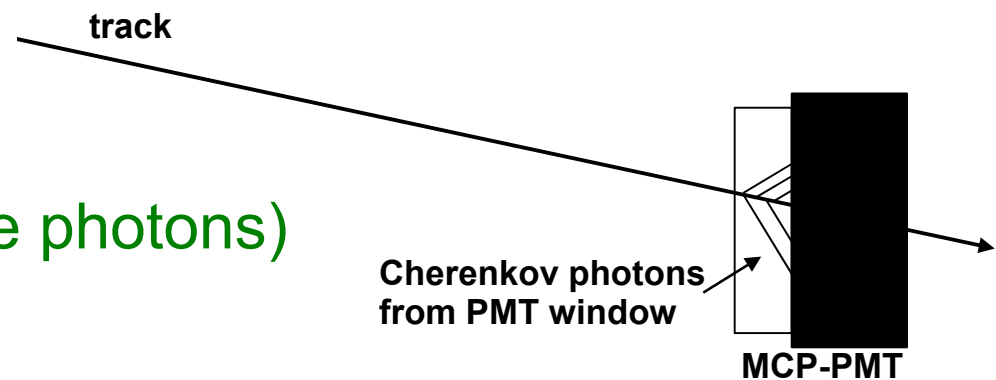


Very fast: MCP-PMT

BURLE 85011 microchannel plate (MCP) PMT: multi-anode PMT with two MCP stages



→ very fast ($\sigma=40\text{ps}$ for single photons)

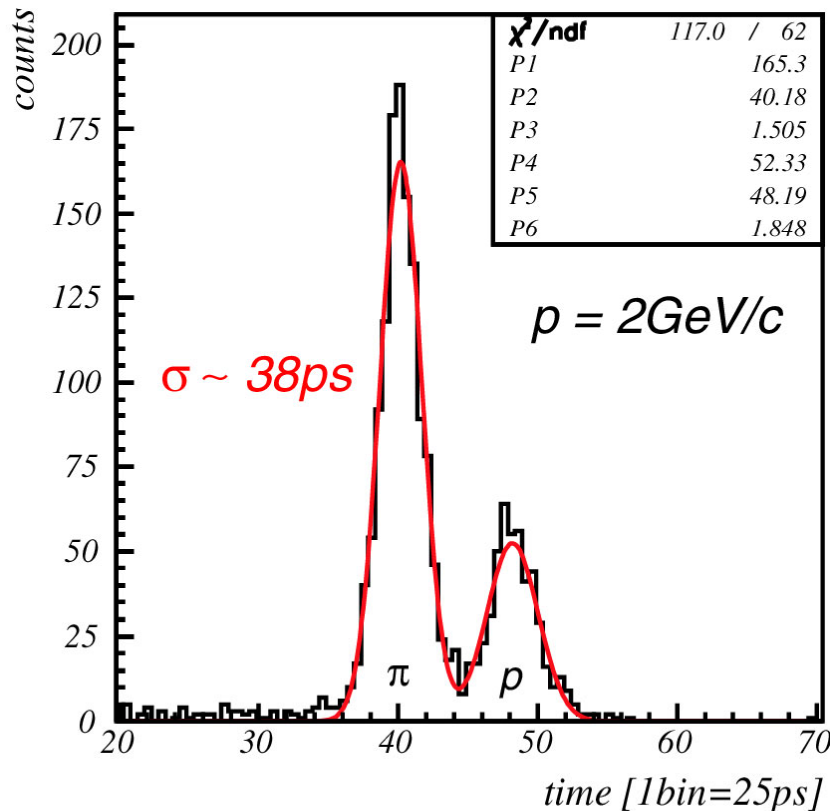




TOF capability: window photons

Expected number of detected Cherenkov photons emitted in the PMT window (2mm) is **~15**

→ Expected resolution **~35 ps**



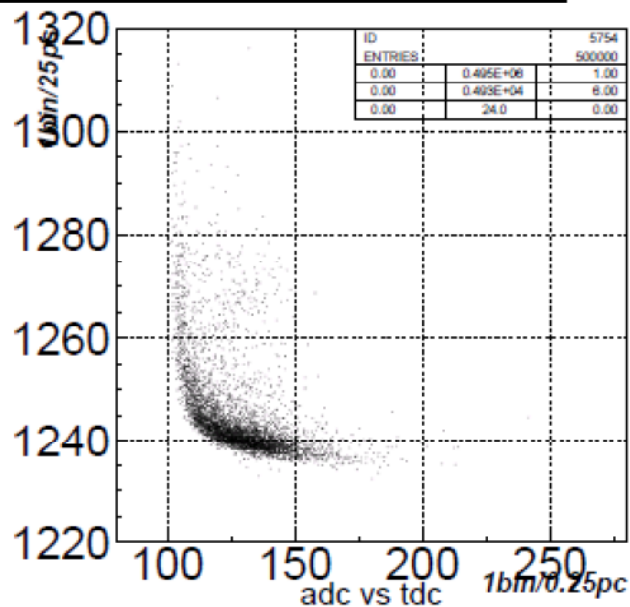
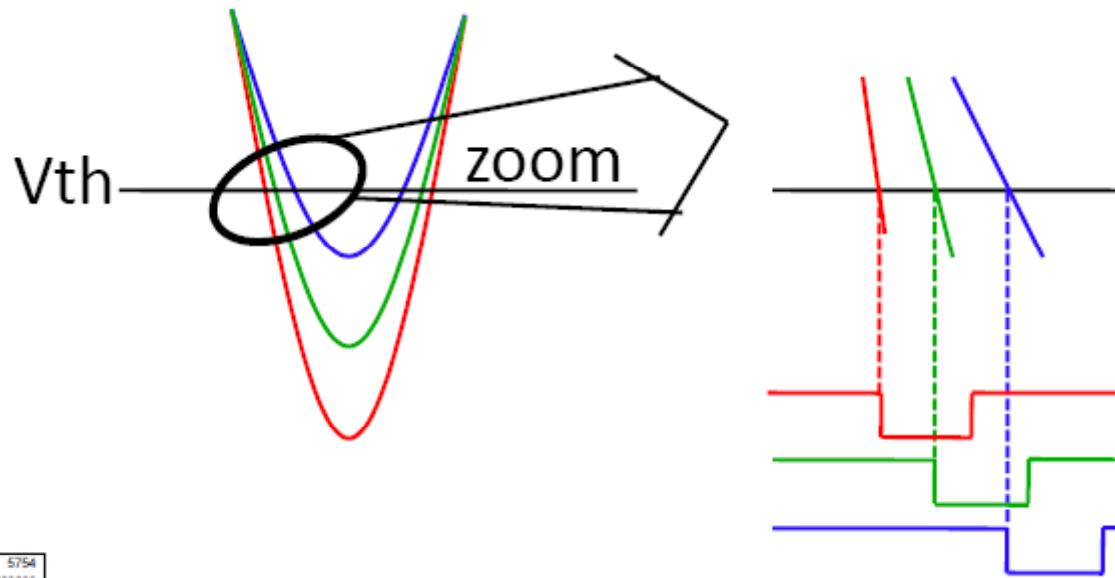
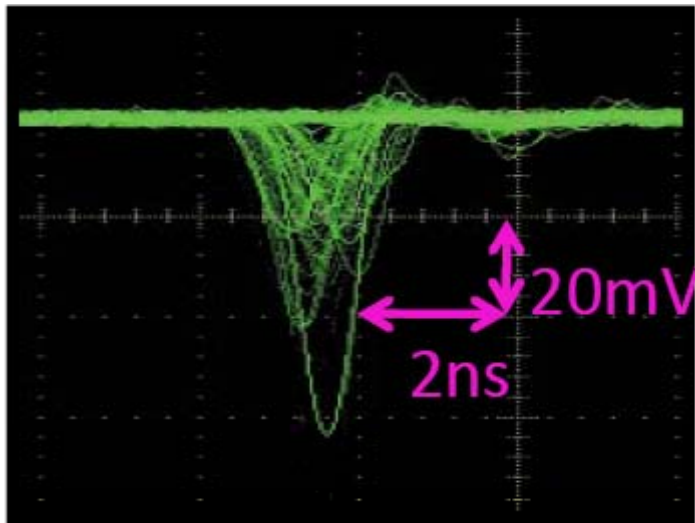
TOF test with pions and protons at $2 \text{ GeV}/c$.

Distance between start counter and MCP-PMT is 65cm

→ In the real detector $\sim 2\text{m}$

→ 3x better separation

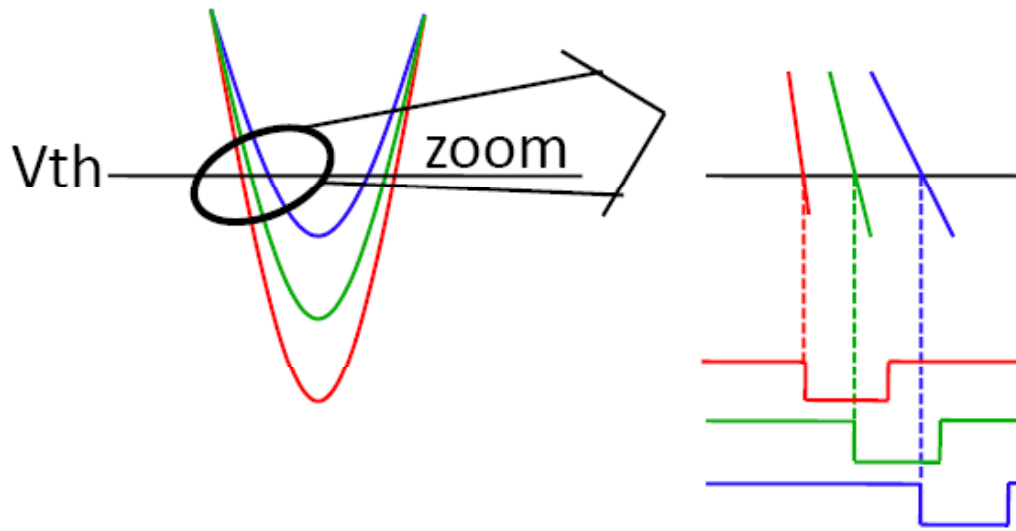
Read out: time walk with a leading edge discriminator



Variation of time determined with a leading edge discriminator: **smaller pulses give a delayed signal**

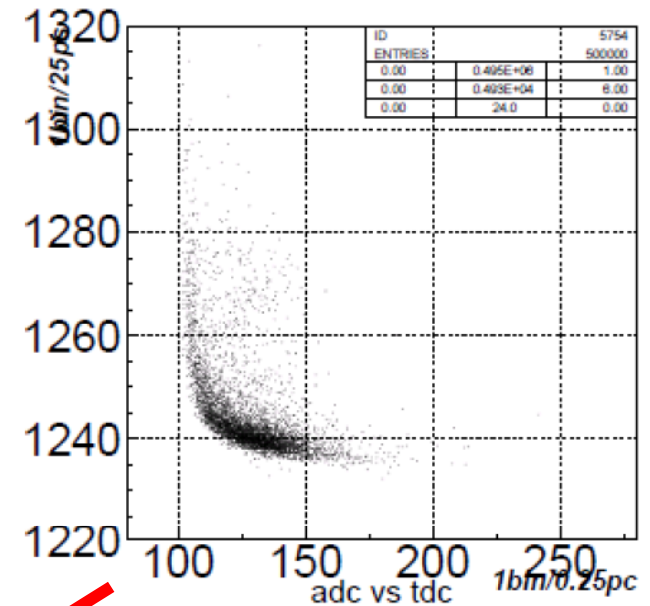
→ Has to be corrected!

Time walk correction 1



One possibility: measure both time (TDC) and amplitude (ADC)

→ Correct time of arrival by using a $\Delta T(\text{ADC})$ correction

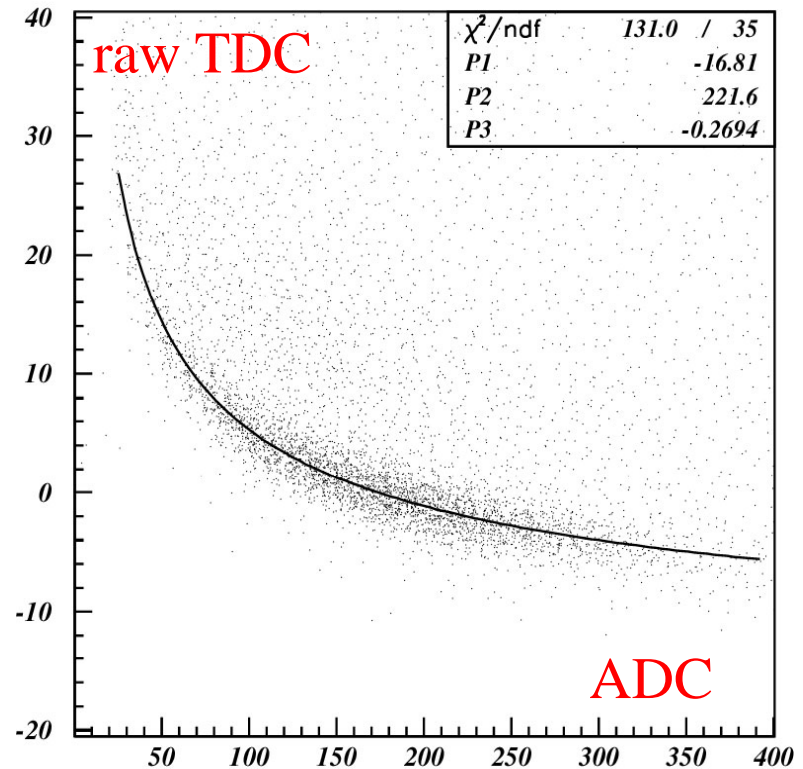
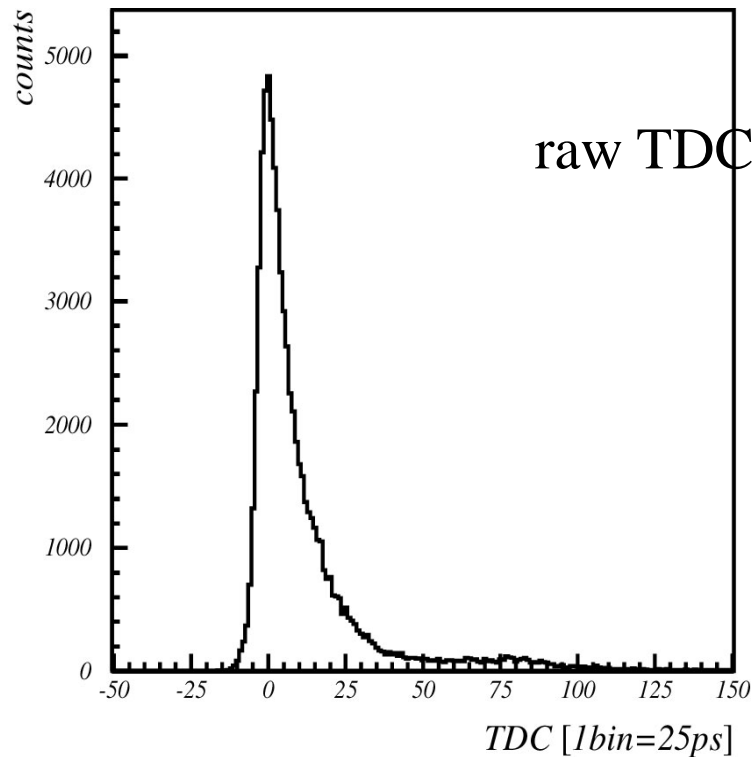
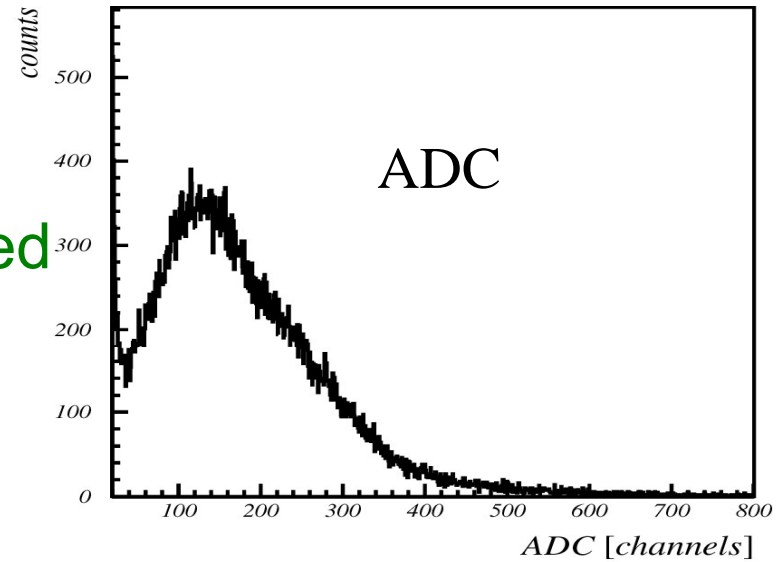


Time walk correction

TDC vs. ADC correlation is fitted with

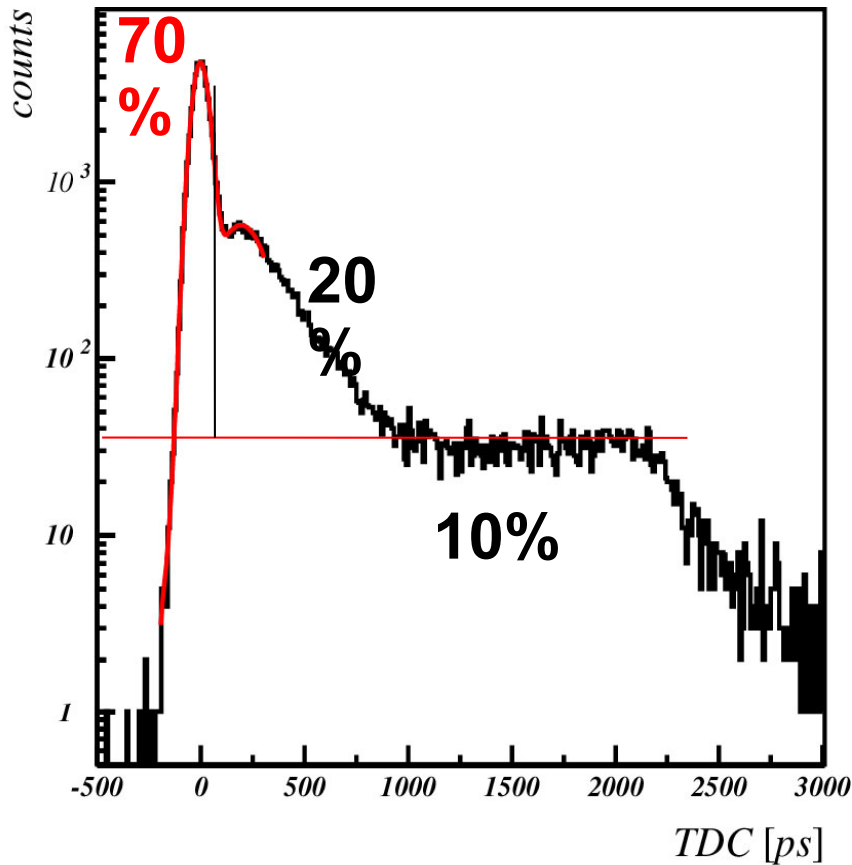
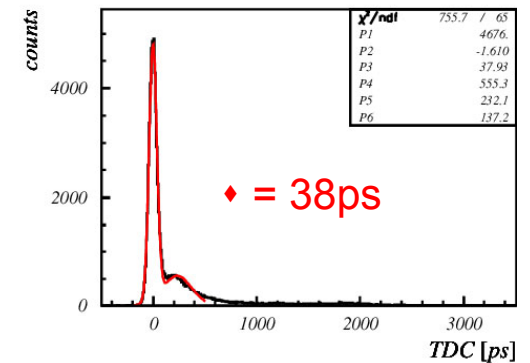
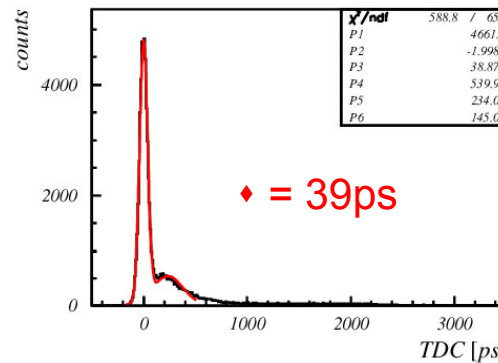
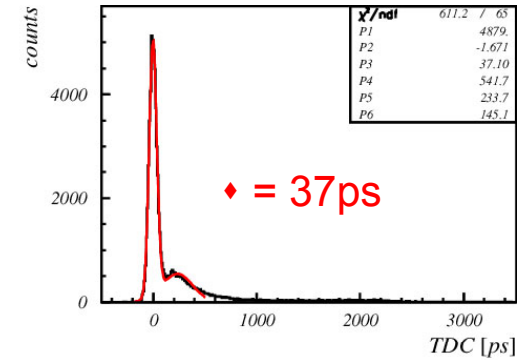
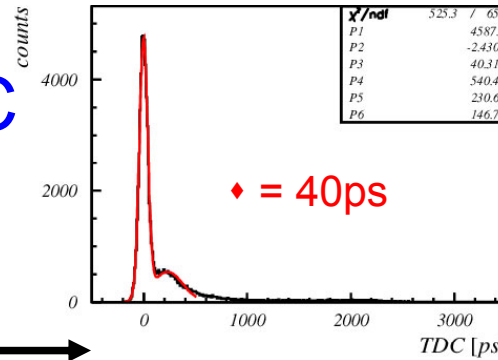
$$TDC = P1 + \sqrt{\frac{P2}{ADC - P3}}$$

and used for TDC correction



Corrected TDC

Corrected TDC distributions for all pads



Response:

- prompt signal ~ 70%
- short delay ~ 20%
- ~ 10% uniform distribution

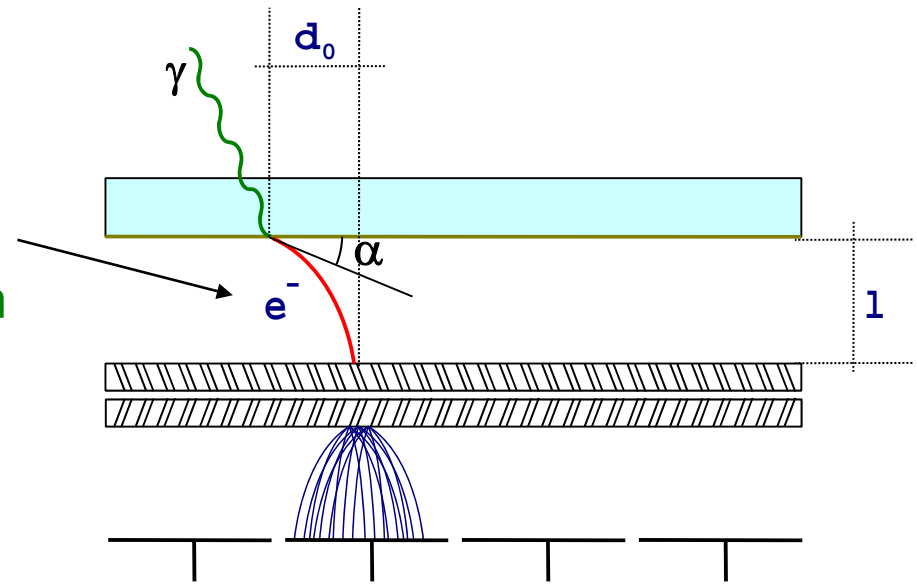
Photon electron detection: modeling

Parameters used:

- $U = 200 \text{ V}$
- $l = 6 \text{ mm}$
- $E_0 = 1 \text{ eV}$
- $m_e = 511 \text{ keV}/c^2$
- $e_0 = 1.6 \cdot 10^{-19} \text{ As}$

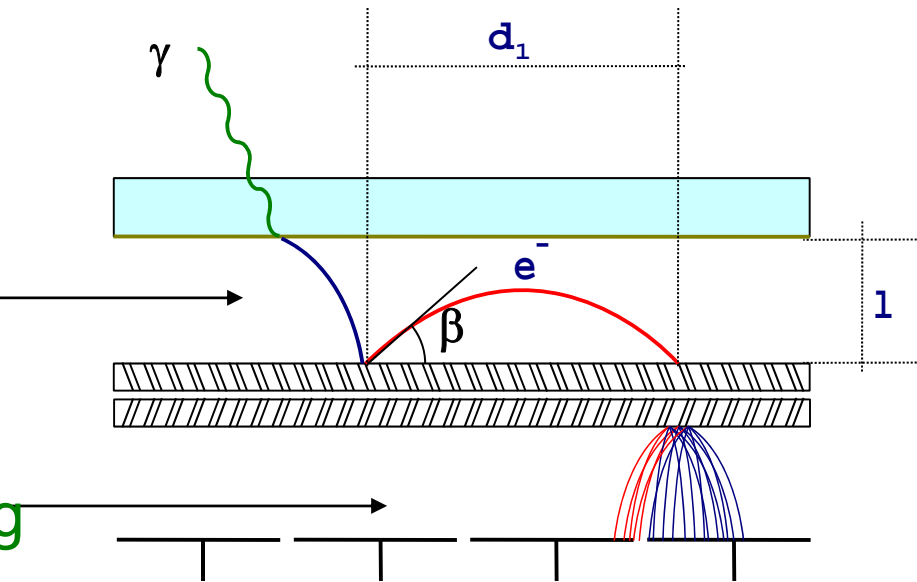
Photo-electron:

- $d_{0,\text{max}} \sim 0.8 \text{ mm}$
- $t_0 \sim 1.4 \text{ ns}$
- $\Delta t_0 \sim 100 \text{ ps}$



Backscattering:

- $d_{1,\text{max}} \sim 12 \text{ mm}$
- $t_{1,\text{max}} \sim 2.8 \text{ ns}$



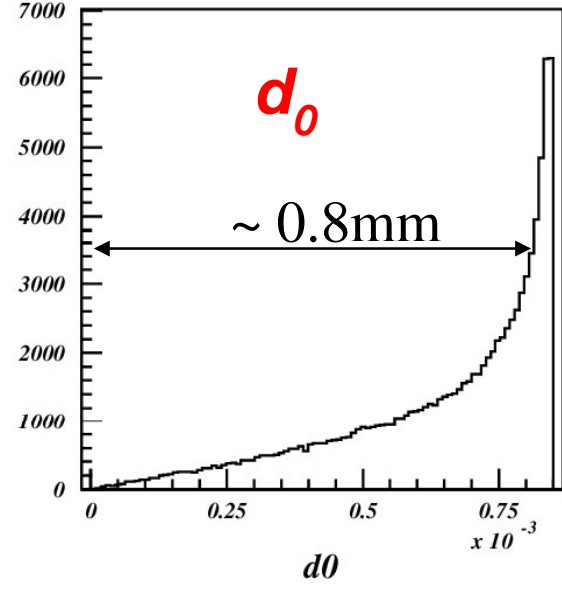
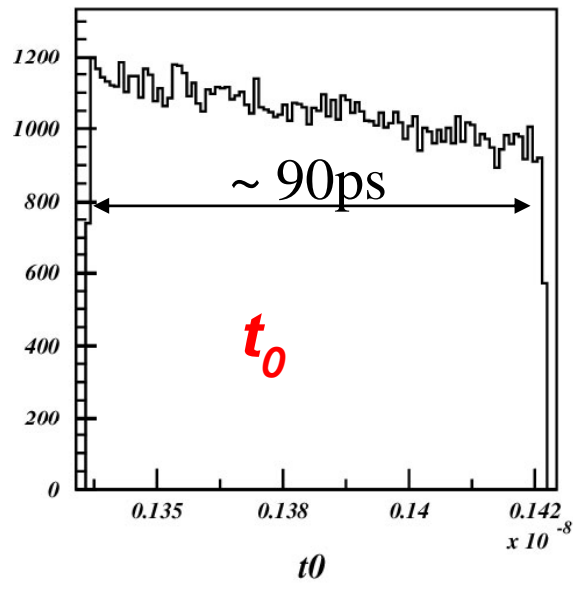
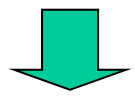
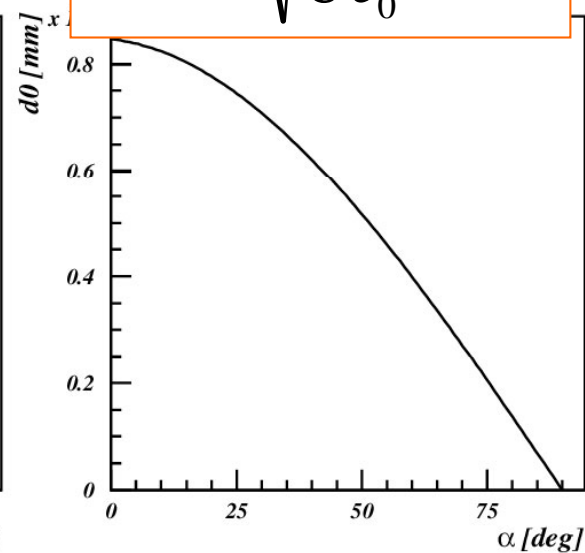
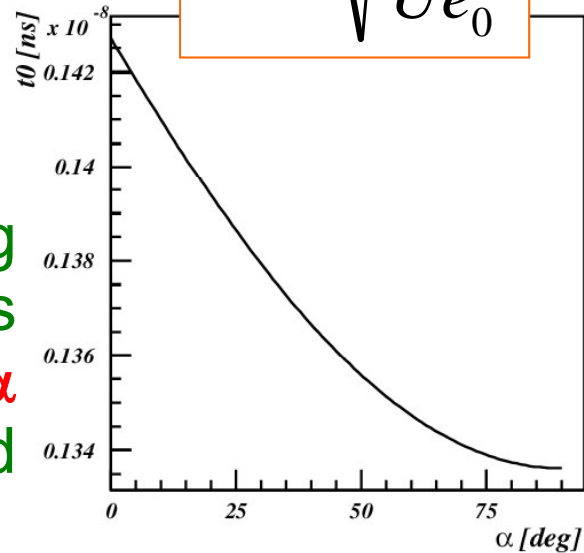
Charge sharing

Photo-electron: simple estimates

Distributions assuming that photo-electron is emitted at angle α uniformly over the solid angle

$$t_0 \approx l \sqrt{\frac{2m_e}{Ue_0}}$$

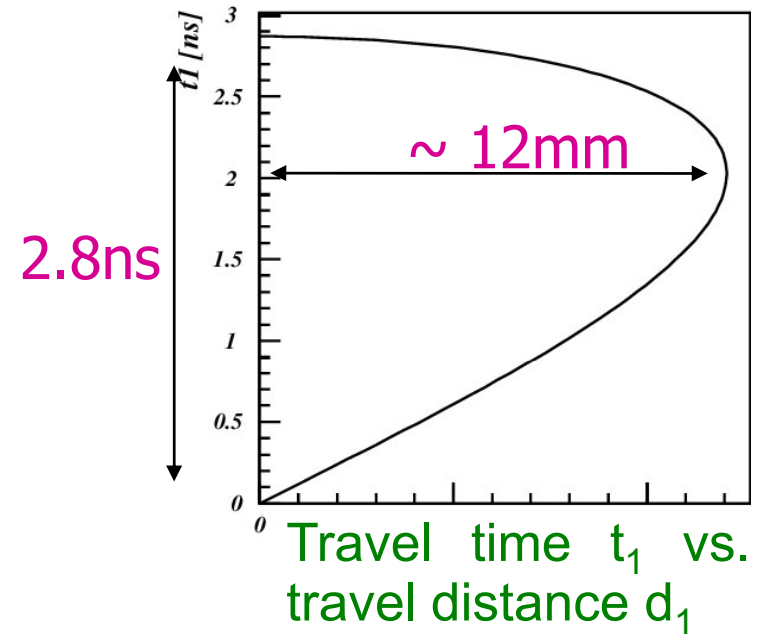
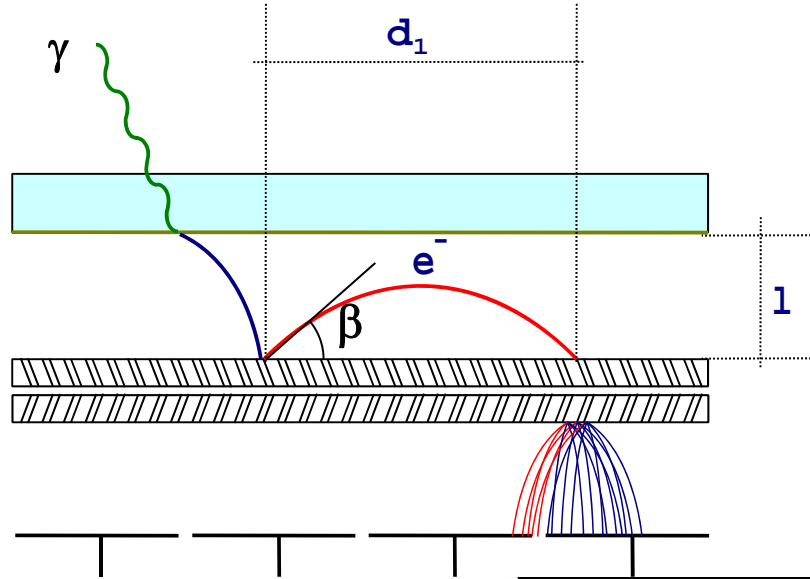
$$d_0 \approx 2l \sqrt{\frac{E_0}{Ue_0}} \cos \alpha$$



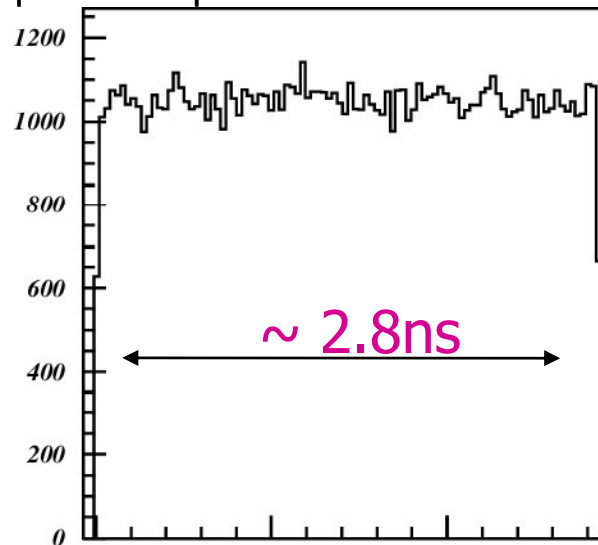
Maximum variation of photo-electron travel time.

$$\Delta t_0 \approx t_0 \sqrt{\frac{E_0}{Ue_0}} \approx \frac{l}{Ue_0} \sqrt{2m_e E_0}$$

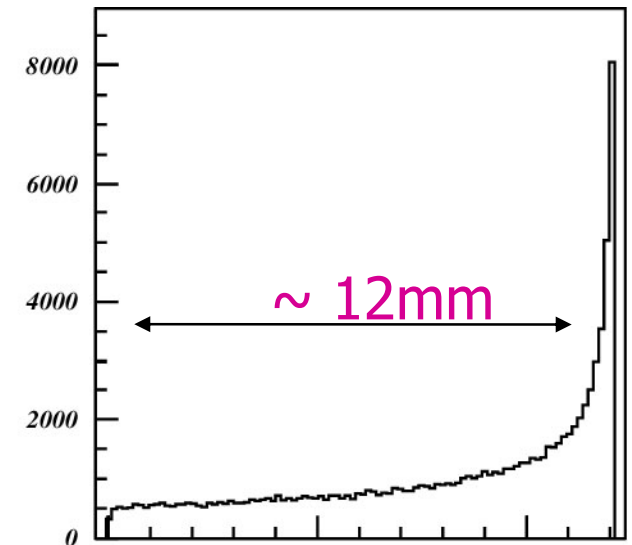
Elastic back-scattering



Distributions assuming that back-scattering by angle β is uniform over the solid angle



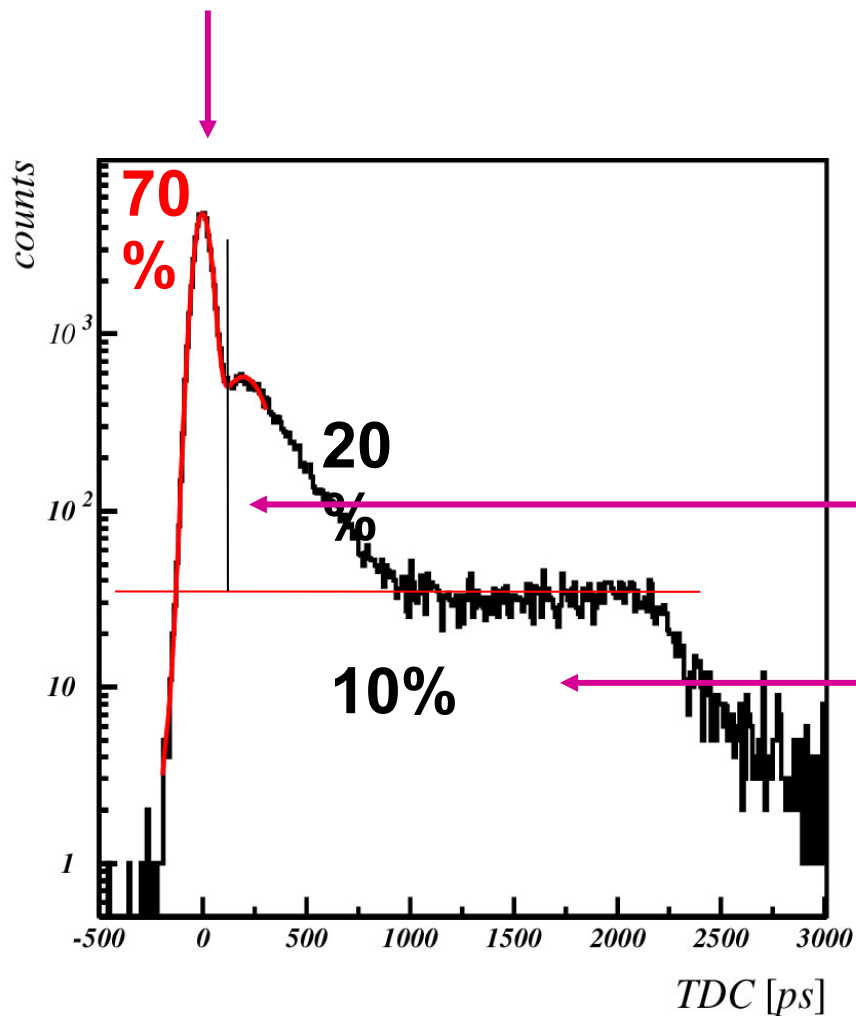
$$t_1 \approx 2t_0 \sin \beta$$



$$d_1 \approx 2l \sin 2\beta$$

Understanding time-of-arrival distribution

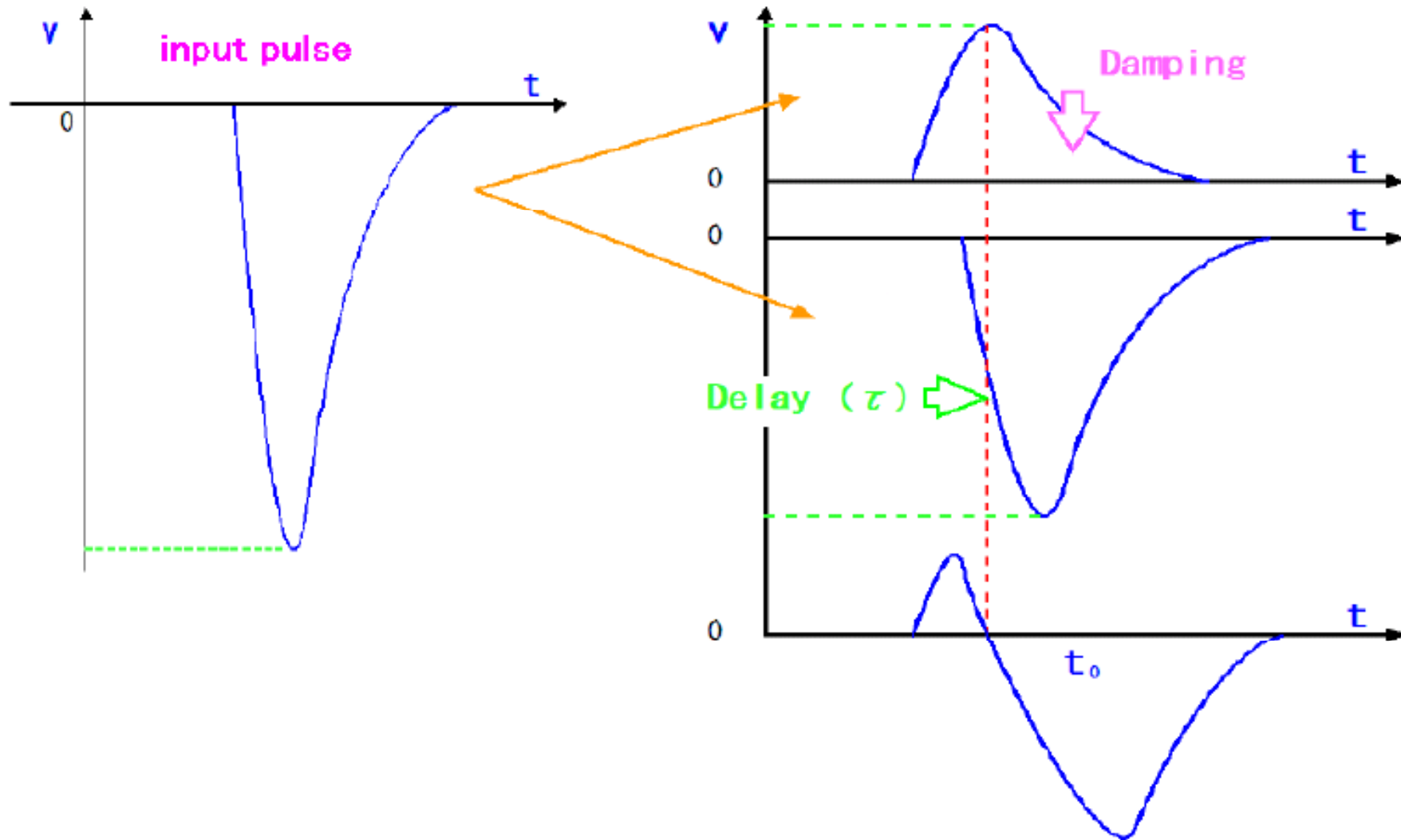
Normal photo-electrons



Inelastically scattered photo-electrons?

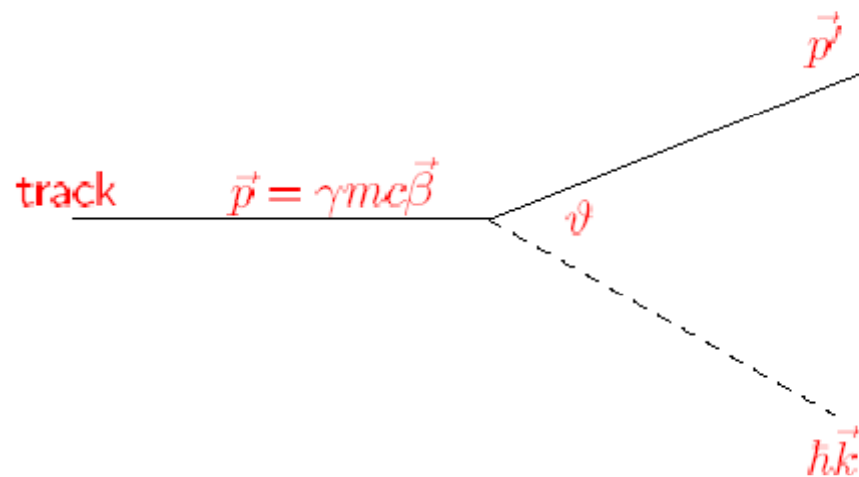
Elastically scattered photo-electrons

Time walk correction 2: constant fraction discriminator



Basics:
dE/dx,
Čerenkov,
Trans. Rad.

Charged particle of mass m and velocity $\vec{\beta}c$ interacts electromagnetically with detector medium via a photon of energy $\hbar\omega$ and momentum $\hbar\vec{k}$



Conservation of energy and momentum gives

$$\hbar\omega\left(1 - \frac{\hbar\omega}{2\gamma mc^2}\right) = \hbar\vec{k} \cdot \vec{\beta}c - \frac{\hbar^2 k^2}{2\gamma m}$$

typically $\hbar\omega \ll \gamma mc^2$ and $\hbar k \ll \gamma mc \rightarrow$

$$\omega = \vec{k} \cdot \vec{\beta}c = \beta ck \cos \vartheta. \quad (1)$$

Basics: dE/dx,
Čerenkov, Trans.
Rad - 2

The photon also has to satisfy the dispersion relation for a given medium with a dielectric constant ϵ

$$\omega^2 - \frac{k^2 c^2}{\epsilon} = 0 \quad (2)$$

From (1) and (2) we get

$$\sqrt{\epsilon} \beta \cos \vartheta = 1$$

which has a solution with a real value of ϑ if

$$\sqrt{\epsilon} \beta = n \beta > 1. \quad (3)$$

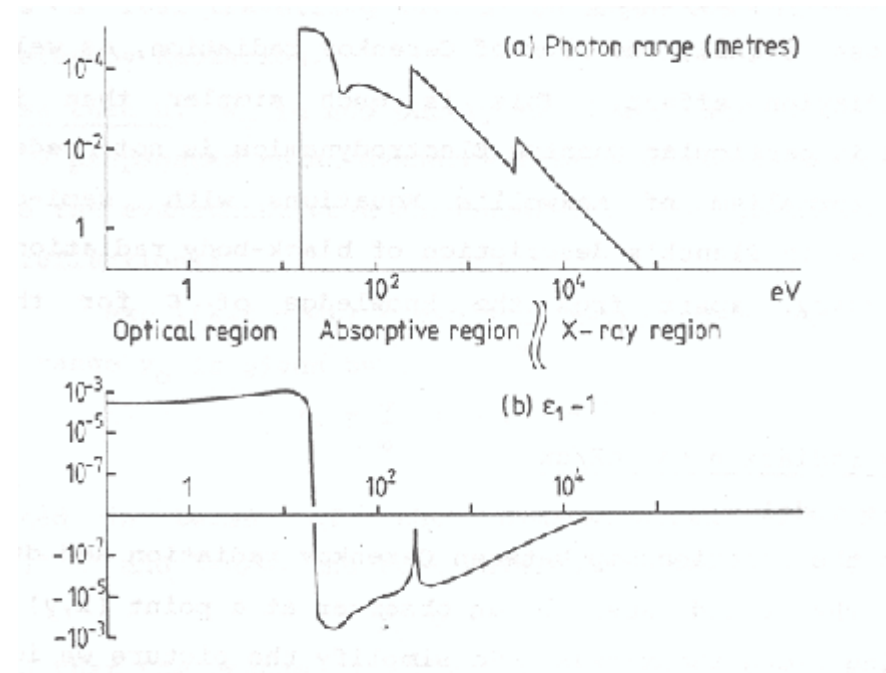
In this case **real** (Čerenkov) photons are emitted, and the emission angle is called Čerenkov angle ϑ_c .

N.B. In discontinuous media diffraction causes real photon emission even if (3) is not fulfilled (transition radiation).

Cross-section for emission (see Appendix for details)

$$\begin{aligned} \frac{d\sigma}{d(h\omega)} &= \frac{\alpha}{\beta^2 \pi} \frac{\sigma_\gamma(h\omega)}{h\omega Z} \log \left[(1 - \beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2 \right]^{-\frac{1}{2}} \\ &+ \frac{\alpha}{\beta^2 \pi} \frac{\sigma_\gamma(h\omega)}{h\omega Z} \log \left[\frac{2m c^2 \beta^2}{h\omega} \right] \quad (\text{ionis., excit.} \rightarrow dE/dx) \\ &+ \frac{\alpha}{\beta^2 \pi} \frac{1}{n_e h c} \left[\beta^2 - \frac{\epsilon_1}{|\epsilon|^2} \right] \Theta \quad (\text{Čerenkov, TRD}) \\ &+ \frac{\alpha}{\beta^2 \pi} \frac{1}{(h\omega)^2} \int_0^{h\omega} \frac{\sigma_\gamma(h\omega')}{Z} d(h\omega') \quad (\delta \text{ electrons}) \end{aligned}$$

Basics: dE/dx , Čerenkov, Trans. Rad – 3

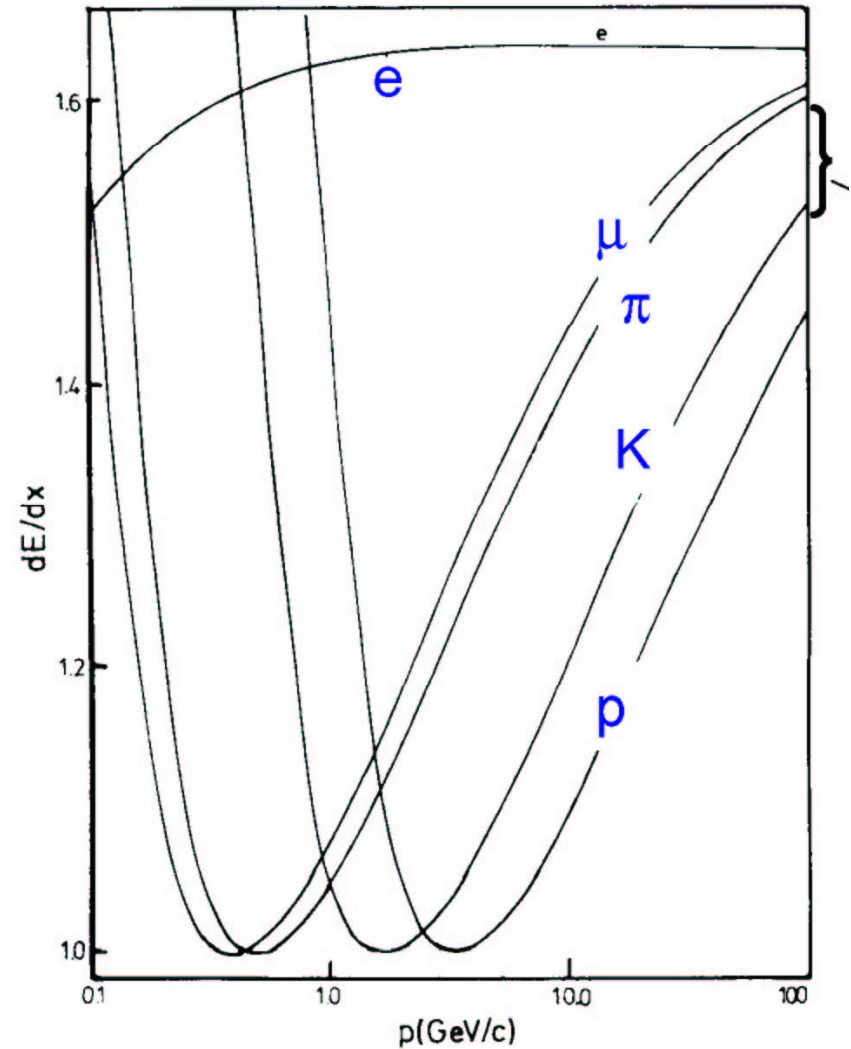


Three frequency ranges: ϵ_1 , ϵ_2 vs frequency

- 1) **Optical region:** ϵ real and >1 . The medium is transparent. and Čerenkov radiation is emitted by particles with velocity above the threshold.
- 2) **Absorptive region:** ϵ complex. Imaginary part makes the range of photons short.
- 3) **X-ray region:** ϵ nearly real and <1 . Čerenkov threshold is greater than c , but sub-threshold Čerenkov radiation can be emitted at discontinuities in the medium --> X-ray Transition Radiation.

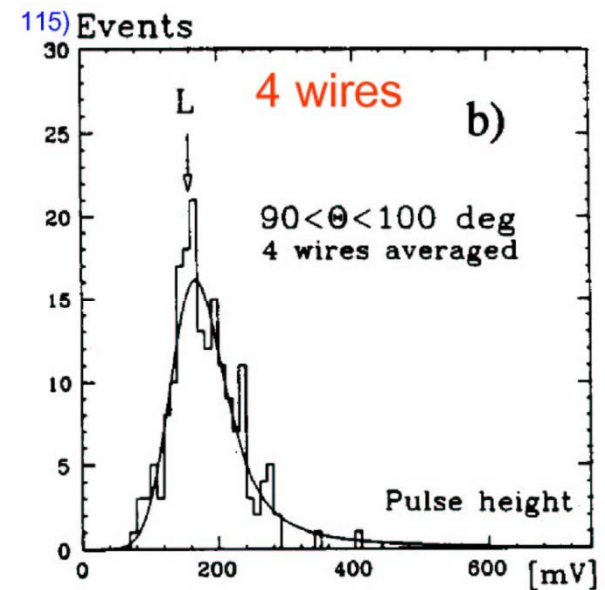
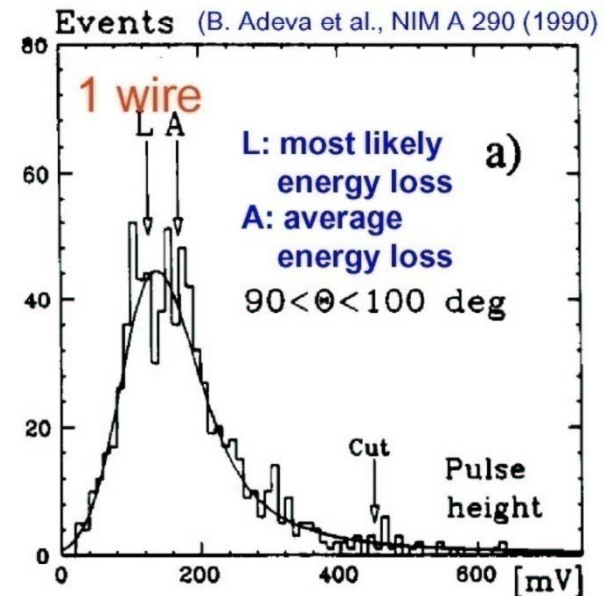
Identification with dE/dx measurement 1

dE/dx is a function of velocity. For particles with different mass the Bethe-Bloch curve gets displaced \rightarrow separation is possible if the resolution is good enough



Identification with dE/dx measurement 2

Problem: long tails (Landau distribution, not Gaussian)

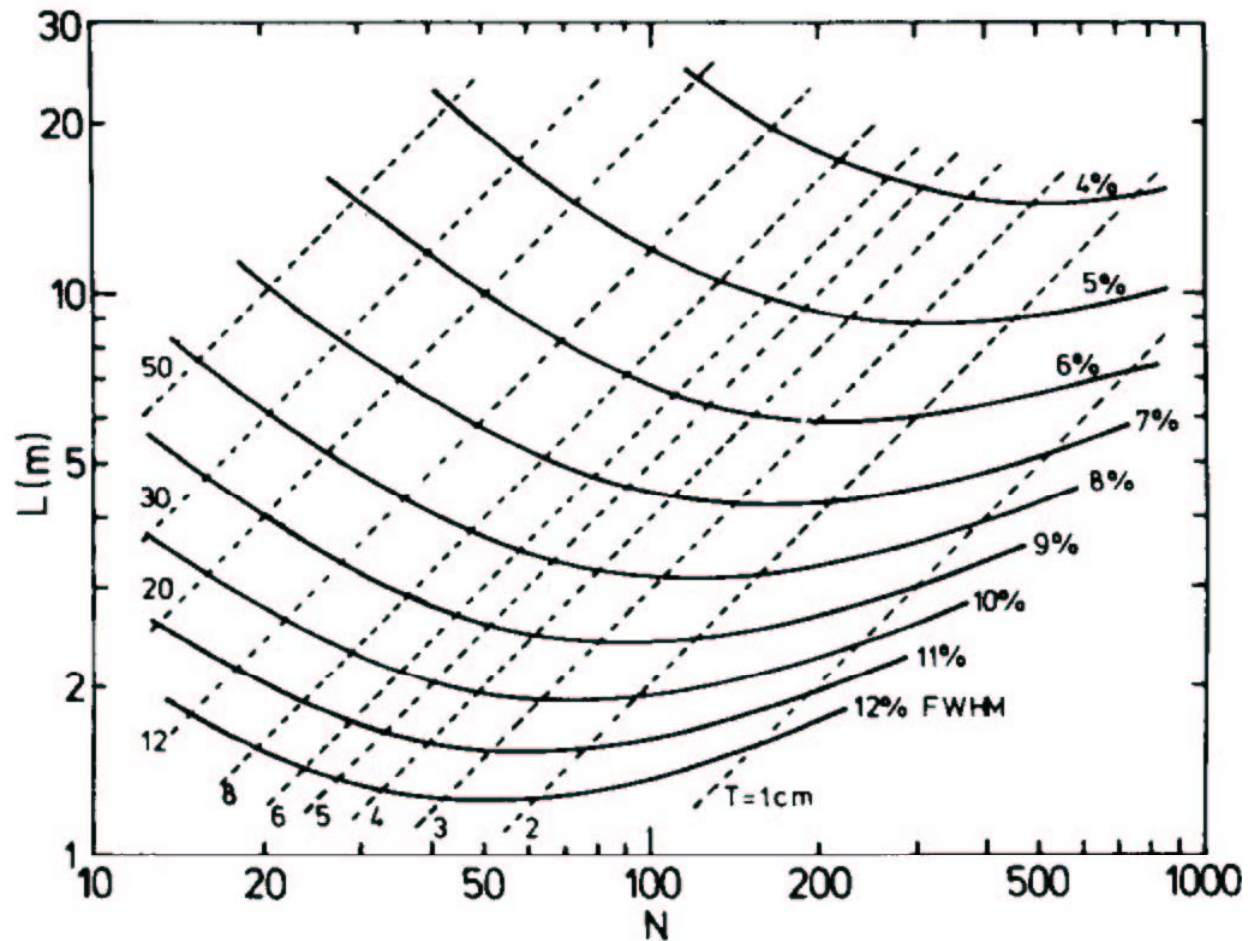


Identification with dE/dx measurement 3

Optimisation of the counter: length L , number of samples N , resolution (FWHM)

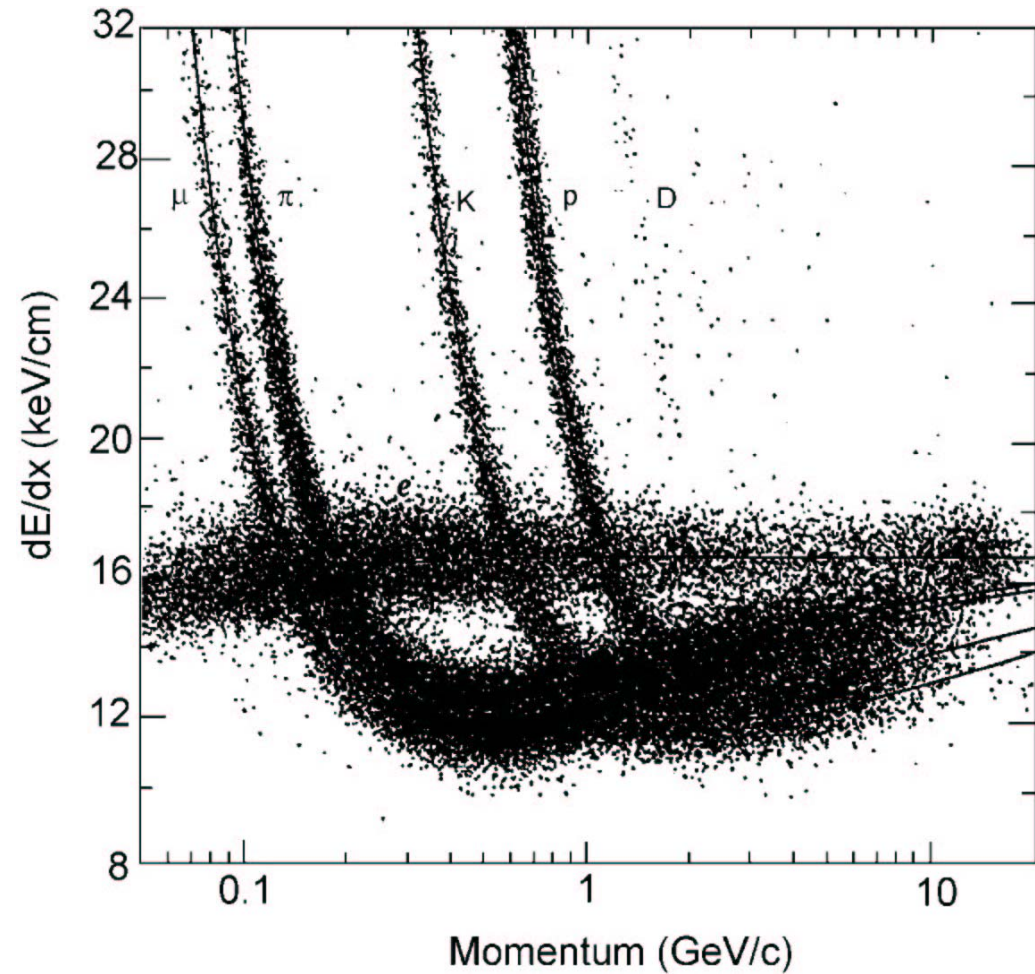
If the distribution of individual measurements were Gaussian, only the sample thickness would be relevant.

Tails: eliminate the largest 30% values \rightarrow the optimum depends also on the number of samples.



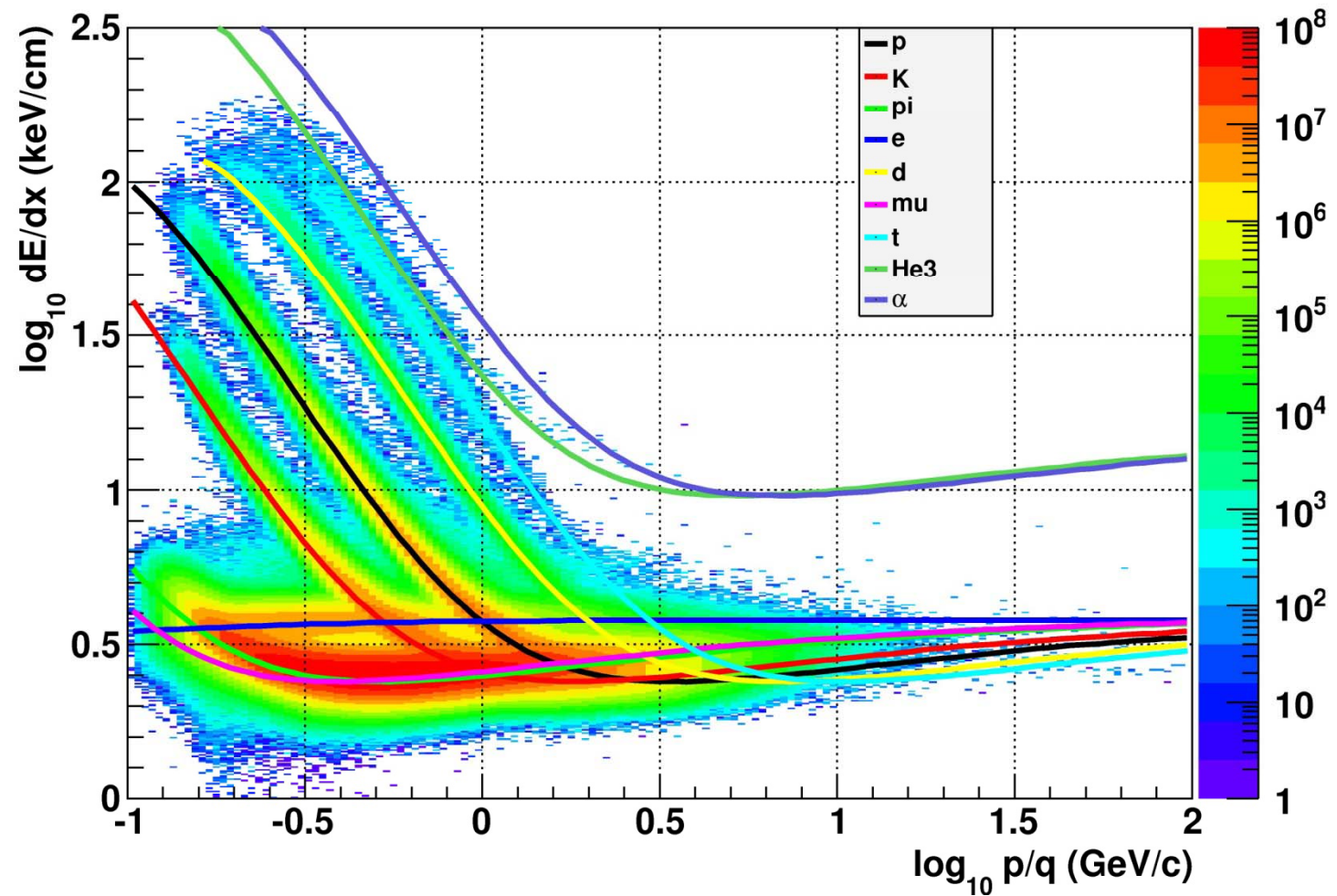
Identification with dE/dx measurement 4

Example of a dE/dx performance in a large drift chamber.



Identification with dE/dx measurement 5

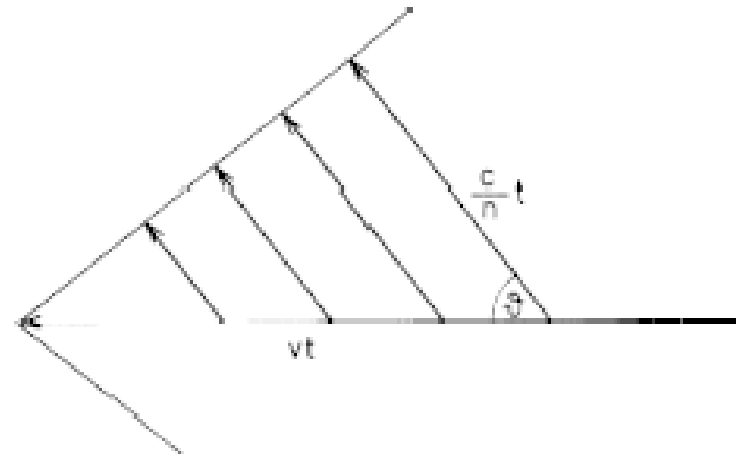
dE/dx performance in the huge STAR TPC



Čerenkov radiation

A charged track with velocity $v = \beta c$ above the speed of light c/n in a medium with index of refraction $n = \sqrt{\epsilon}$ emits **polarized light** at a characteristic (Čerenkov) angle,

$$\cos\theta = c/nv = 1/\beta n$$



Two cases:

- 1) $\beta < \beta_t = 1/n$: below threshold no Čerenkov light is emitted.
- 2) $\beta > \beta_t$: the number of Čerenkov photons emitted over unit photon energy $E = h\nu$ in a radiator of length L amounts to

$$\frac{dN}{dE} = \frac{\alpha}{\hbar c} L \sin^2 \theta = 370(\text{cm})^{-1} (\text{eV})^{-1} L \sin^2 \theta$$

Number of detected photons

Example: in 1m of air ($n=1.00027$) a track with $\beta=1$ emits $N=41$ photons in the spectral range of visible light ($\Delta E \sim 2$ eV).

If Čerenkov photons were detected with an average detection efficiency of $\varepsilon=0.1$ over this interval, $N=4$ photons would be measured.

In general: number of detected photons can be parametrized as

$$N = N_0 L \sin^2\theta$$

where N_0 is the figure of merit,
$$N_0 = \frac{\alpha}{\hbar c} \int Q(E)T(E)R(E)dE$$

and $Q T R$ is the product of photon detection efficiency, transmission of the radiator and windows and reflectivity of mirrors employed.

Typically: $N_0 = 50 - 100/\text{cm}$

Rewrite the basic relations:

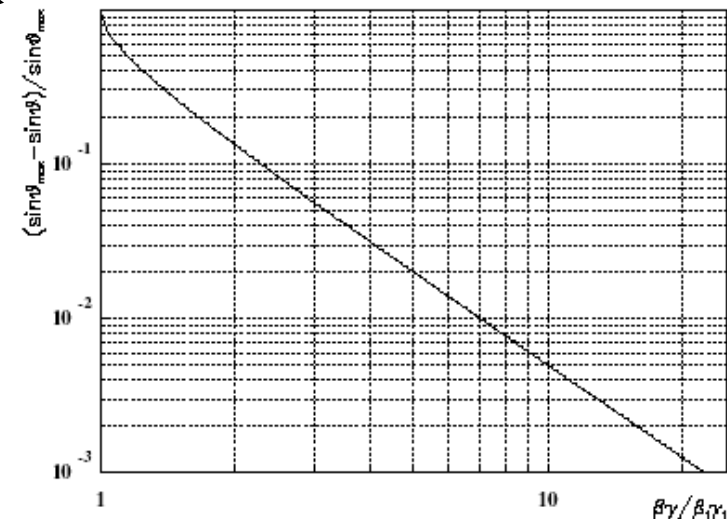
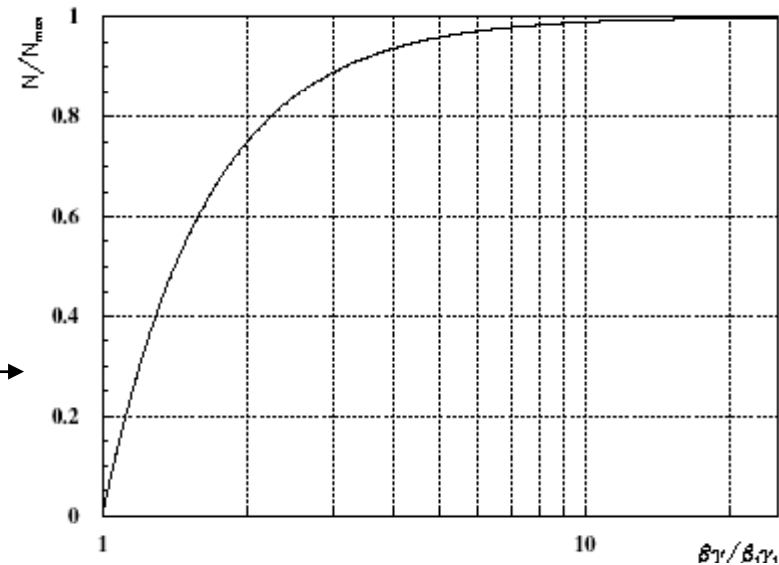
- the threshold Lorentz factor $\gamma_t = (1 - 1/n^2)^{-1/2}$
- the asymptotic value of Čerenkov angle (for $\beta = 1$)

$$\sin^2 \theta_{max} = \frac{1}{\gamma_t^2}$$
- the asymptotic number of Čerenkov photons:

$$N_{max} = \frac{N_0 L}{\gamma_t^2}$$
- number of photons, non-asymptotic case:

$$\frac{N}{N_{max}} = 1 - \frac{(\beta_t \gamma_t)^2}{(\beta \gamma)^2}$$
- Čerenkov angle, non-asymptotic case:

$$\frac{\sin^2 \theta}{\sin^2 \theta_{max}} = \sqrt{1 - \frac{(\beta_t \gamma_t)^2}{(\beta \gamma)^2}}$$



The basic relations are functions of the ratio $\frac{\beta\gamma}{\beta_t\gamma_t} = \frac{p}{p_t}$

Types of Čerenkov counters

Threshold counters --> count photons to separate particles below and above threshold

Ring Imaging (RICH) --> measure Čerenkov angle and count photons

Short historical excursion

- 1934 Čerenkov characterizes the radiation
- 1938 Frank, Tamm give the theoretical explanation
- 50-ties - 70-ties Čerenkov counters are developed and are being used in nuclear and particle physics experiments, as differential and threshold counters
- 1958: Nobel prize for Čerenkov
- 1977 Ypsilantis, Seguinot introduce the idea of a RICH counter with a large area wire chamber based photon detector
- 1981-83 first use of a RICH counter in a particle physics experiment (E605)
- 1992--> first results from the DELPHI RICH, SLD CRID, OMEGA RICH

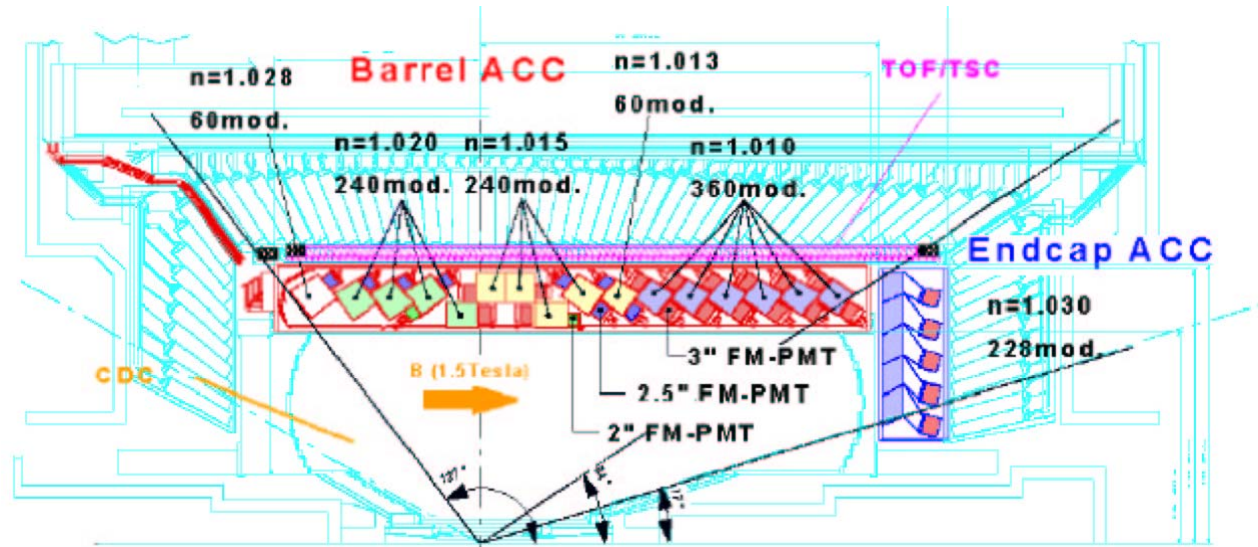
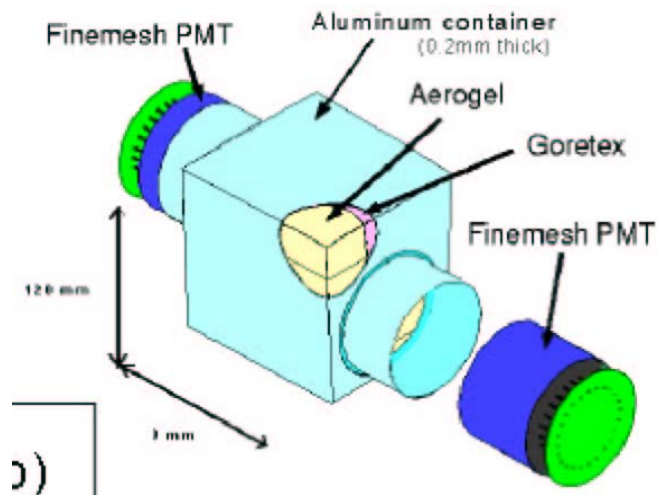
Threshold Čerenkov counters

- Beam veto counters
- Detection of sub-threshold particles in a RICH
- Aerogel Čerenkov counter in Belle: K (below) vs. π (above thr.)
by properly choosing n for a given kinematic region

Threshold Čerenkov counter: Belle ACC (aerogel Cherenkov counter)

Aerogel Čerenkov counter Belle: K (below) vs. π (above thr.) by properly choosing n for a given kinematic region (more energetic particles fly in the 'forward region')

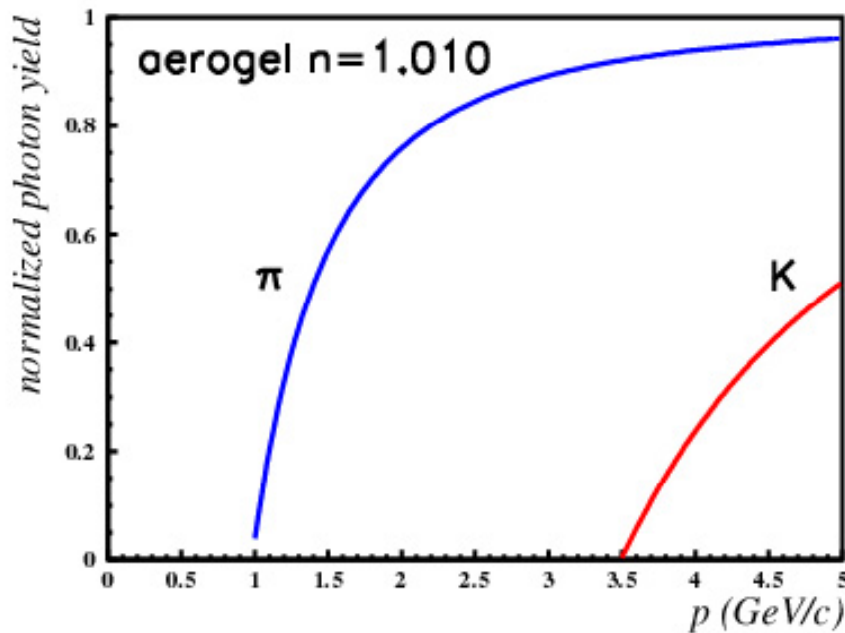
Detector unit: a block of aerogel and two



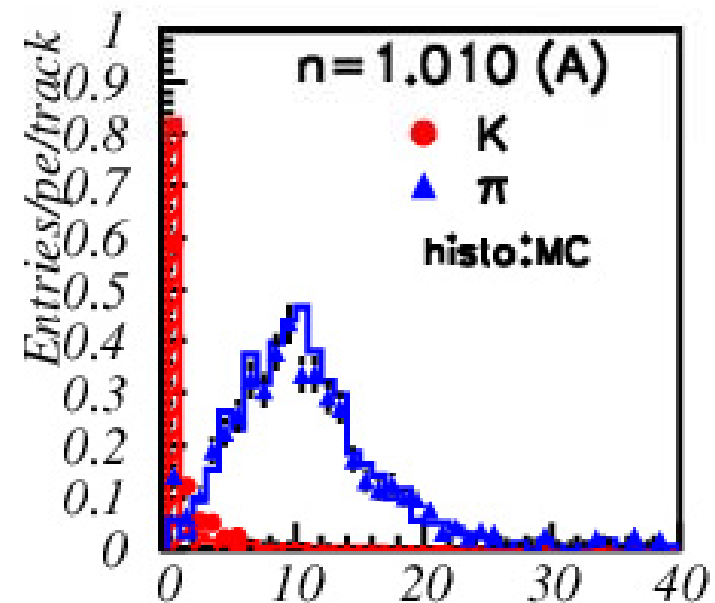
Fine-mesh PMT: works at high B fields

Threshold Čerenkov counter: Belle ACC (aerogel Cherenkov counter)

expected yield vs p



measured for $2 \text{ GeV} < p < 3.5 \text{ GeV}$
expected and measured
number of hits

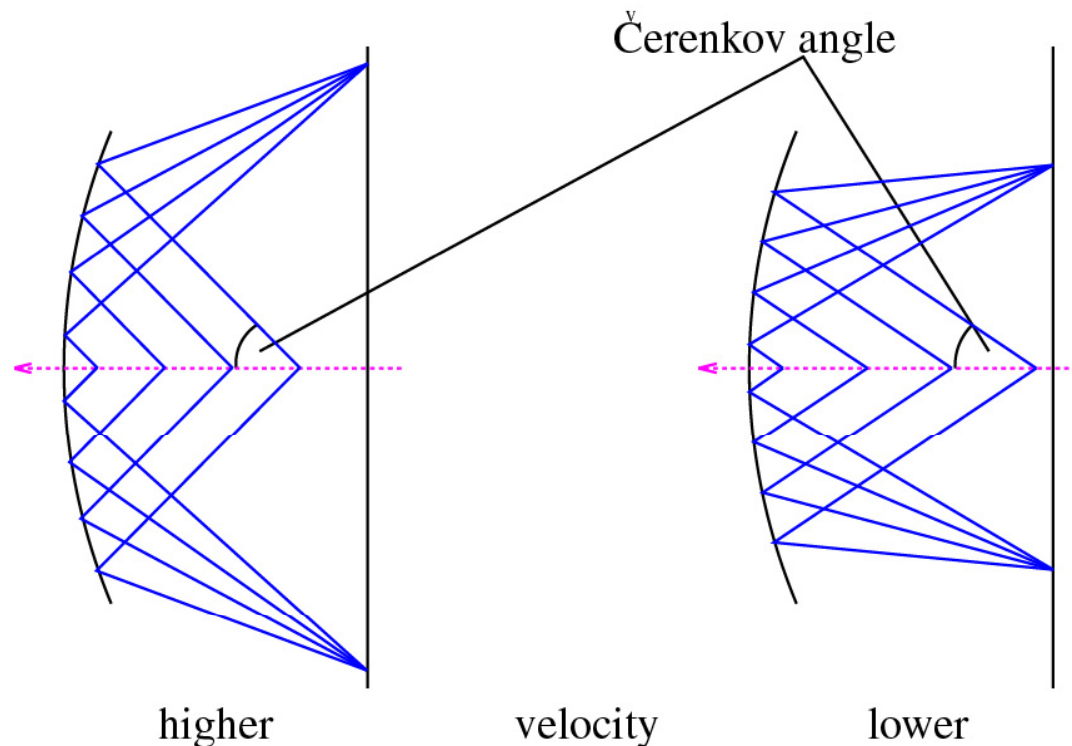


RICH counter

Aim: measure the direction of Čerenkov photons emitted by a charged track.

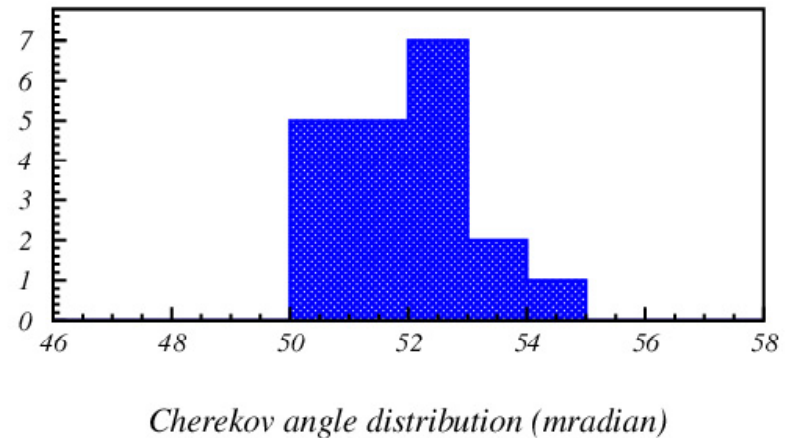
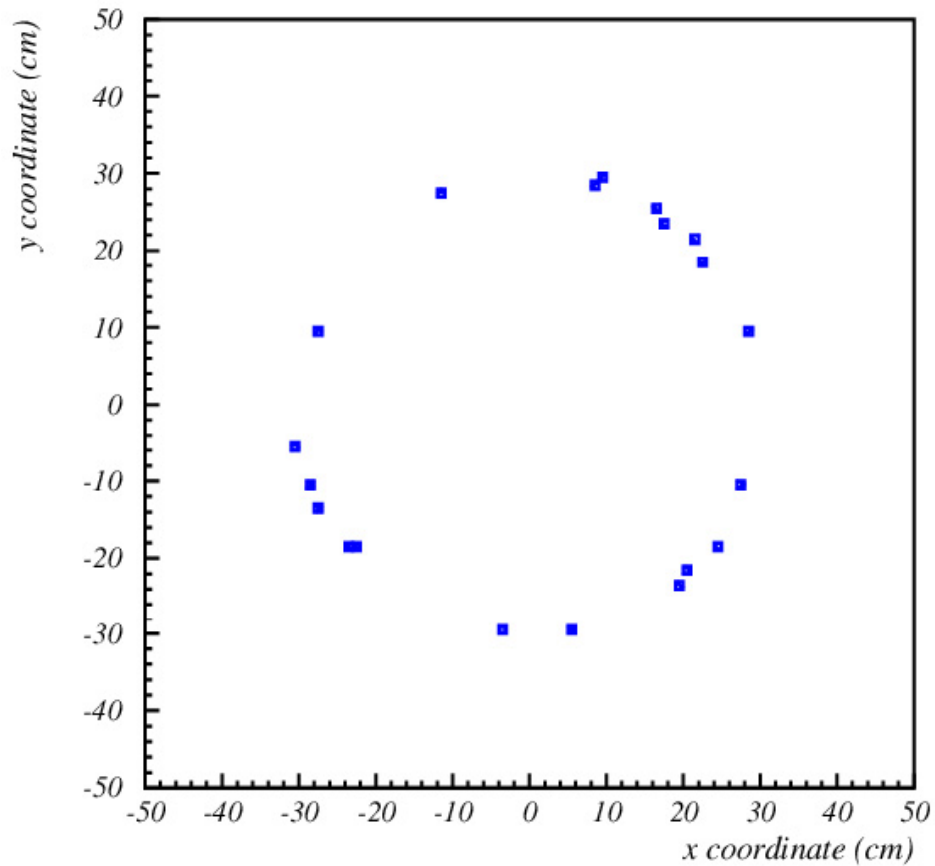
Idea: transform the **direction** into a **coordinate**.

Take a **spherical mirror**: parallel rays intersect on the focal surface. Since photons are emitted uniformly over the azimuthal angle around the track, a ring is formed on the focal plane. With a position sensitive photon detector in the focal plane we get a **Ring Imaging Čerenkov counter (RICH)**.



RICH counter 2

From the image on the photon detector, the Čerenkov angle of the track can be reconstructed, i.e. from the known track direction (ring center) and hit coordinate the angle is calculated and plotted



Analysis of RICH data

Rings are accompanied by noise hits and other rings

How to choose between the hypotheses e , μ , π , K , p ?

- 1) Count photons within 2.5σ of each hypothesis
- 2) Determine likelihood for each track and each hypothesis independently (extended maximum likelihood)
- 3) Global max. likelihood: maximize likelihood for all tracks in the event at the same time
- 4) Iterative pattern analysis: associate each photon predominately with a single track
- 5) If only poor tracking information is available: look for rings in a stand-alone mode, use some form of Hough transform Essential:

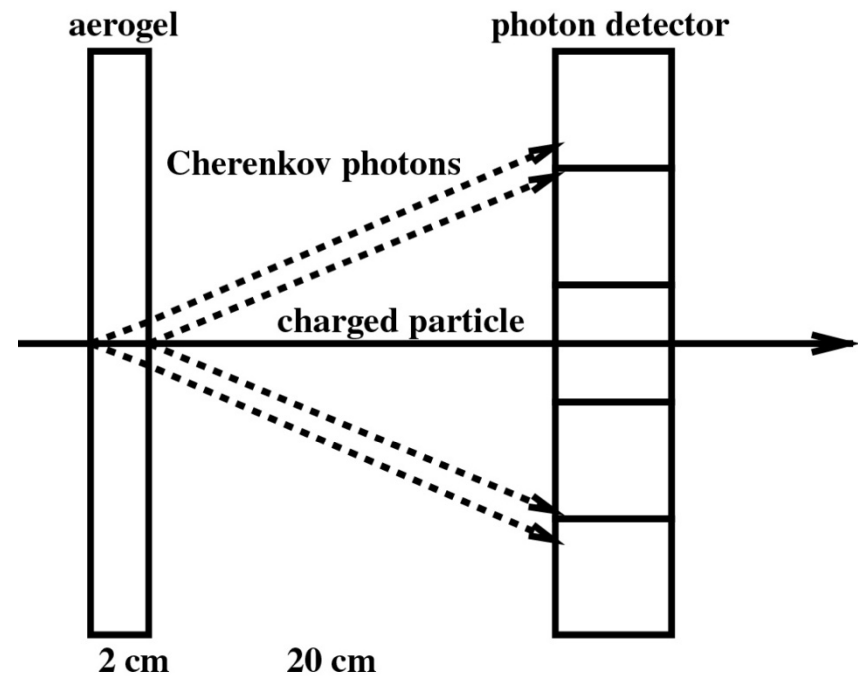
->number of photons

->single photon resolution

Summary: need about 10 for isolated rings, more than 20 for high density cases.

Resolution of a RICH counter

- Photon impact point resolution (photon detector resolution)
- Emission point uncertainty
- Dispersion: $n=n(\lambda)$ in $\cos\theta = 1/\beta n$
- Track parameters
- Errors of the optical system



Resolution of a RICH counter

Sources of error in the \check{C} . angle measurement

- finite coordinate resolution of the photon detector

$$\sigma_{\theta} = \frac{a}{f\sqrt{12}}$$

for detector pad size a and mirror focal length f ,
e.g. $\sigma_{\theta} = 0.45$ mrad for $a = 9$ mm, $f = 5.75$ m.

- dispersion, variation of the refractive index (dn/dE) over the energy range with RMS width σ_E of detected Čerenkov photons,

$$\sigma_{\theta} = \frac{1}{\beta n^2 \sin \theta} \frac{dn}{dE} \sigma_E$$

solid radiators: 5 – 7 mrad, liquids: 3 – 4 mrad,
gases: 0.1 – 0.5 mrad (depends on the radiator and detector!).

- optical error due to the imperfections on the mirror surface and mirror misalignment, typically $\approx 0.1 - 0.2$ mrad
- finite precision in track slope parameters as determined by the tracking system

Resolution of a RICH counter

- spread in track slope parameters as caused by multiple scattering within the radiator

$$\sigma_\theta = \frac{1}{\sqrt{6}} 15 \text{MeV}/c \frac{\sqrt{L/X_0}}{\beta p}$$

- spread in track slope parameters due to stray magnetic fields in the radiator

$$\sigma_\theta = \frac{1}{\sqrt{12}} \frac{B_\perp L}{p} \frac{0.3 \text{GeV}/c}{T_m}$$

- the optical error due to the finite angle of incidence of photons upon the spherical mirror (spherical aberration)

In a typical case the first two dominate since they are hardest to overcome.

The combined single photon error σ_1 is a quadratic sum of the contributions.

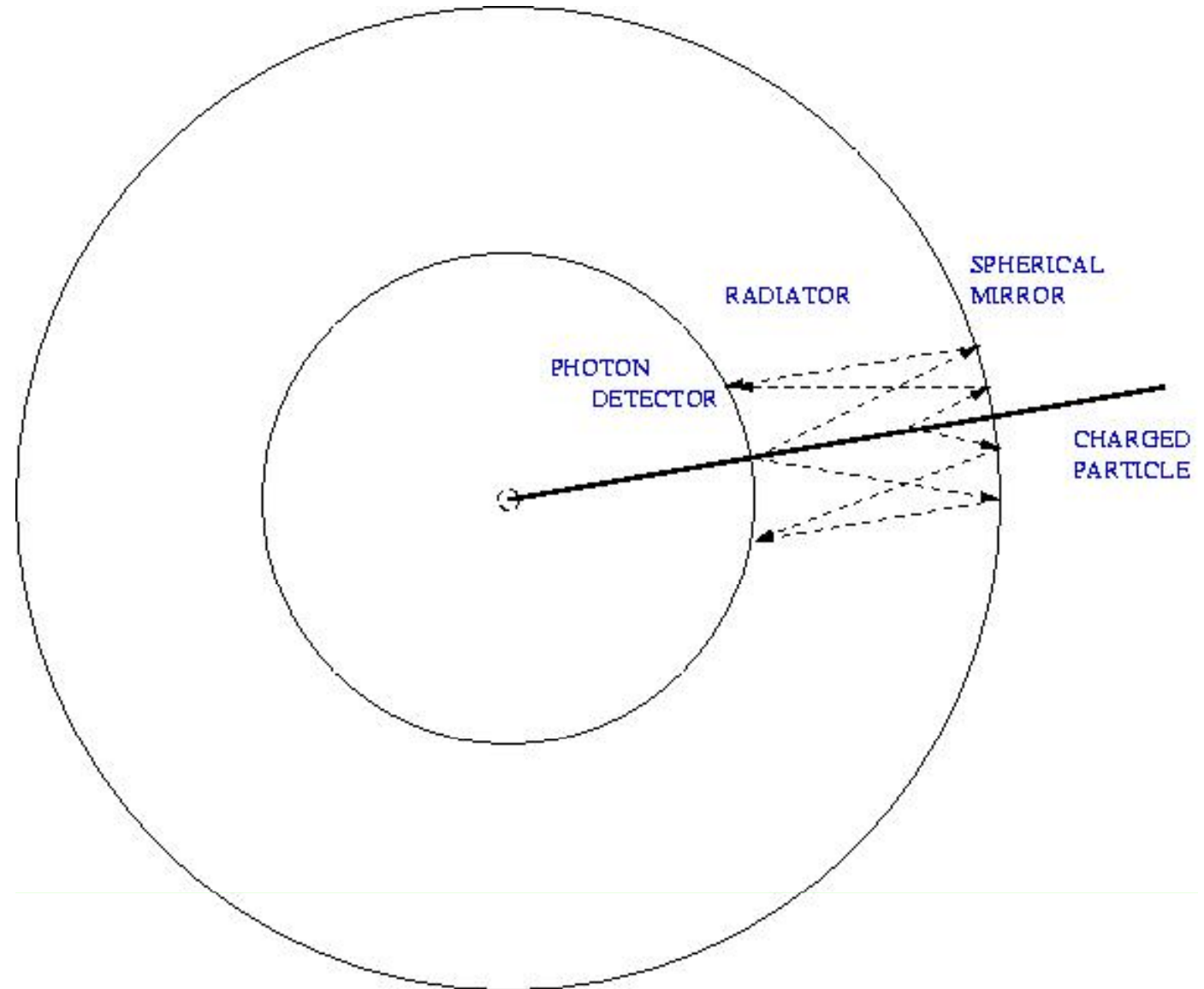
Assuming N detected photons, the overall resolution is

$$\sigma_N = \frac{\sigma_1}{\sqrt{N}}$$

for isolated tracks.

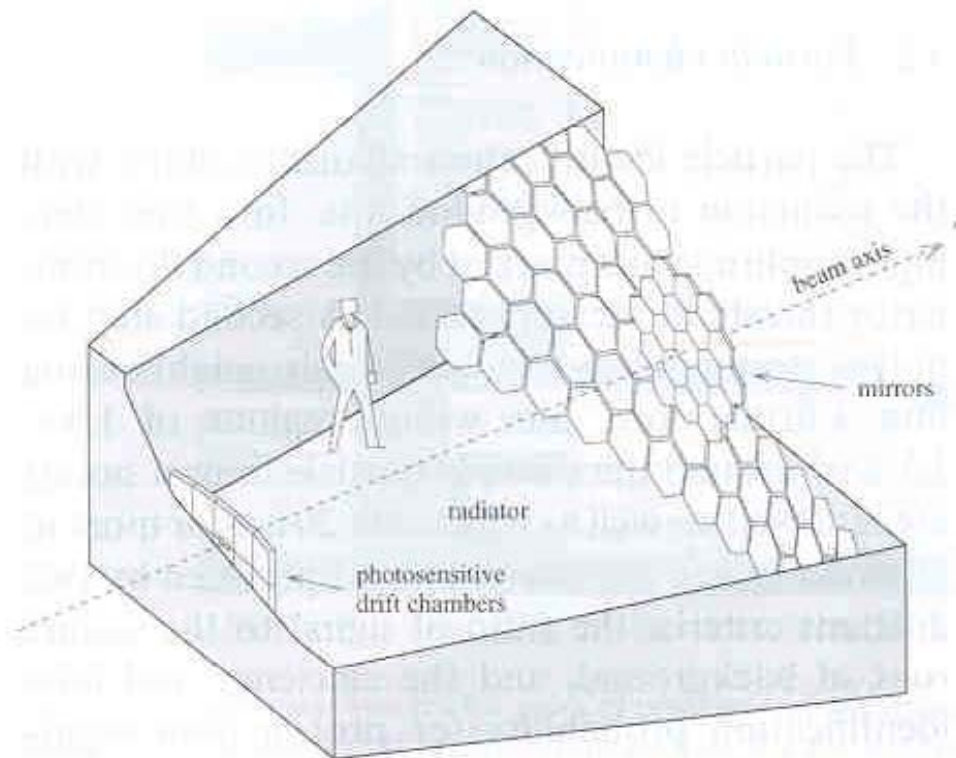
RICH Designs: mirror focused 1

Original mirror
focused set-up

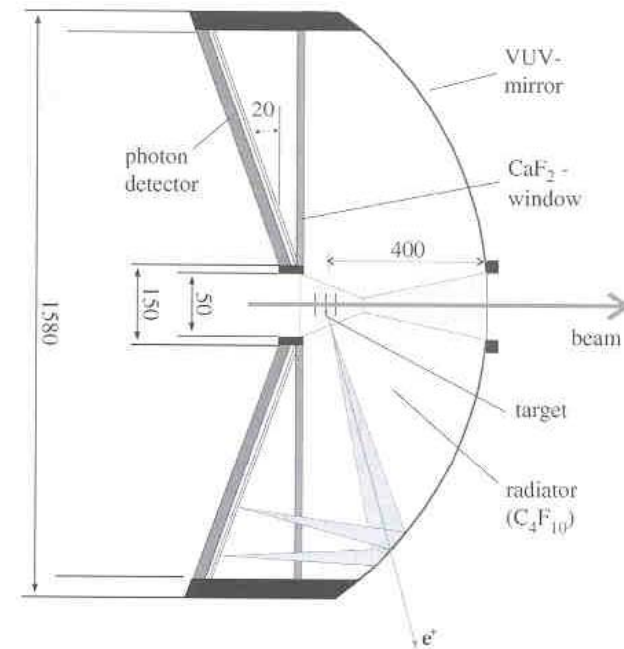


RICH Designs: mirror focused 2

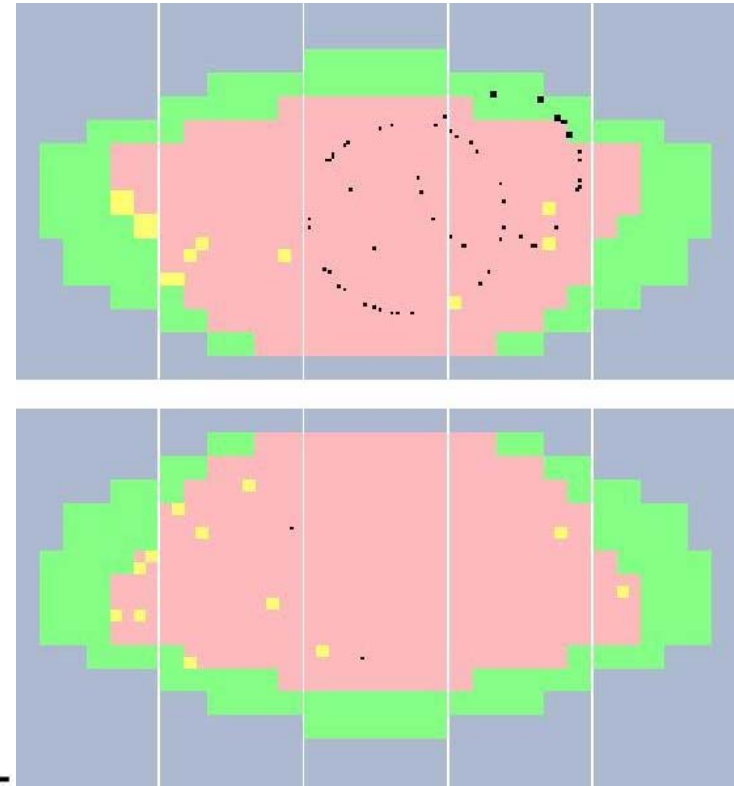
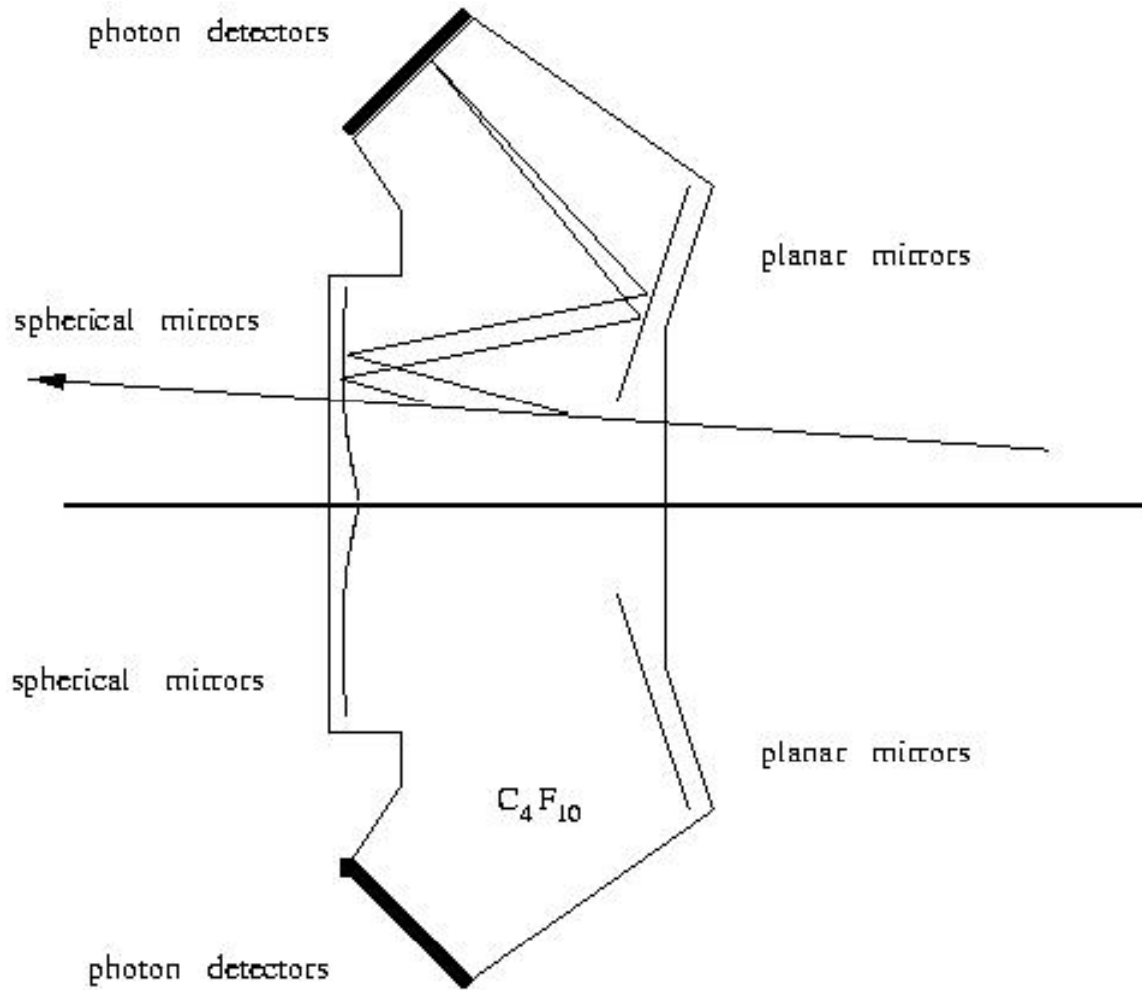
OMEGA



HADES



RICH Designs: HERA-B



The light source is of particular nature: one has to be careful about the position, orientation, form of the photon detector plane: impact on resolution!

→ T. Ypsilantis, J. Seguinot, NIM A343 (1994) 30

→ P. Križan, M. Starič, NIM A379 (1996) 124

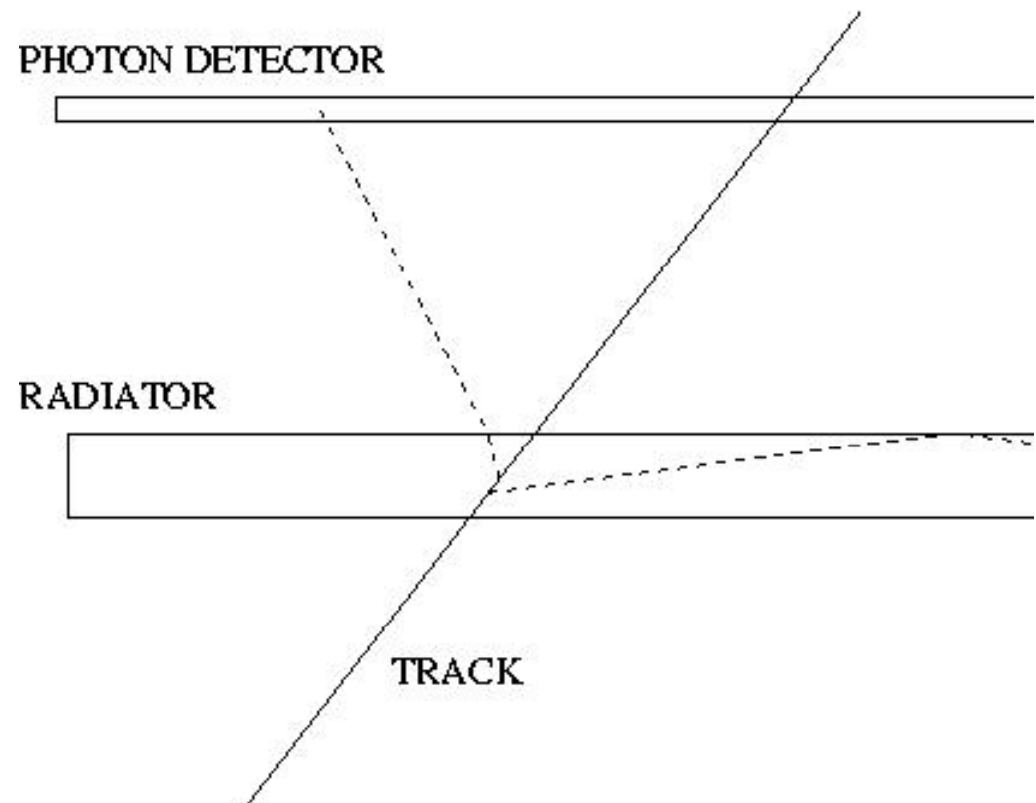
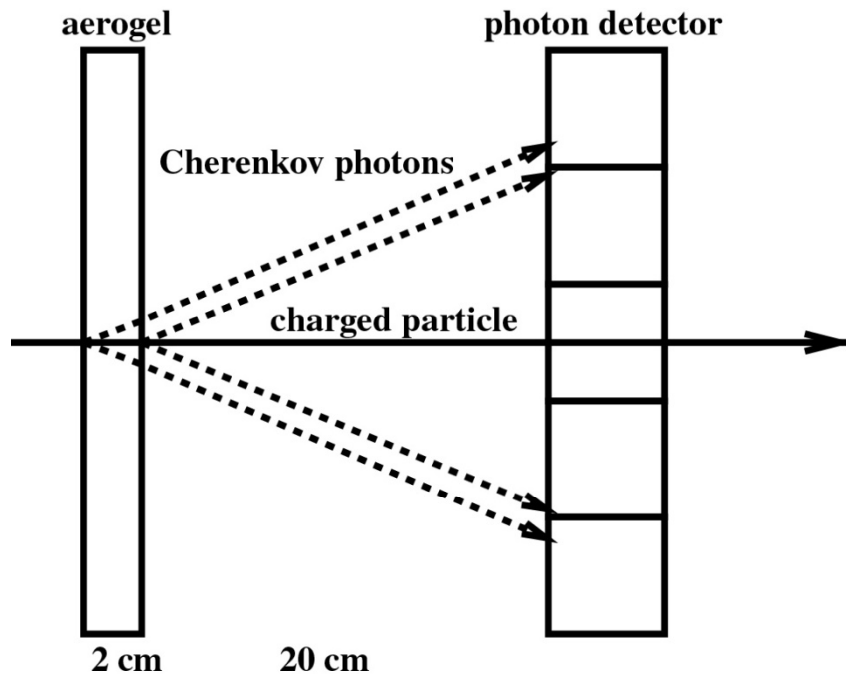
Proximity focusing RICH

Geometry variation: in case of a solid or liquid radiator, the radiator can be rather thin (only about 1 cm) - one does not need a spherical mirror.

Photons are led to propagate over a distance of 10-20~cm, until they reach the photon detector.

Errors in the proximity focusing case depend on the photon angle:

- detector granularity
- emission point error
- dispersion



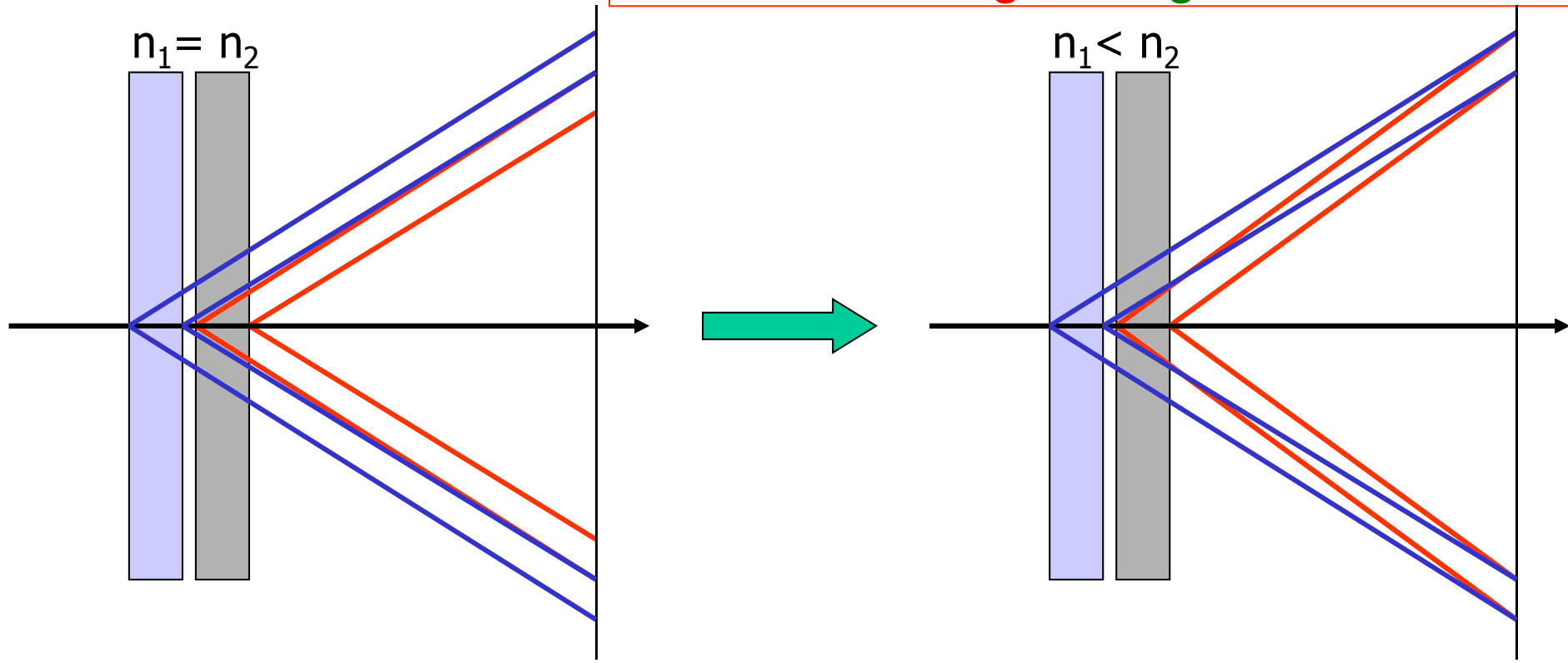


Radiator with multiple refractive indices

How to increase the number of photons without degrading the resolution?

normal

→ stack two tiles with different refractive indices: “focusing” configuration

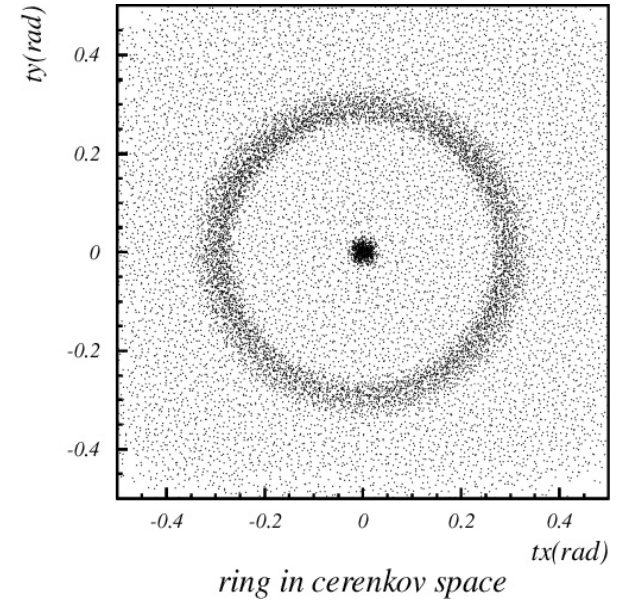
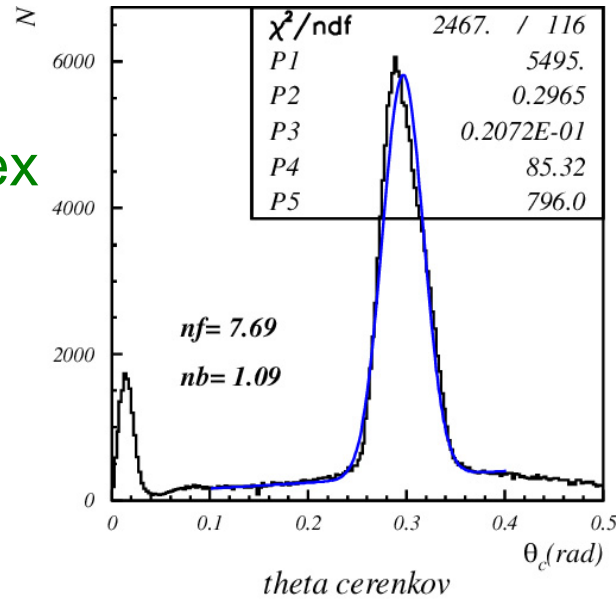
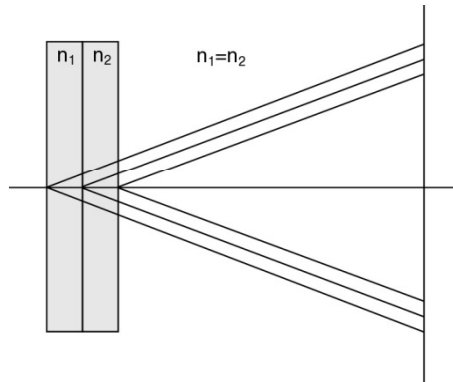


→ focusing radiator

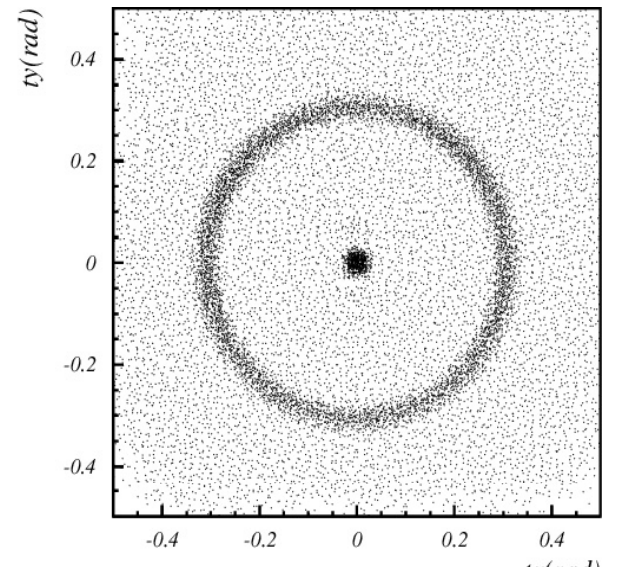
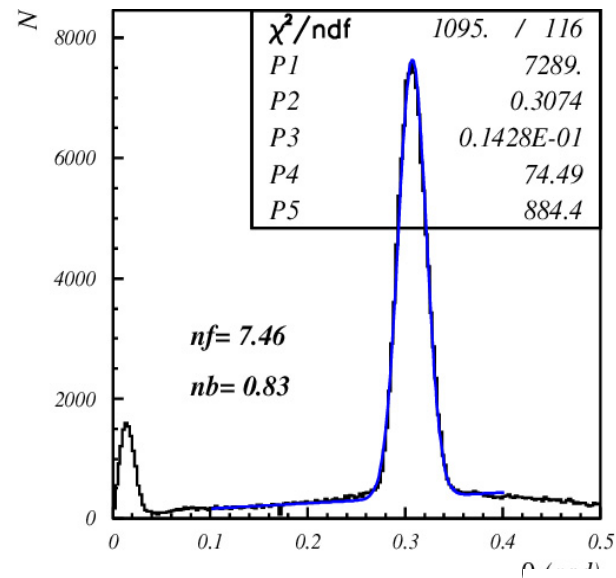
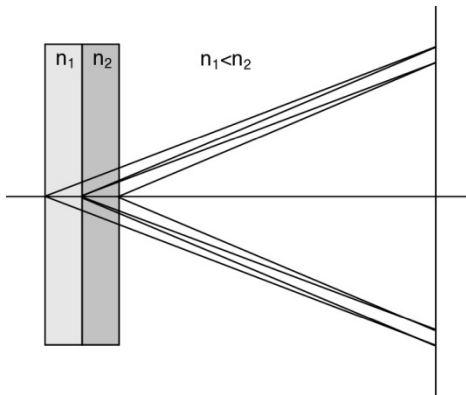


Focusing configuration – data

4cm aerogel single index



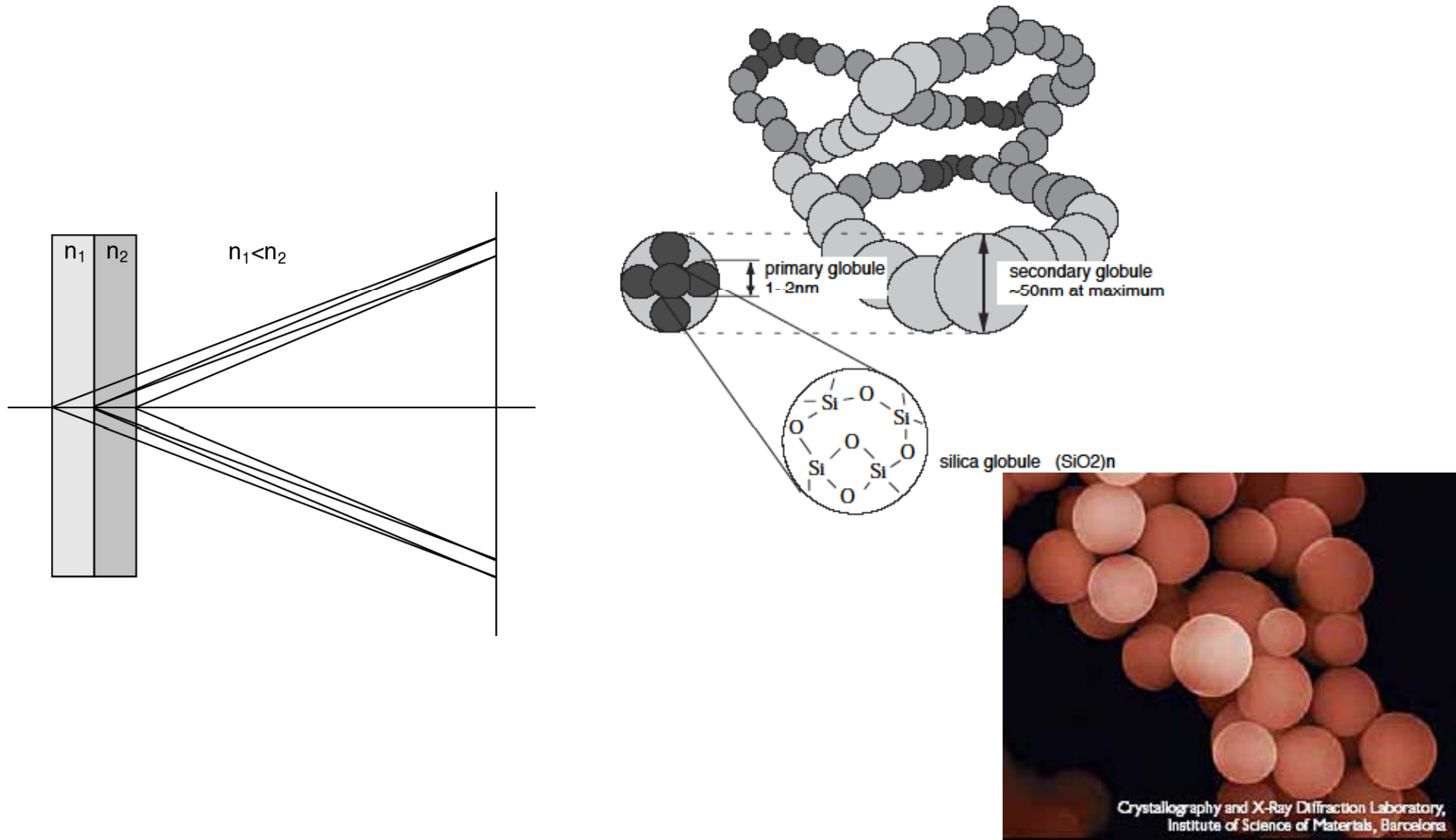
2+2cm aerogel



→ NIM A548 (2005) 383

Radiator with multiple refractive indices

Such a configuration is only possible with aerogel (a form of Si_xO_y)
– material with a **tunable** refractive index between **1.01** and **1.13**.



Aerogel production

Two production centers: Boreskov Institute of Catalysis, Novosibirsk, and KEK+Matsushita

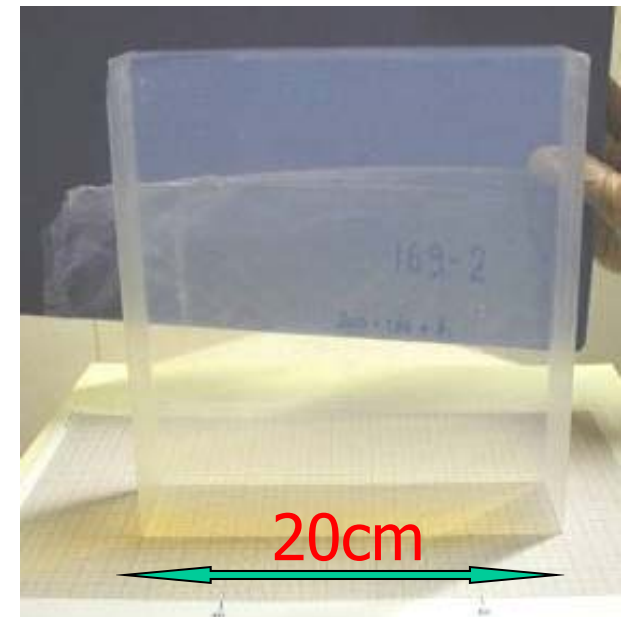
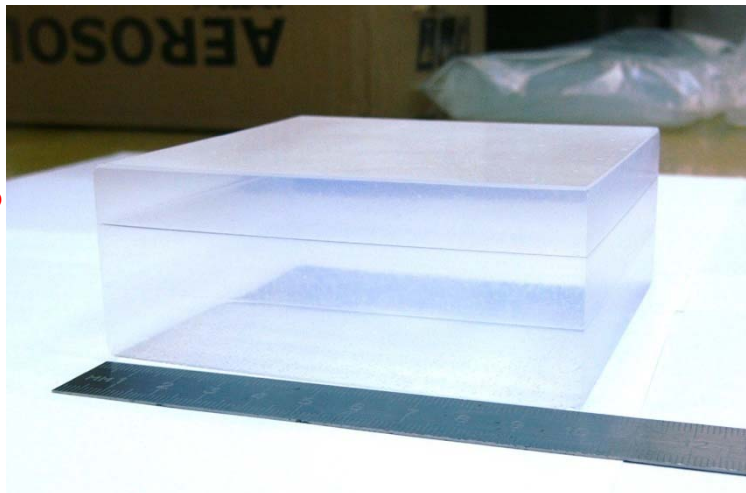
Considerable improvement in aerogel production methods:

- Better transmission (>4cm for hydrophobic and ~8cm for hydrophylic)
- Larger tiles (LHCb: 20cmx20cmx5cm)
- Tiles with multiple refractive index

$n_1=1.046$

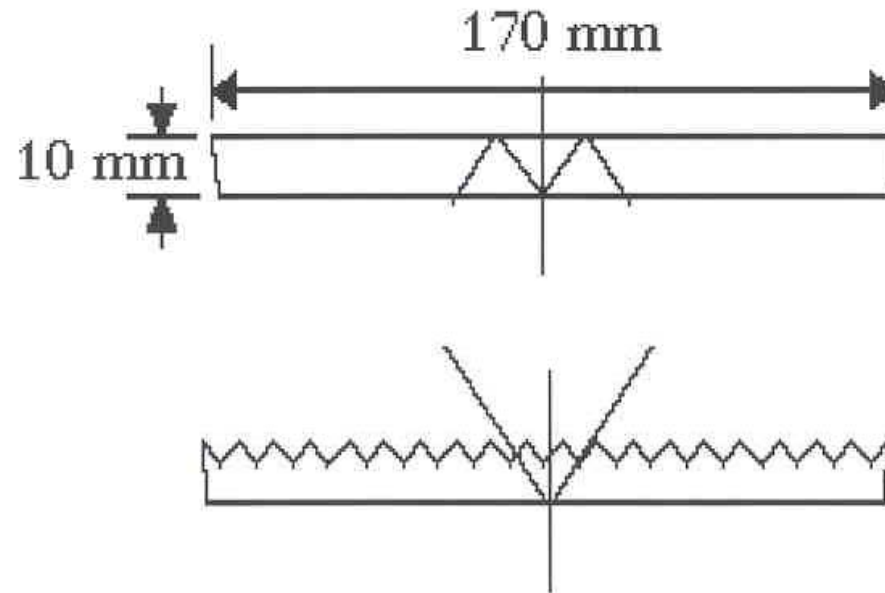
$n_2=1.041$

$n_3=1.037$



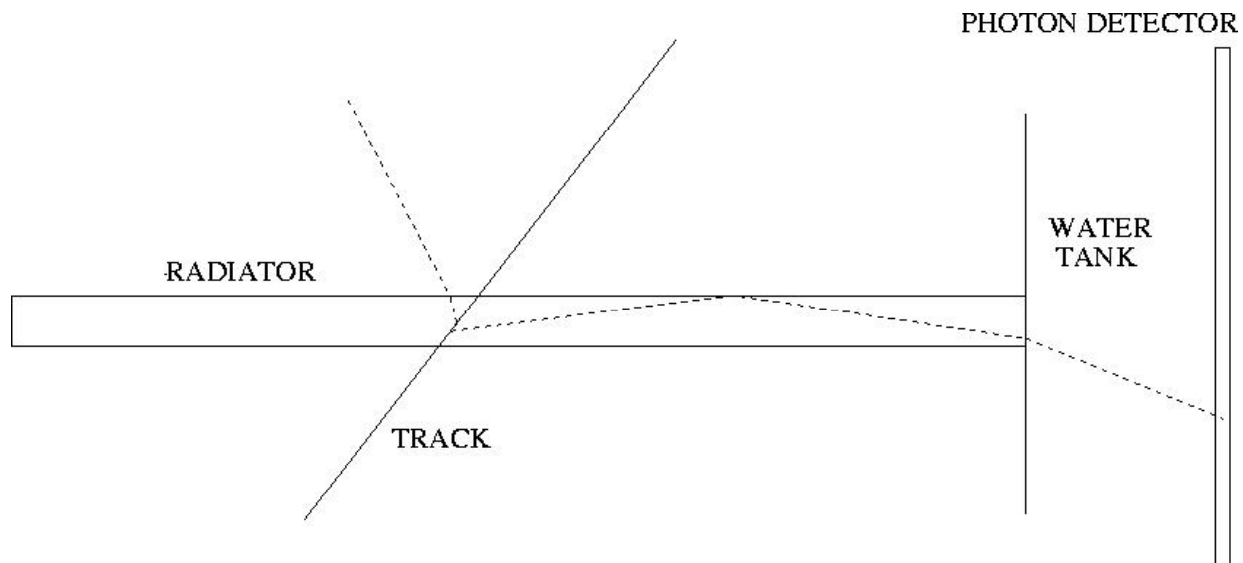
Proximity focusing RICH 2

Problem with perpendicular incidence in case of $n > \sqrt{2}$: tilt the radiator or form it as a sawtooth (CLEO).

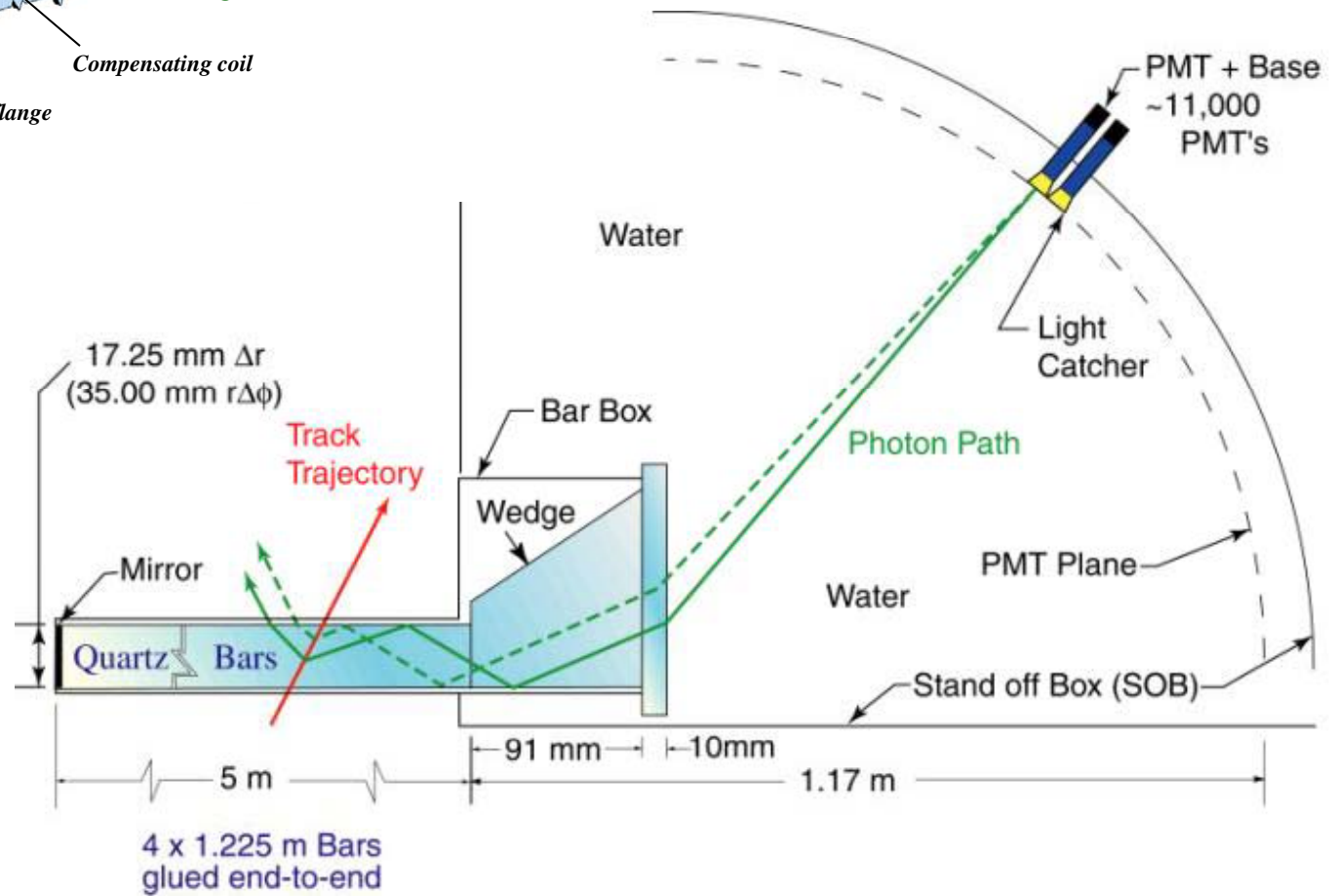
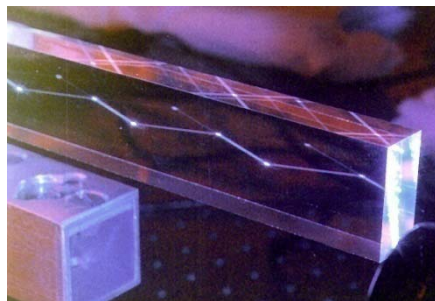
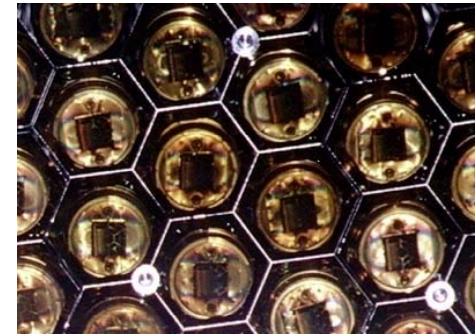
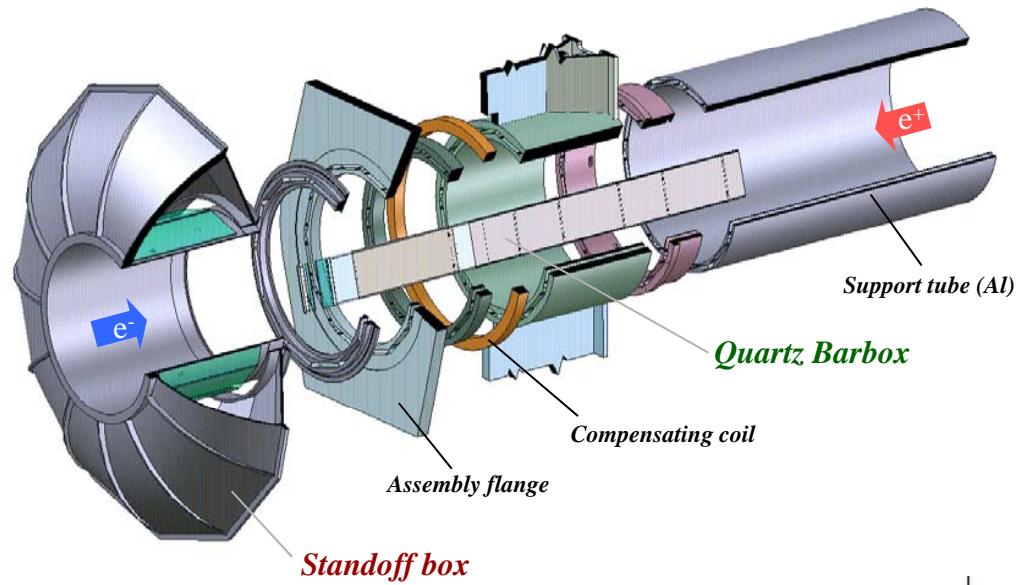


DIRC

Geometry variation: photons trapped in a solid radiator are propagated along the radiator bar to the side, and detected as they exit and traverse a gap.

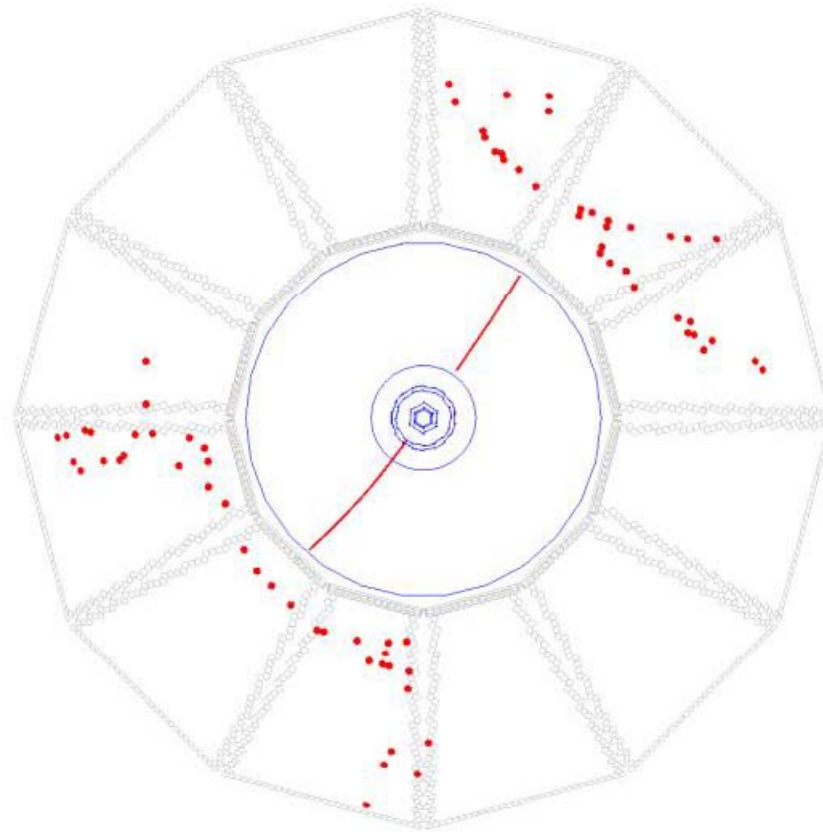


DIRC Layout



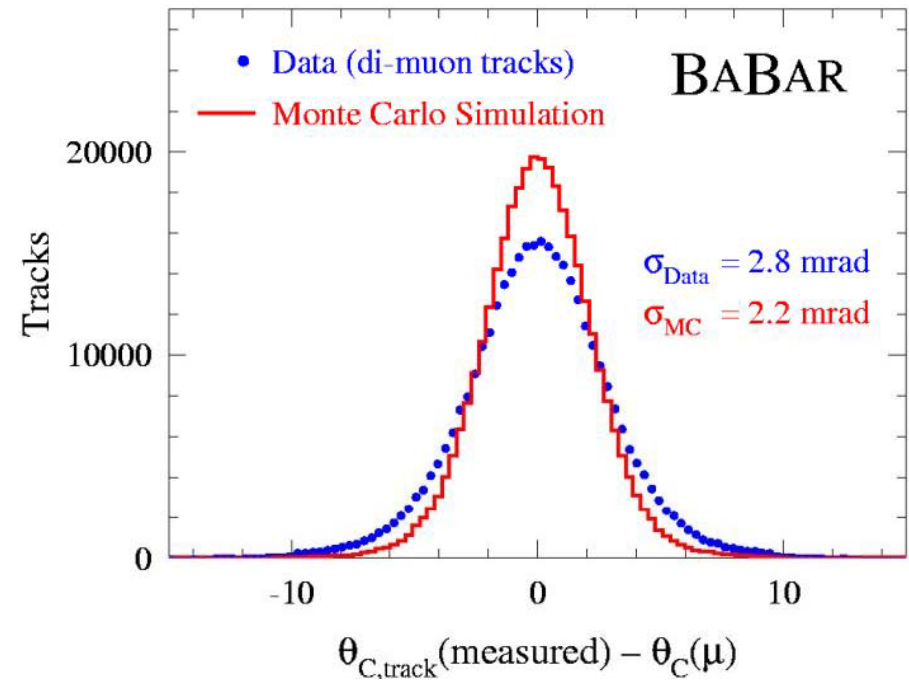
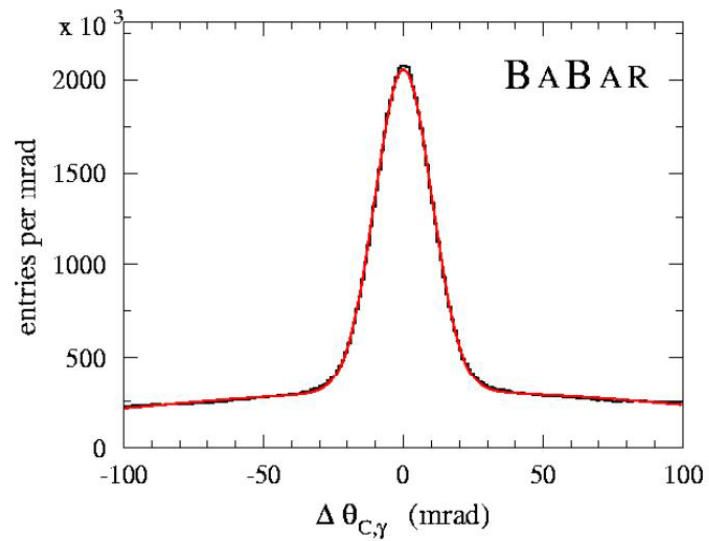
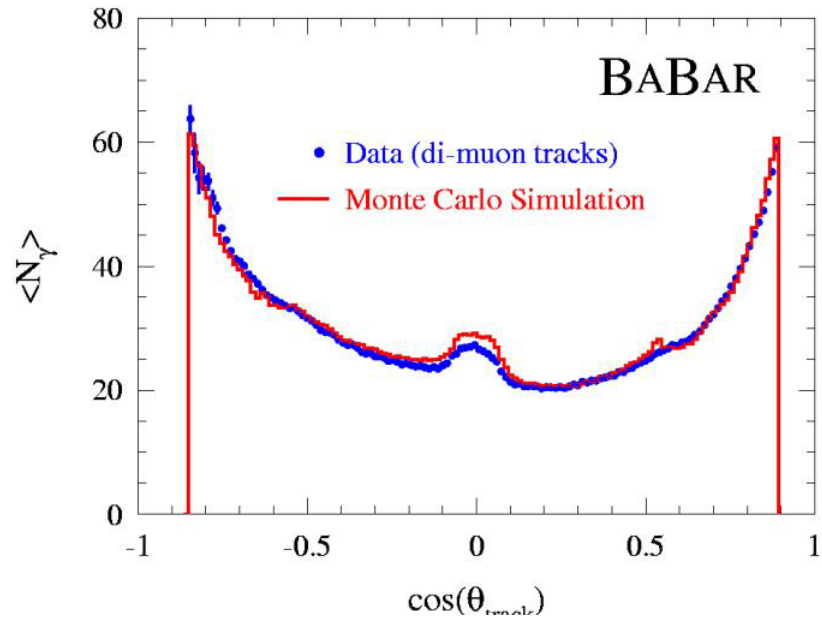
DIRC

Babar DIRC: a Bhabha event $e^+ e^- \rightarrow e^+ e^-$



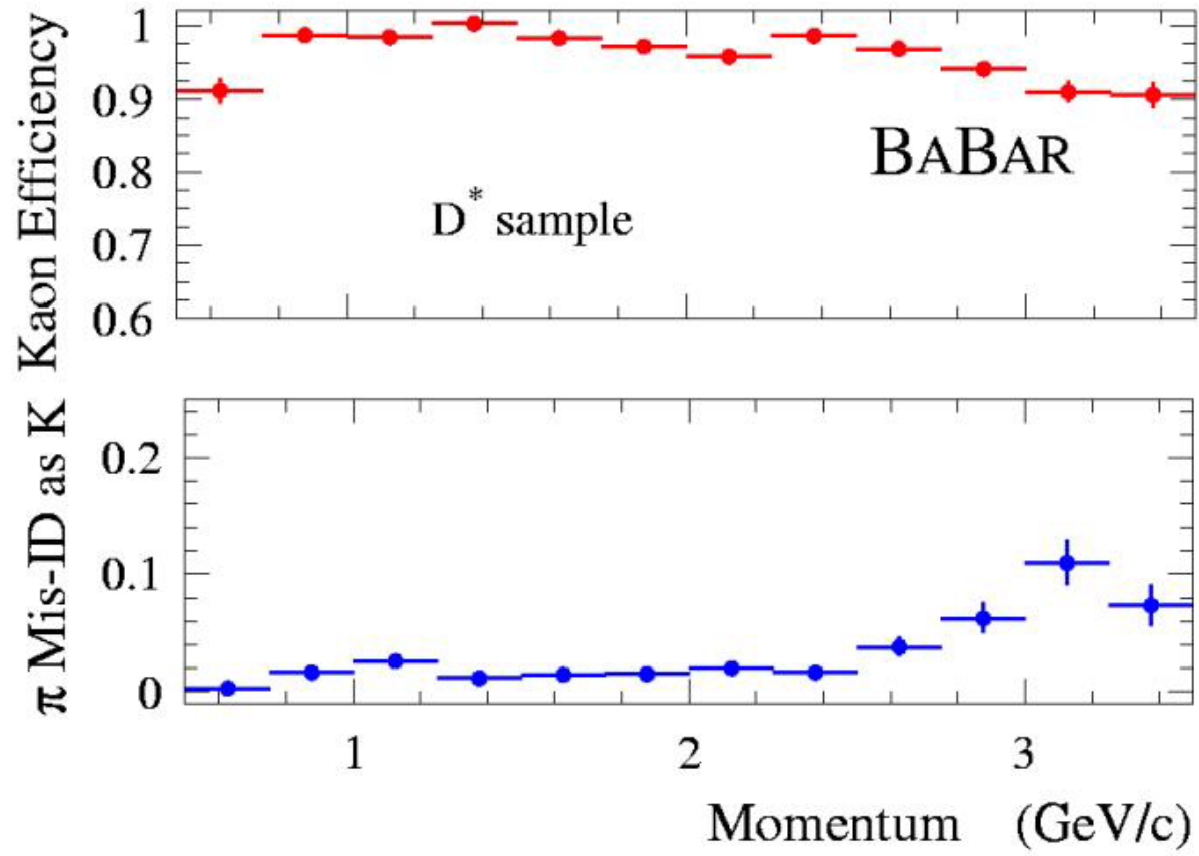
DIRC

Performance

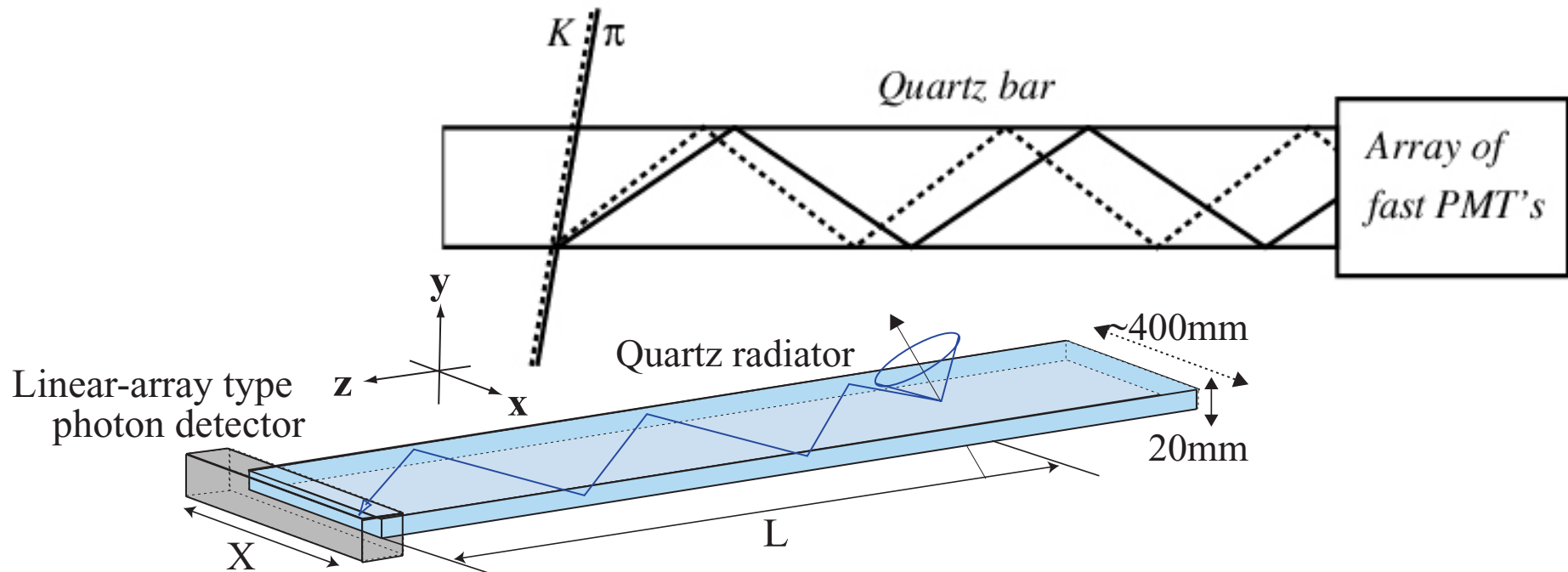


DIRC

Performance



Time-Of-Propagation (TOP) counter



Similar to DIRC, but instead of two coordinates measure:

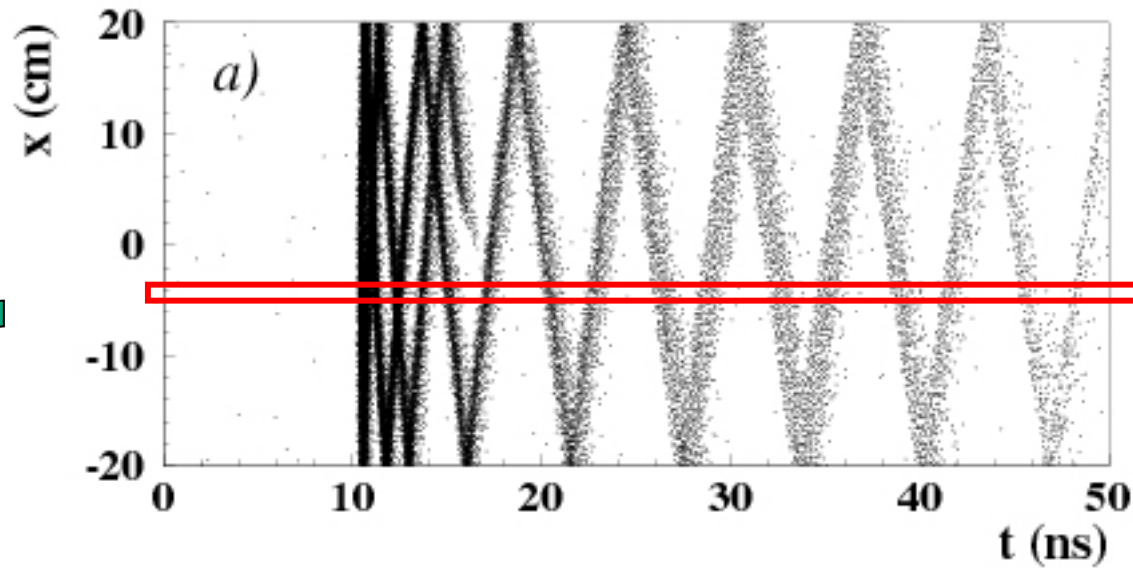
- One (or two coordinates) with a few mm precision

- Time-of-arrival

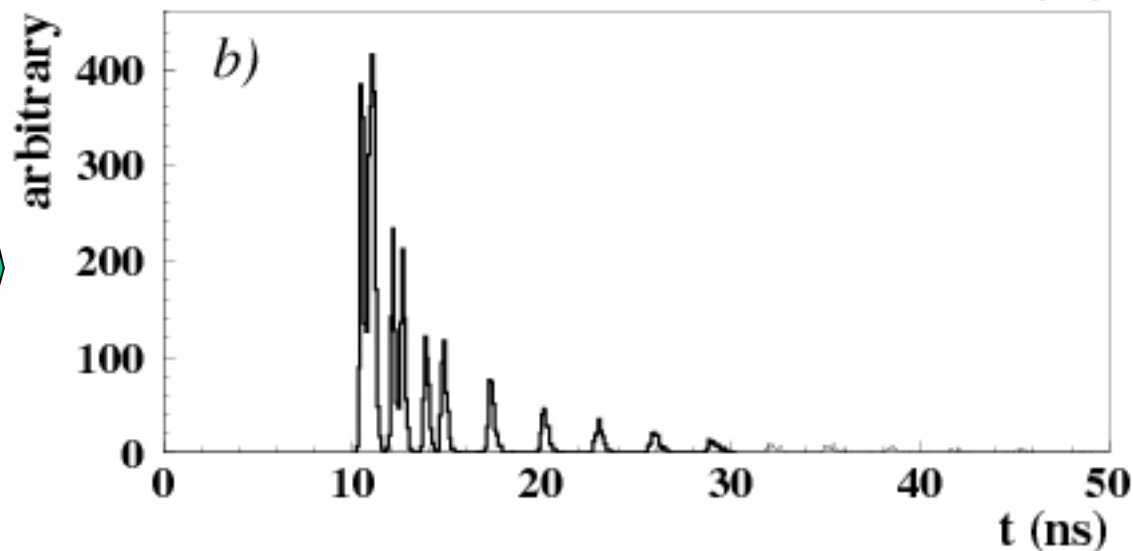
- Excellent time resolution $< \sim 40\text{ps}$

- required for single photons in 1.5T B field

TOP image



Pattern in the coordinate-time space ('ring') of a pion hitting a quartz bar with ~ 80 MAPMT channels



Time distribution of signals recorded by one of the PMT channels: different for π and K

Limits of the RICH technique

The choice of RICH radiator medium in case of a specific experiment depends on the particles we would like to identify, and their kinematics:

- the threshold momentum for the lighter of the two particles we want to separate: $\mathbf{p}_t = \beta_t \gamma_t \mathbf{m} \mathbf{c}$, $\beta_t = 1/n$ should coincide with the lower limit of momentum spectrum \mathbf{p}_{\min} . Typically

$$\mathbf{p}_{\min} = \sqrt{2} \mathbf{p}_t$$

- the resolution in Čerenkov angle should allow for a separation up to the upper limits of kinematically allowed momenta \mathbf{p}_{\max}

π/K separation example:

Limiting performance at the high momentum side: irreducible contribution to the resolution - dispersion.

radiator	LiF solid	C ₆ F ₁₄ liquid	C ₅ F ₁₂ gas	N ₂ gas	He gas
σ_θ (mrad)	7.0	3.9	0.45	0.40	0.13
σ_N (mrad)	2.2	1.2	0.14	0.13	0.04
p_{max} (GeV/c) for $3\sigma \pi/K$	3.5	6.9	50	100	330
p_{min} (GeV/c)	0.6	0.9	11	28	83

photon detector: TMAE, 10 det. photons assumed

Summary:

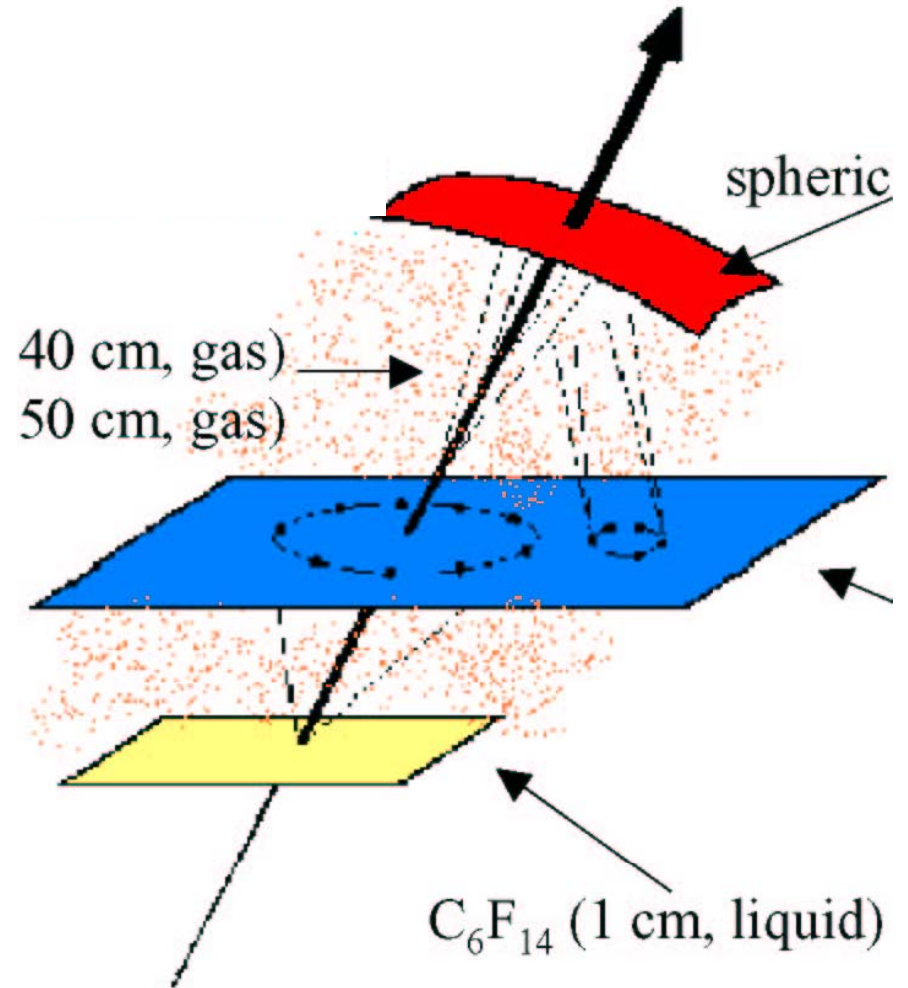
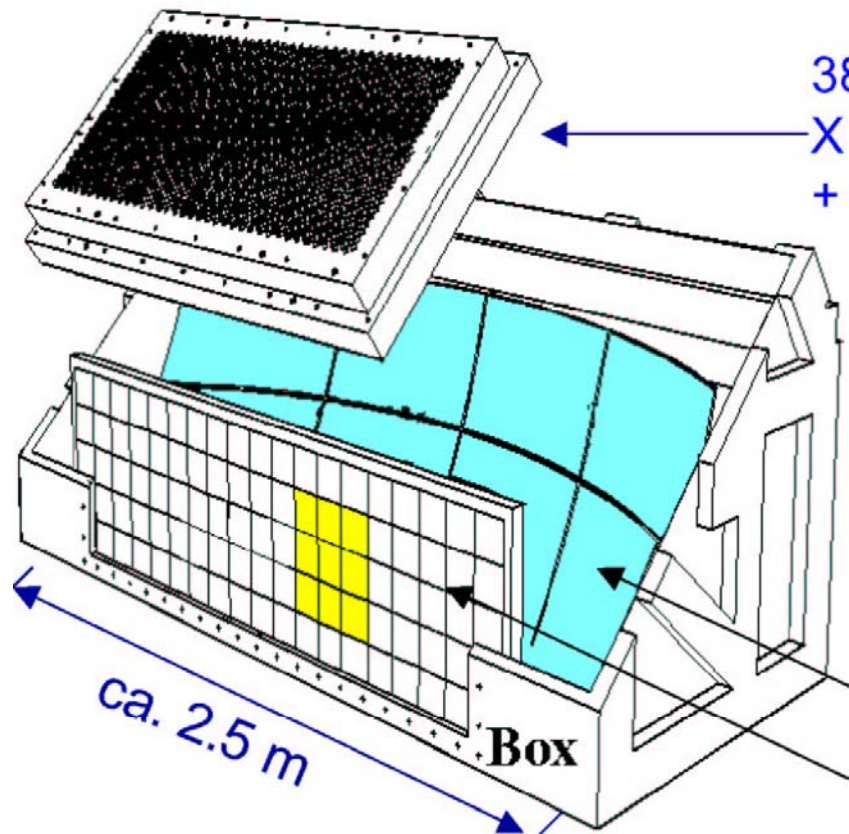
$$p_{max} / p_{min} \sim 4-7$$

for a 3σ separation between the two particles

For a larger kinematic region **2 radiators are needed!**

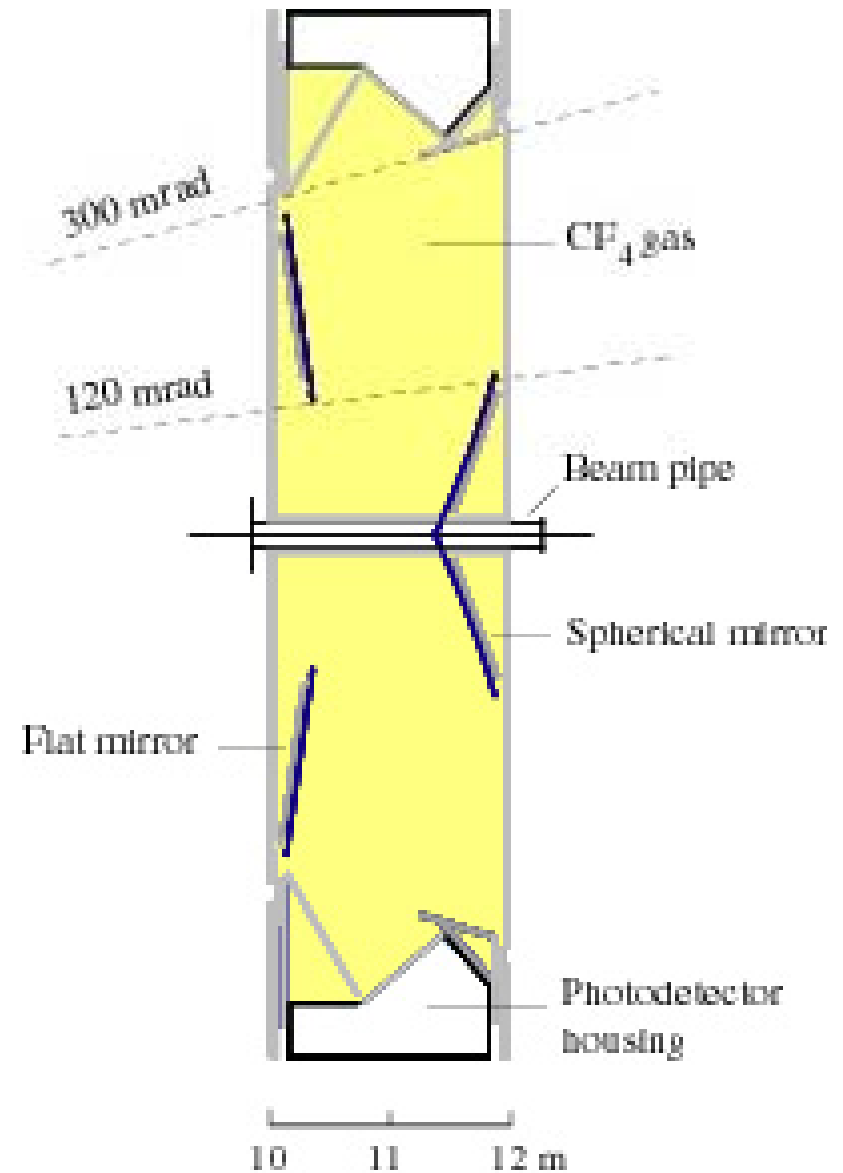
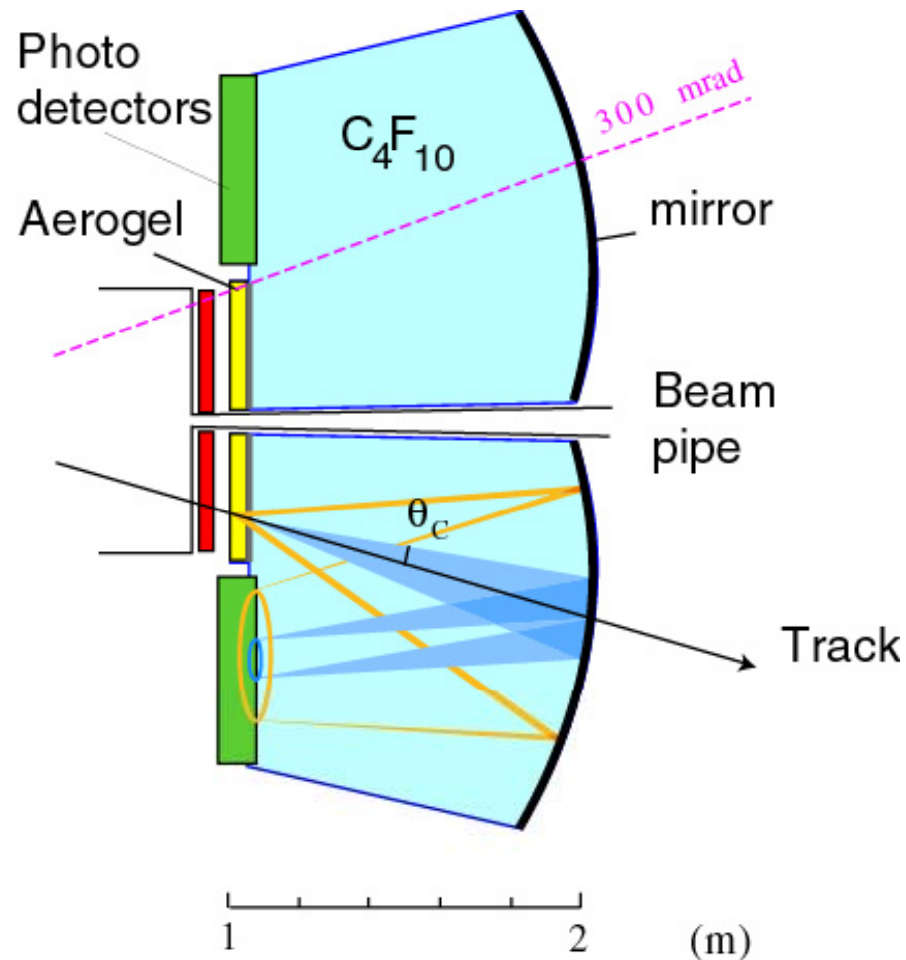
RICHes with several radiators

- --> DELPHI, SLD (liquid+gas)
- --> HERMES (aerogel+gas)



RICHes with several radiators 2

- > LHC-b (aerogel+2 gases)



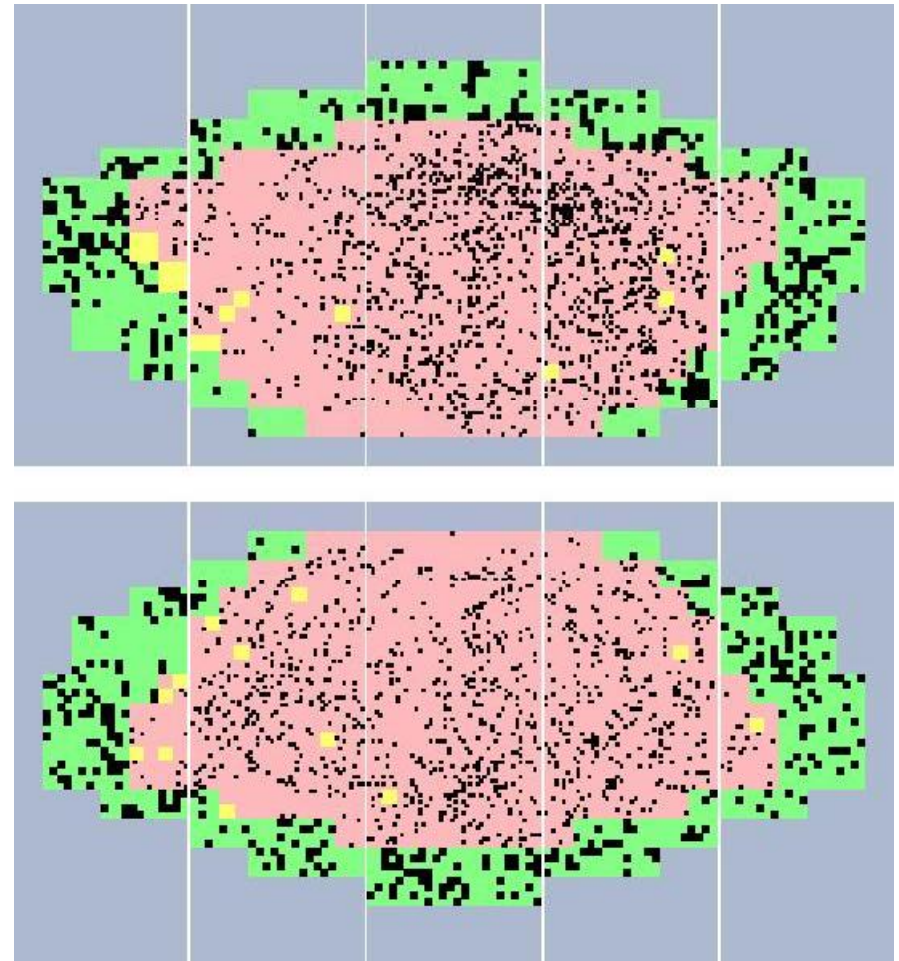
High occupancies

In case of a sizeable background
under the Čerenkov angle peak:
effective resolution for N detected
photons

$$\sigma_N > \sigma_1 / \sqrt{N}$$

HERA-B case: for isolated rings a 3σ
 π/K separation should be possible
up to $p_{\max} \sim 100 \text{ GeV}/c$.

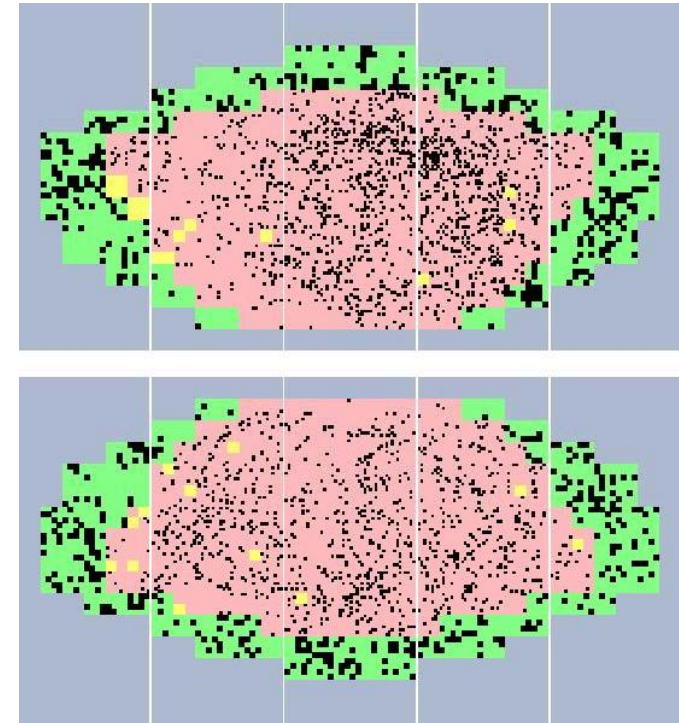
At high track densities, however, we
get $p_{\max} \sim 50 \text{ GeV}/c$.



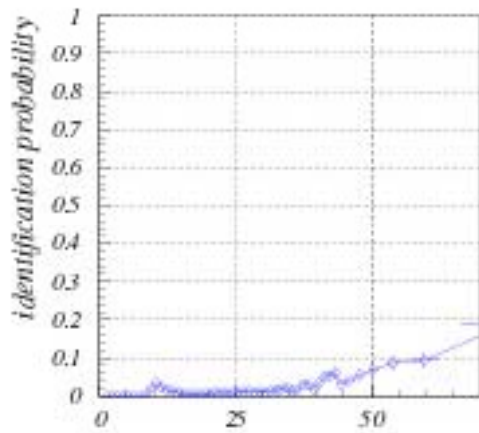
High occupancies

Still: it works actually very well!

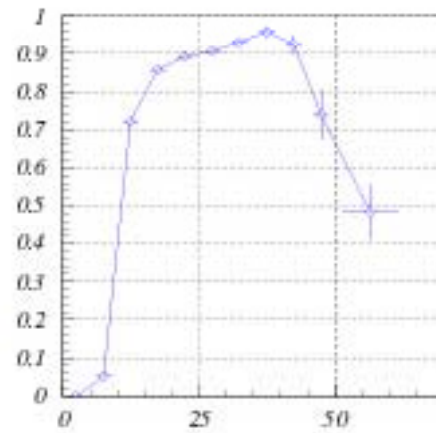
Kaon efficiency and pion, proton fake probability



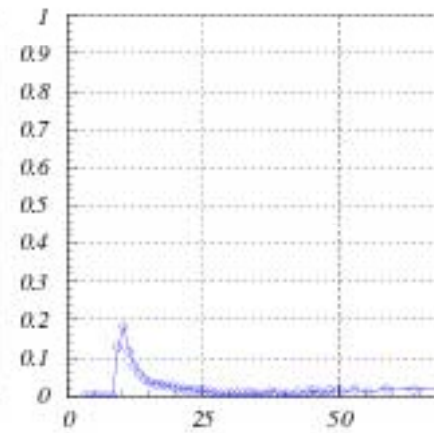
$l_{rk} > 0.50$



Pions



Kaons



Protons

Overview: RICH Building Blocks

Need 10 - 20 detected photons and a good angular resolution:

How do we get there?

Very carefully design, build and run a RICH counter

RICH Building Blocks

- Radiators
- Photon detectors
- Light collectors
- Large system aspects

Radiators

Radiator length needed for **20** detected photons in case of $\beta=1$ particles

Number of detected photons: $N = N_0 L \sin^2\theta$ with $N_0 = 50-100/\text{cm}$

N = 20:

- Solid radiators: example $n=1.5$, $\sin^2\theta=1-(\beta n)^{-2}=0.55$
-> $L=0.7\text{cm}$ (for $N_0 = 50/\text{cm}$)
- Gaseous radiators: example $n=1.001$, $\sin^2\theta=1-(\beta n)^{-2}\sim 2(n-1)=0.002$
-> $L=200\text{cm}$ (for $N_0 = 50/\text{cm}$)
- Aerogel radiator: example $n=1.05$, $\sin^2\theta=1-(\beta n)^{-2}\sim 2(n-1)=0.1$
-> $L=4.3\text{cm}$ (for $N_0 = 50/\text{cm}$)

Photon detectors

Need:

- photosensitive substance
- amplification/multiplication of the photoelectron

Photosensitive substances:

- solids (SbCs, Sb-K-Cs, CsI, ...)
- gases (TMAE, TEA)

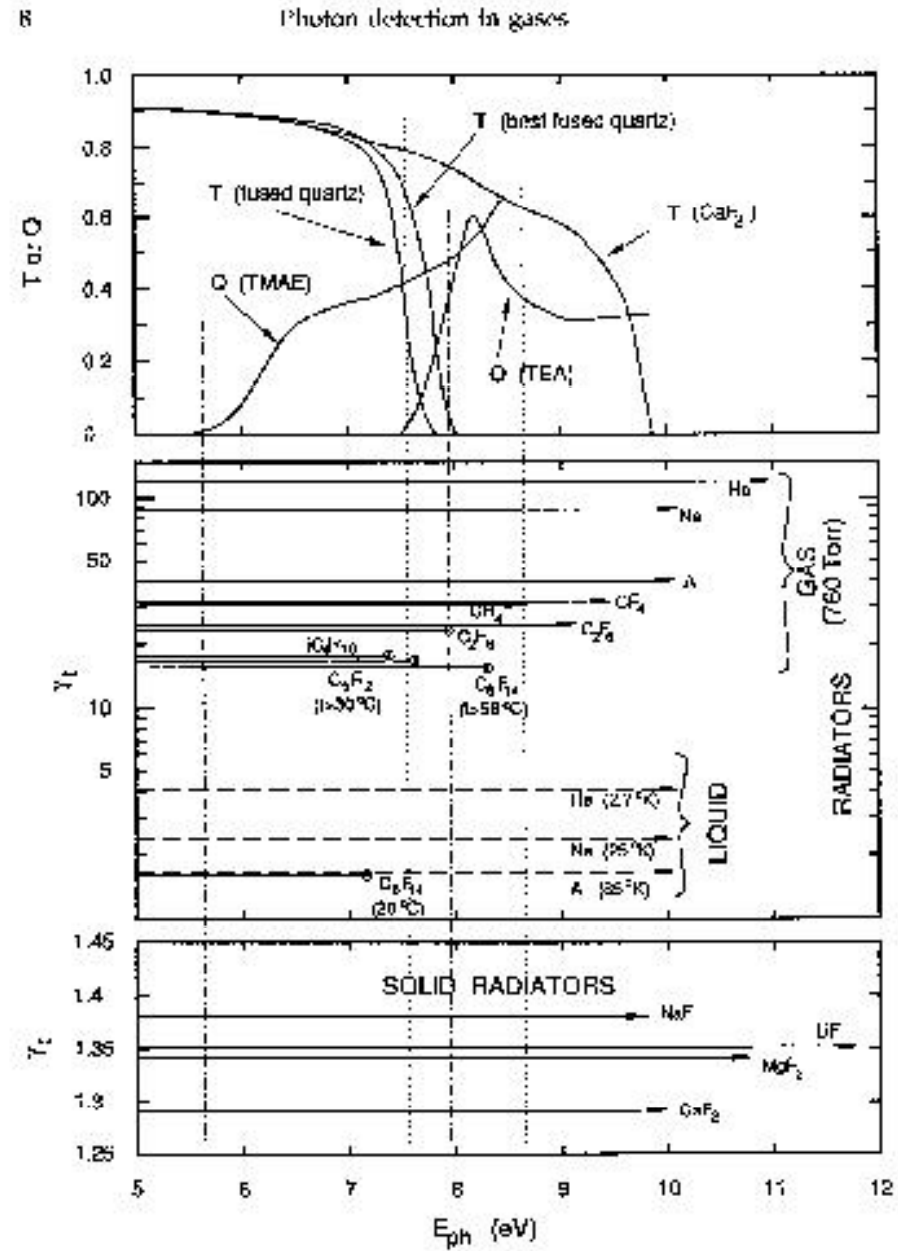
Multiplication through:

- multiplication in a dynode structure in vacuum
- avalanche amplification in a gas
- avalanche amplification in silicon
- photoelectron acceleration in electric field in vacuum, detection in Si

Combined to

- gas based photon detectors (wire chambers with 2d read-out)
- vacuum based photon detectors (PMT, HPD)
- silicon based photon detectors (APD, VLPC)

Detection efficiency,
radiator and window
transmission.



Mirrors

Good reflectivity in UV: not trivial to produce and maintain (water!).

But: was done and worked very well (DELPHI RICH, SLD CRID) for years

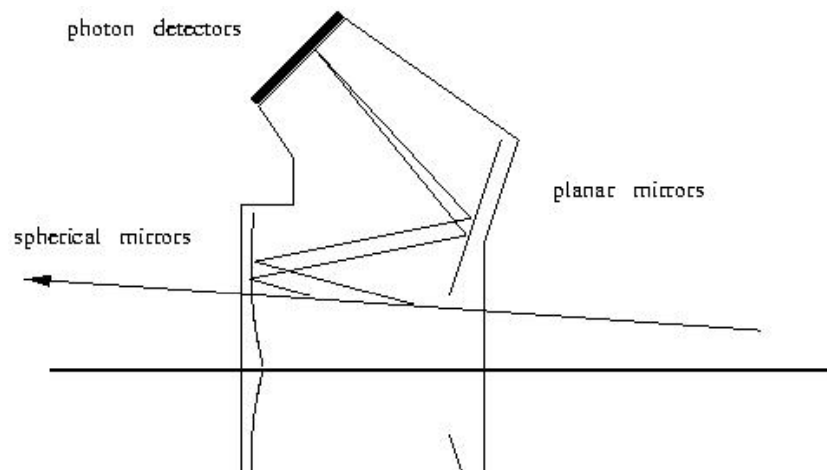
DELPHI mirror system →

HERA-B mirror system →

Typically: 0.5-1 cm of glass, coated with Al, and a protective layer (MgF_2 etc).

Light mirrors: on composite substrate (expensive!)

System of mirrors of the HERA-B RICH



Mirrors 2

Sistem of mirrors of the
DELPHI RICH



Windows

Transmission of radiator exit windows has to match the sensitivity of the photosensitive substance.

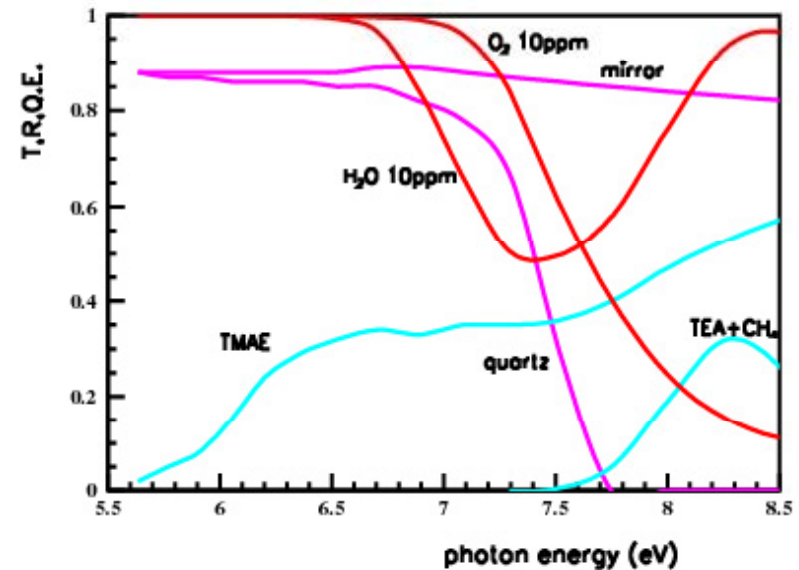
- quartz for TMAE, CsI,
- CaF_2 for TEA,
- UVT plexiglass for PMTs

For comparison: mylar is only transparent above ~ 400 nm, UVT plexiglass above 300 nm

Large system aspects

- Water and oxygen content in the radiator (order of one meter to a few meters of gas, or a centimeter of liquid) have to be kept **very low** in case of photon detectors for **UV light**.

Example: take a RICH with photon path of 7.5 m, plot the influence of 10 ppm of water or oxygen (or 100 ppm for 0.75 m).



Gas based detectors (TMAE, TEA, CsI)

Why bother to develop gas chamber based light sensitive detectors instead of photomultiplier tubes?

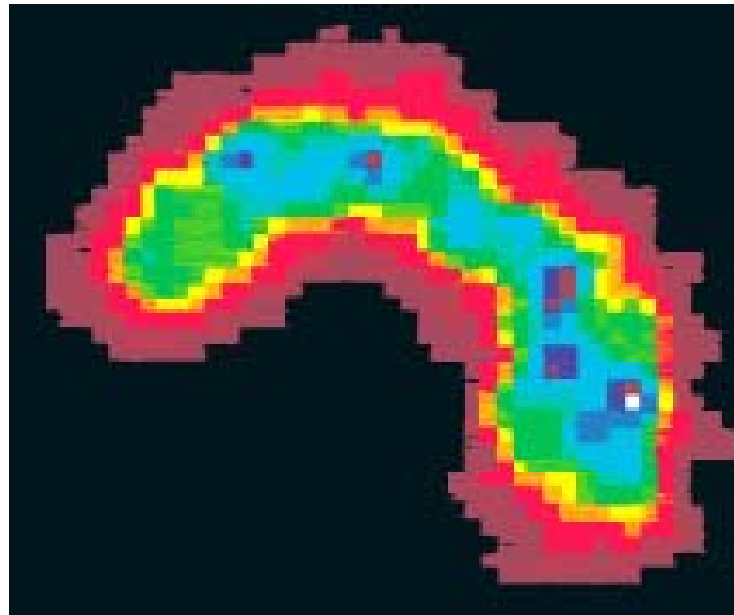
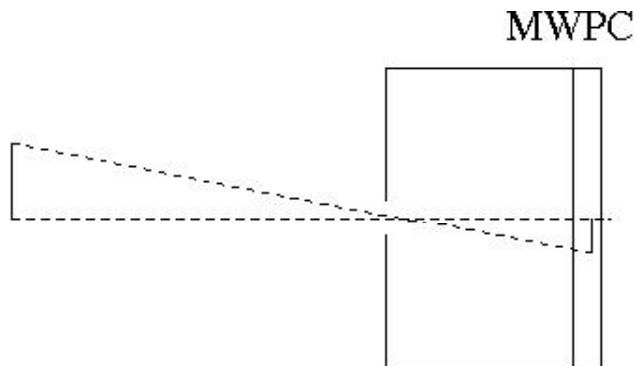
Virtues:

- good spatial resolution (order few mm),
- coverage of large surfaces (square meters) with a large fraction of active area at reasonable cost,
- highly efficient single photon detection.

These virtues made gas based detectors boost the RICH identification method.

Example: metal surface

Note that a polished metal surface is sensitive to visual light, although with a very low efficiency.



The 'photograph' of the tungsten filament in a light bulb was taken with a pinhole camera (a black box with a small hole) equipped with a small 5cm x 5cm MWPC with a two dimensional delay line read-out.

UV photon detection in wire chambers: photosensitive materials

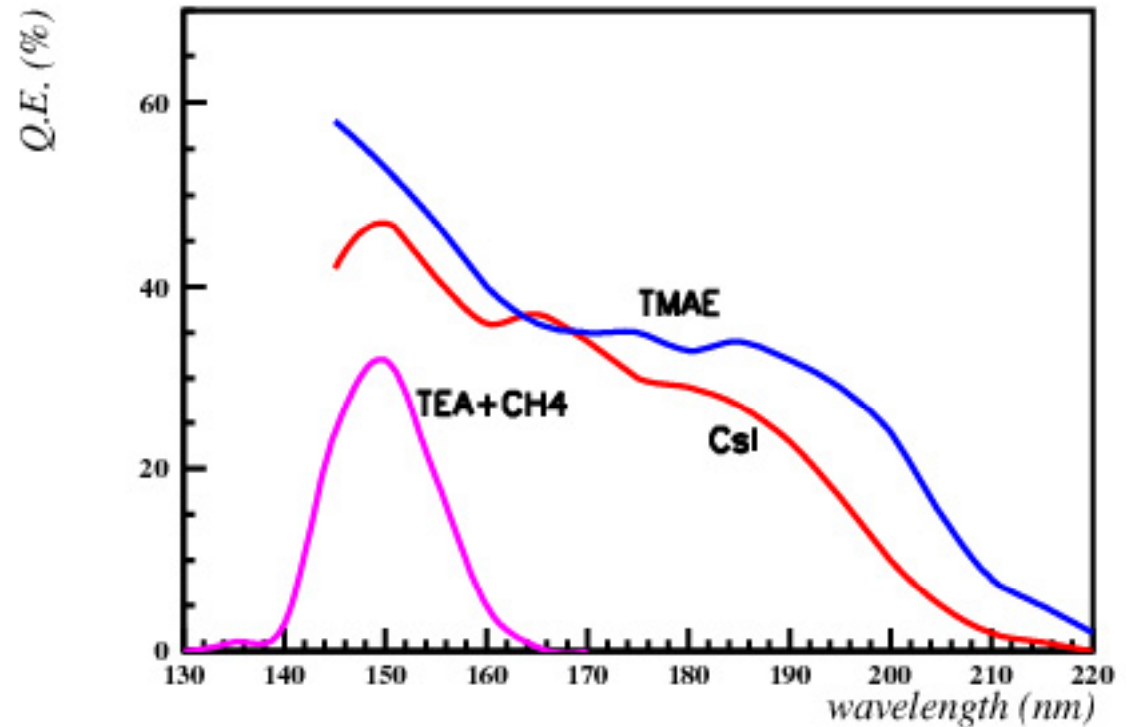
Either added to the gas mixture

- TMAE

- TEA

or a layer on one of the cathodes

- CsI on a Sn-Pb substrate

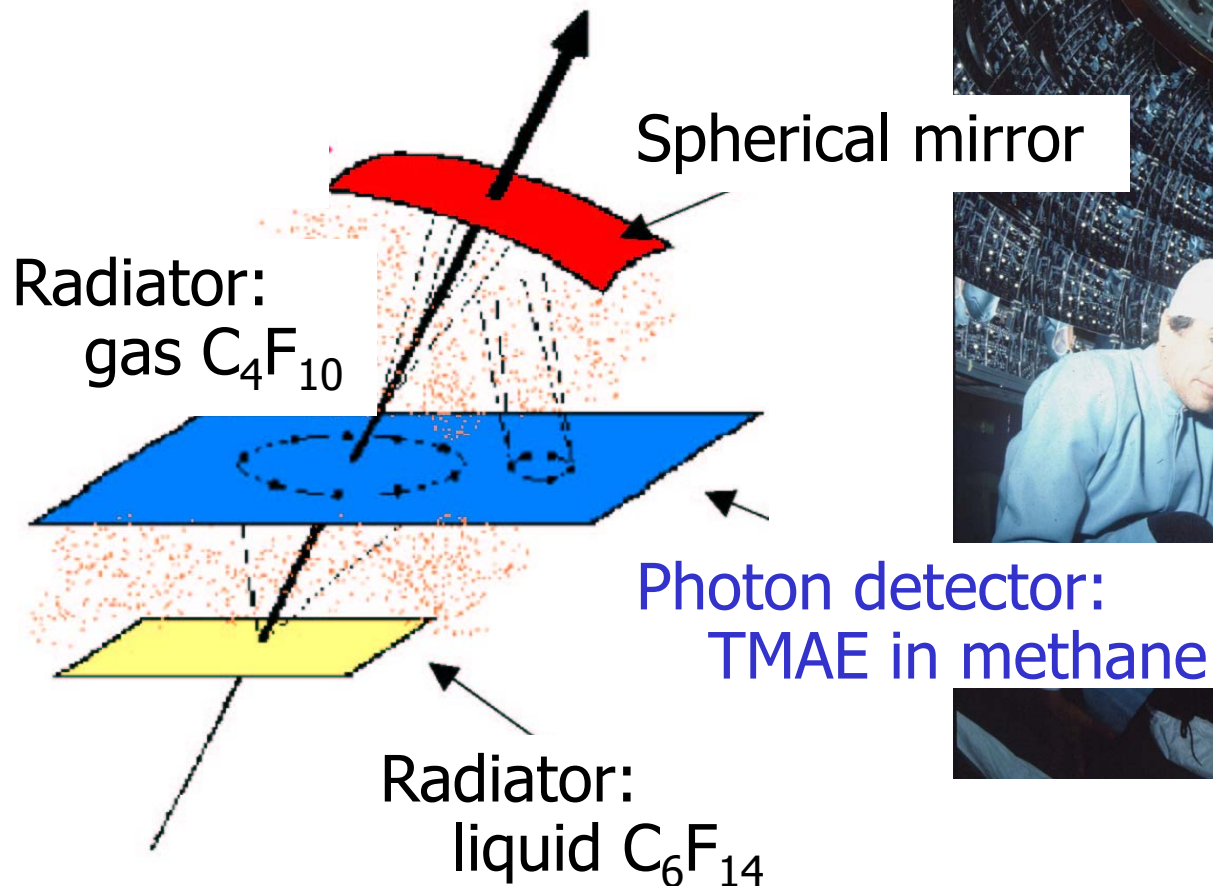


quantum efficiency vs λ

All chemically very active!

DELPHI RICH

Inside the DELPHI RICH:
segmented spherical mirror



TMAE

A liquid at room temperature, with a vapour pressure of 0.30 torr (at 20 C)

Absorption length of about 3~cm at room temperature

Typical chamber: a thick conversion volume (~10cm is needed to enable an efficient absorption, usually combined with a TPC type chamber)

Gas purity and chamber materials : very clean system needed (TMAE reacts intensely with oxygen!), stainless steel pipes and valves, oxysorb

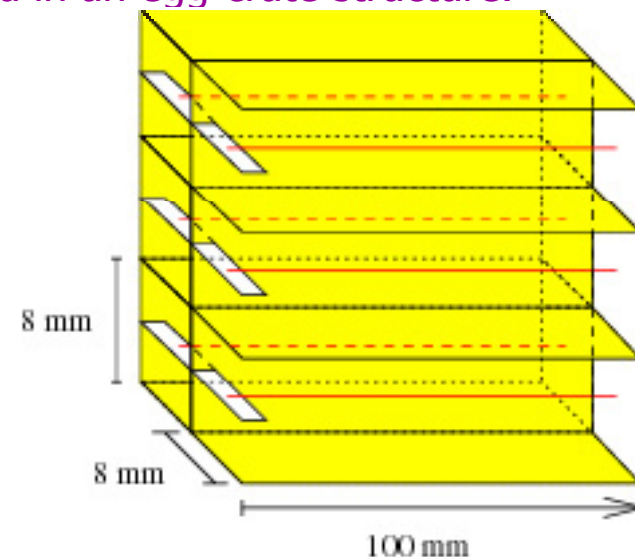
Examples:

TPC (Omega, DELPHI, SLD)

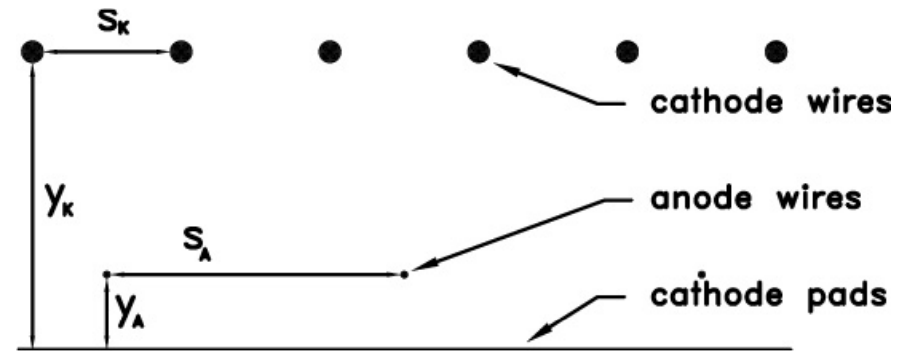
multiwire chamber with pad read-out (Caprice): two parallel plate stages, coupled to a multiwire chamber with pad read-out (CERES)

Higher rates: a different geometry is needed. An example is the JETSET/HERA-B prototype geometry with anode wires embedded in an egg-crate structure.

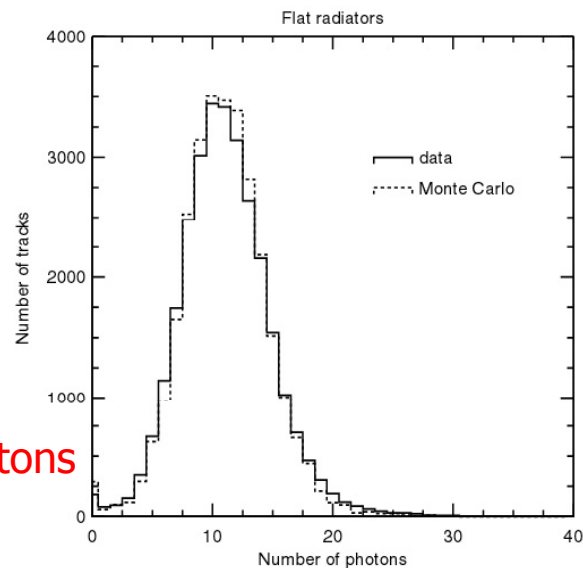
Photons enter the chamber from the left side.



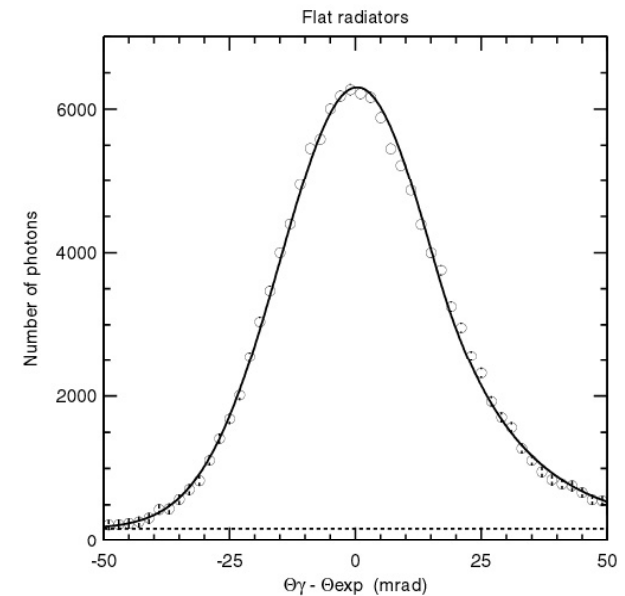
TEA



- A liquid at room temperature, with a vapour pressure of 52 torr (at 20 C); added to methane by bubbling it through the liquid.
- Absorption length of 0.61mm (at 20 C).
- Typical chamber: a multiwire chamber with pad read-out.
- Typical dimensions: few mm thick.
- First used in the pioneering RICH experiment E605.
- Examples: Cl



Number of detected photons



Resolution

CsI

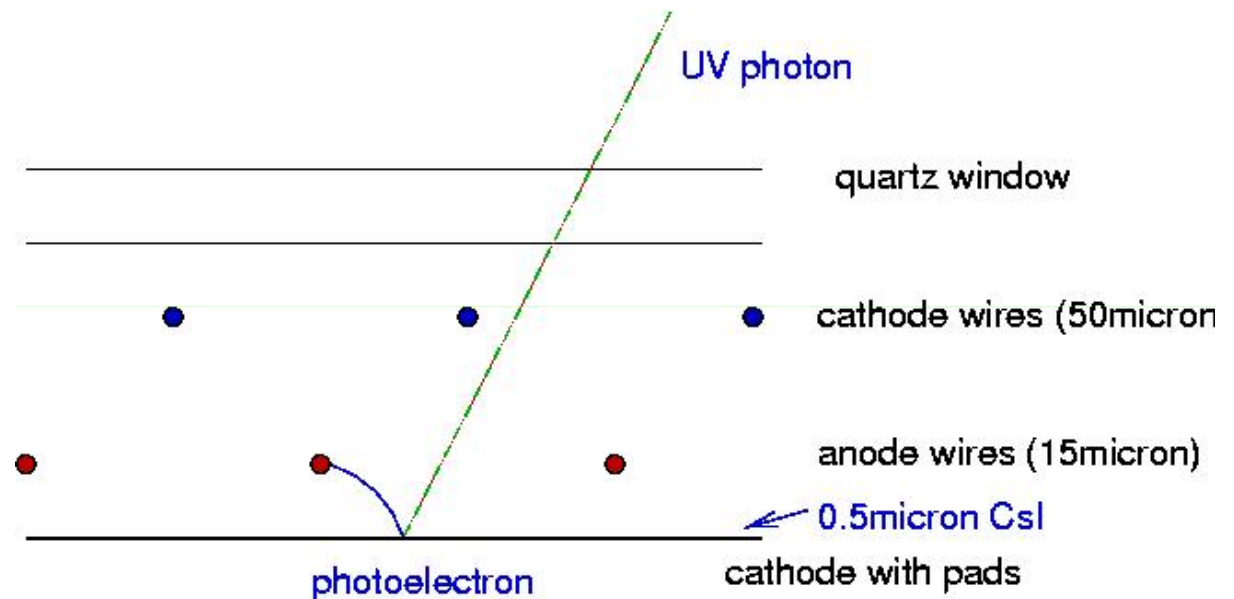
A solid layer, 100nm – 1000nm thick, evaporated in vacuum on one of the cathodes (or on the entrance window)

Needs a high purity chamber gas, usually methane with a water and oxygen content of order ppm.

Typical chamber: a multiwire chamber with pad read-out, reflective photocathode.

Electric field: voltage on cathode wires has to be adjusted to guarantee a uniform amplification around the anode wire.

Chamber variation: photocathode evaporated on the entry window, chamber can then be a MSGC, MGC, can also have a GEM amplification structure and multiwire chamber with pad read-out.



UV photon detection in wire chambers: photo-electron detection

Distribution of pulse heights due to individual photoelectrons is exponential!

Dramatic consequence for photo-electron detection probability (=efficiency). For a given electronics threshold U_{th} the efficiency is

$$\varepsilon = \int_{U_{th}}^{\infty} \frac{1}{U} e^{-\frac{U}{U}} dU = e^{-\frac{U_{th}}{U}}$$

-> efficient detection of single photons is only possible with a low noise electronics!

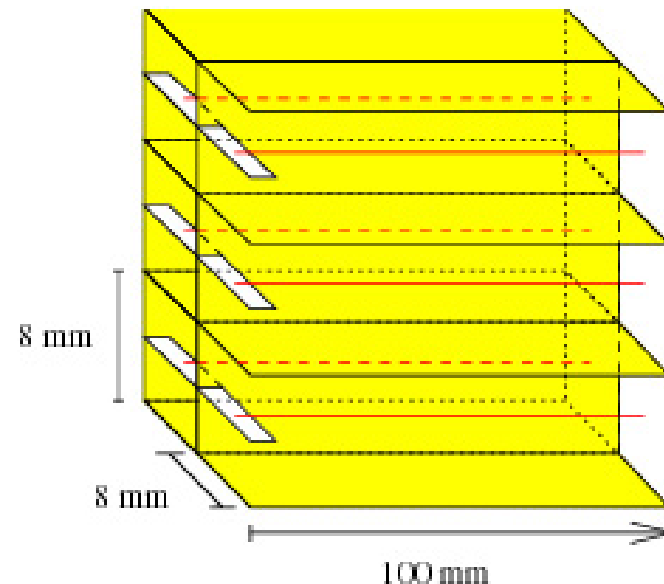
How low is low? The visual charge is about 20% (for integration times of order $\tau=20\text{ns}$) of the avalanche charge, i.e. at a gas amplification of $2 \cdot 10^5$ the average detected signal corresponds to $4 \cdot 10^4$ electrons.

If we want to cut noise at 4σ , and keep a 90% efficiency ($U_{th} = 0.1 U$), the electronics noise has to be kept at $4 \times 10^4 \times 0.1 / 4 = 1000 \text{ e- ENC}$

Problems of wire chamber based photon detectors 1

Feedback photons: **Avalanche photons cause emission of secondary photons (feedback photons)**. The process is enhanced because chamber is light sensitive no or little quenching gas is used. The result is more background (best case) or chamber instability (in particular in cases when a lot of primary electrons are liberated by a charged track which passed the chamber).

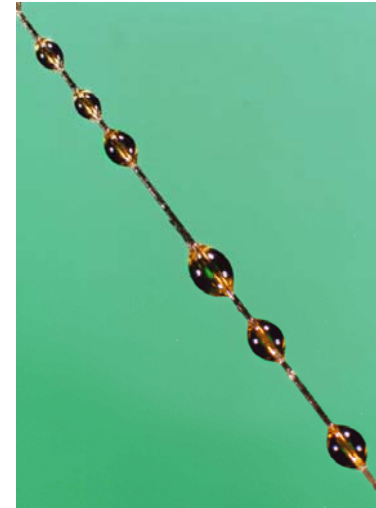
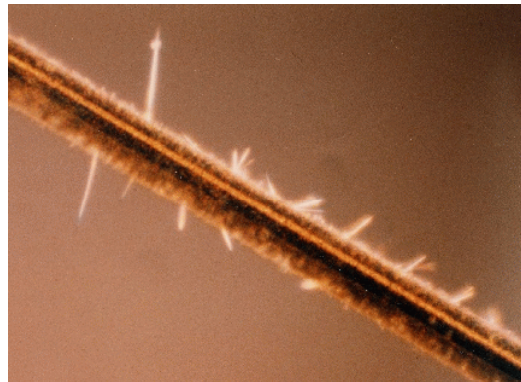
- Solution: use isolated cell geometry or isolate anode wires from each other by using blinds, or work at low gain, preferably in a region with no charged tracks



Problems of wire chamber based photon detectors 2

Anode related effects: ageing due to **accumulation of polymerisation deposits on the anode wires**, particularly in TMAE loaded gases. Consequence: gas amplification drops as a function of deposited charge. A drop of amplification exponentially decreases the efficiency.

- Recovery: Remove deposits by washing the wires with alcohols, or heating. **Attractive: in situ heatable anode wires for high rate applications.**



Cathode related effects: Accumulation of polymerisation deposits on the cathode planes (good insulators) can cause a large electric field after enough charge has accumulated, emission of electrons from the cathode - Malter effect.

Summary on RICHes with wire chamber based photon detectors.

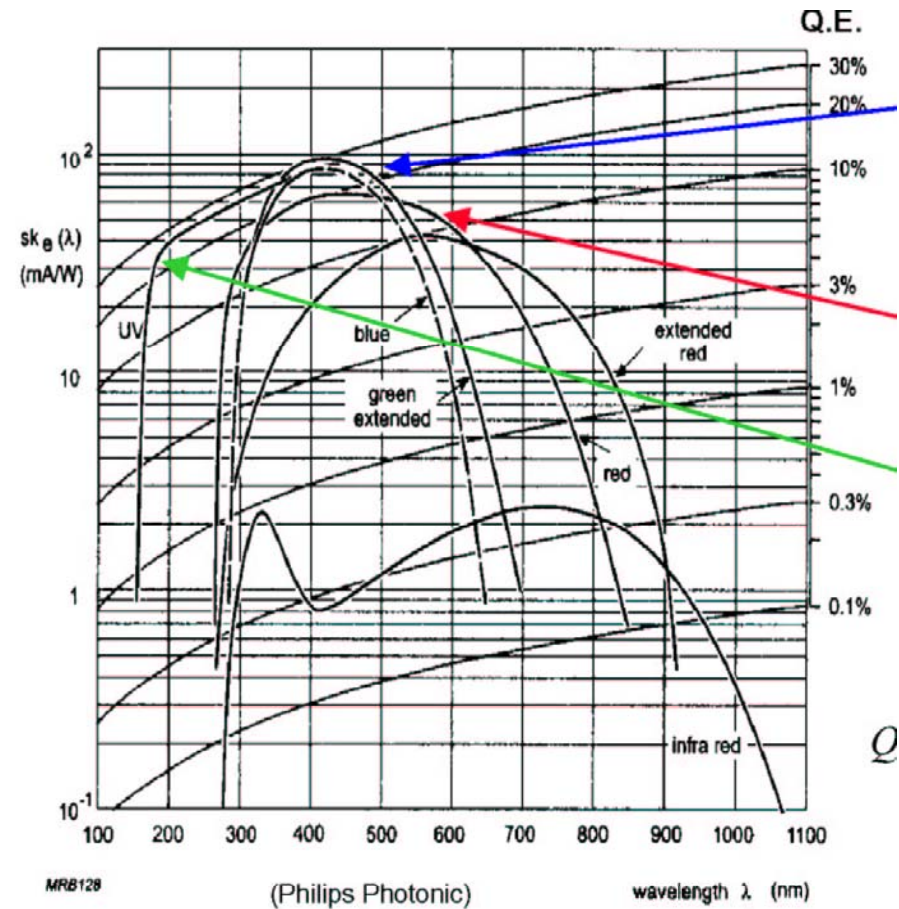
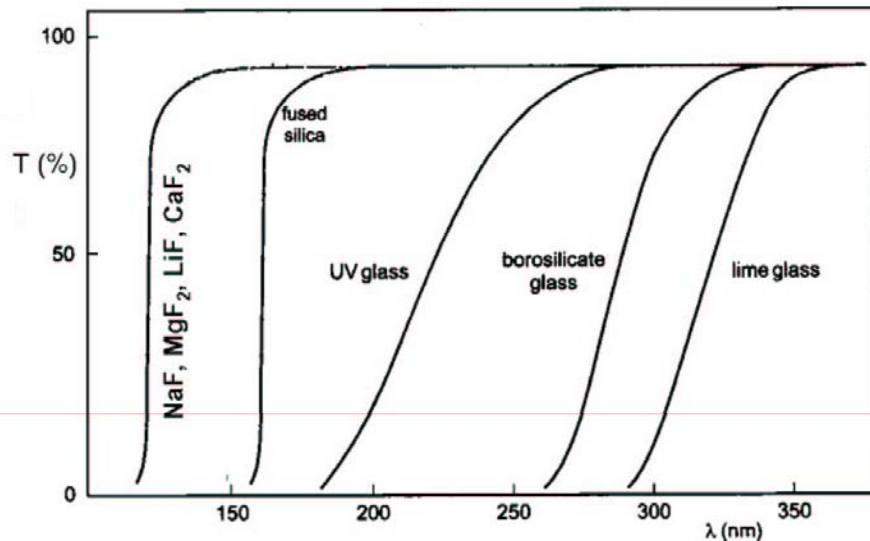
- Only UV photons can be detected, their detection is not trivial.
- However: large system have been successfully operated for years.
- Visual photon detection in gas chambers: still being developed.
- High rate operation: problematic, in particular if long term stability is required – much more R+D would be needed to become competitive with PMTs

Vacuum base photon detectors

Vacuum operation: a large variety of photosensitive material in visible.

RICH counters: preferably highest sensitivity in blue.

Low wavelength cut-off: defined by the window.

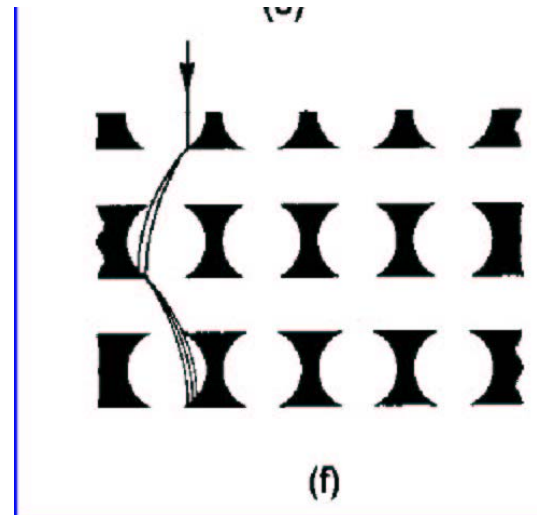
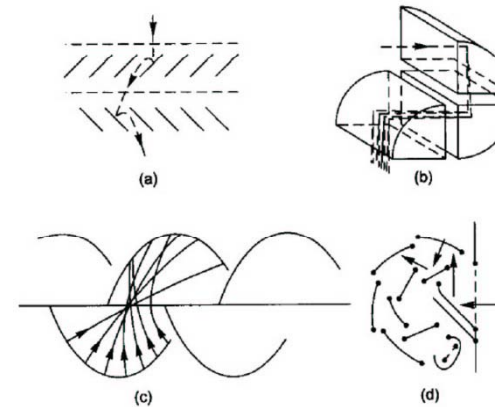
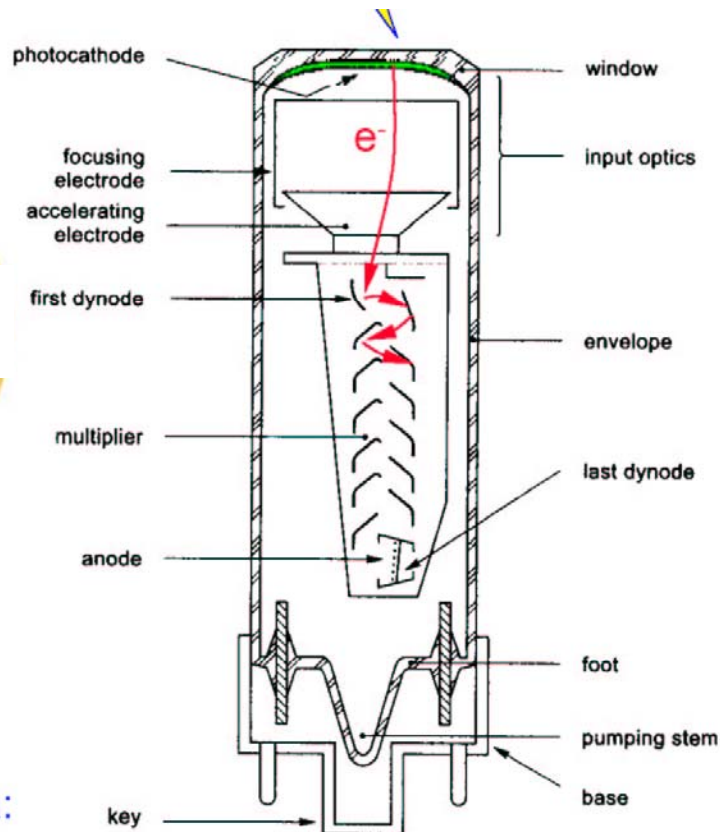


Photomultiplier tubes (PMTs)

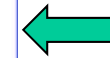
Photoelectron signal is amplified by the multiplication at each dynode stage.

Several issues:

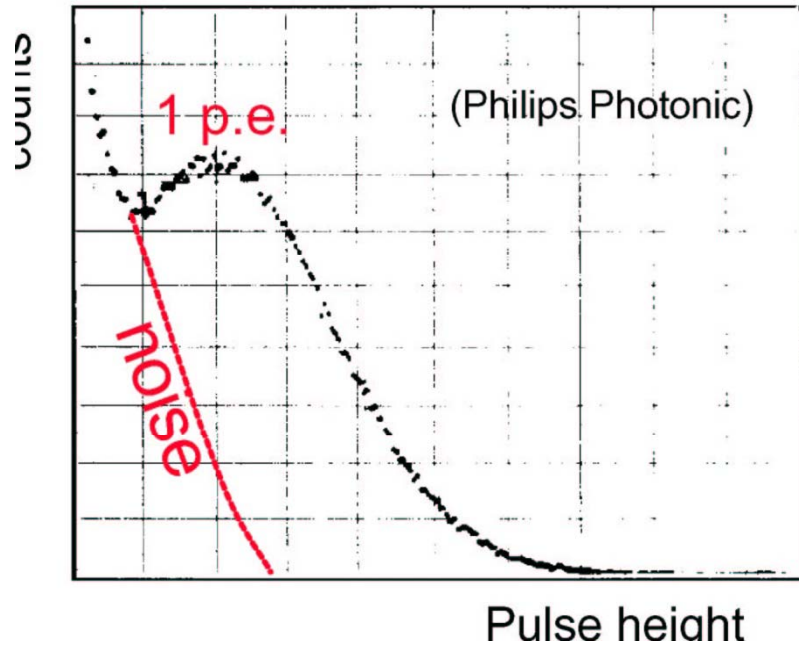
- Single photon response
- More than one channel per tube
- Low dead area fraction



Best structure to implement many anodes with very little cross-talk



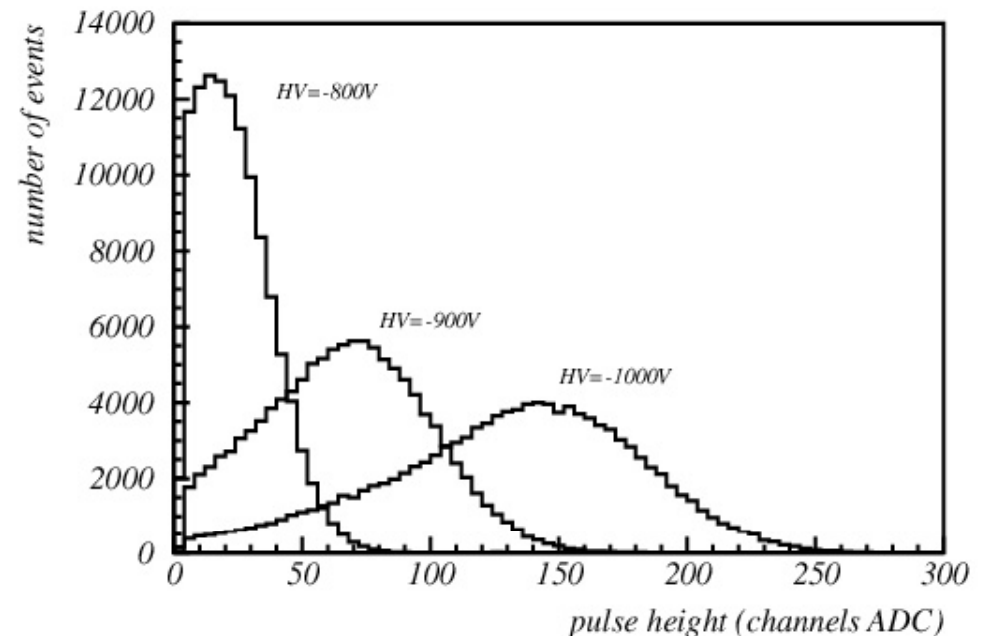
Single photon response



Multianode Hamamatsu
R5900-M16 PMT: excellent

Not trivial to detect single photons in PMTs, most do not have that capability

<- not bad on that scale

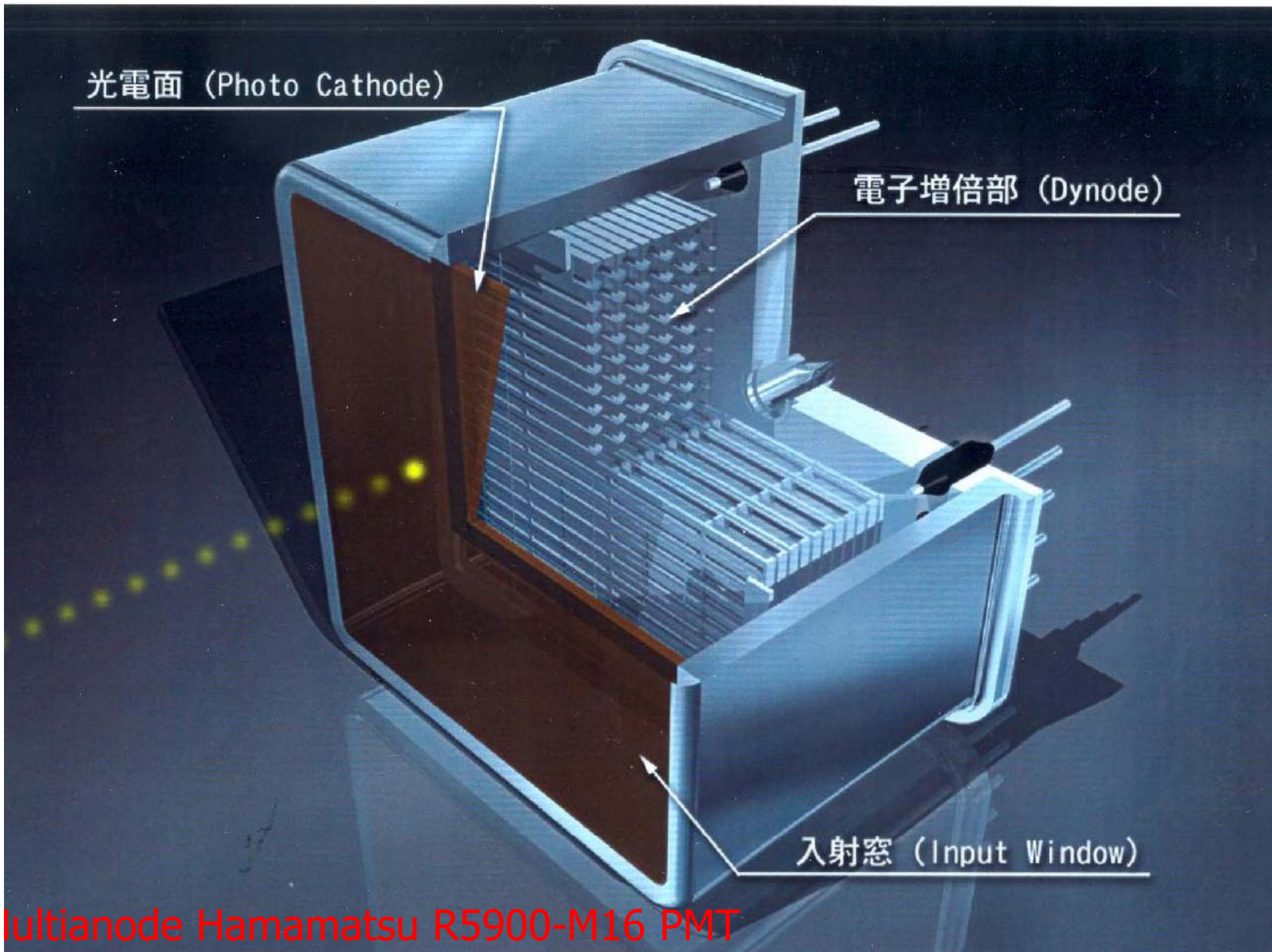


光電面 (Photo Cathode)

電子増倍部 (Dynode)

入射窓 (Input Window)

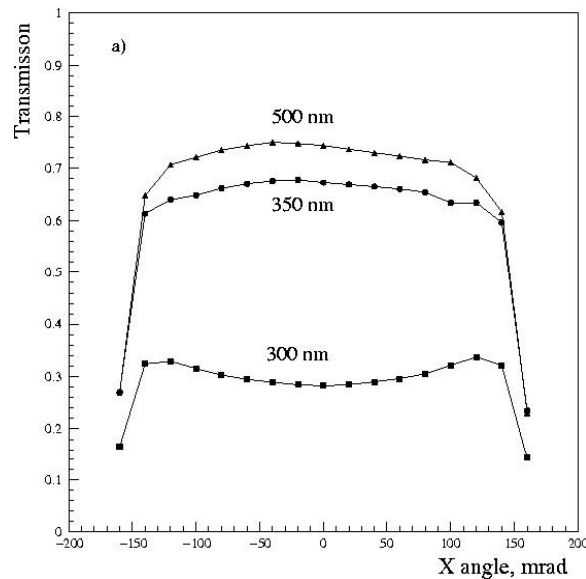
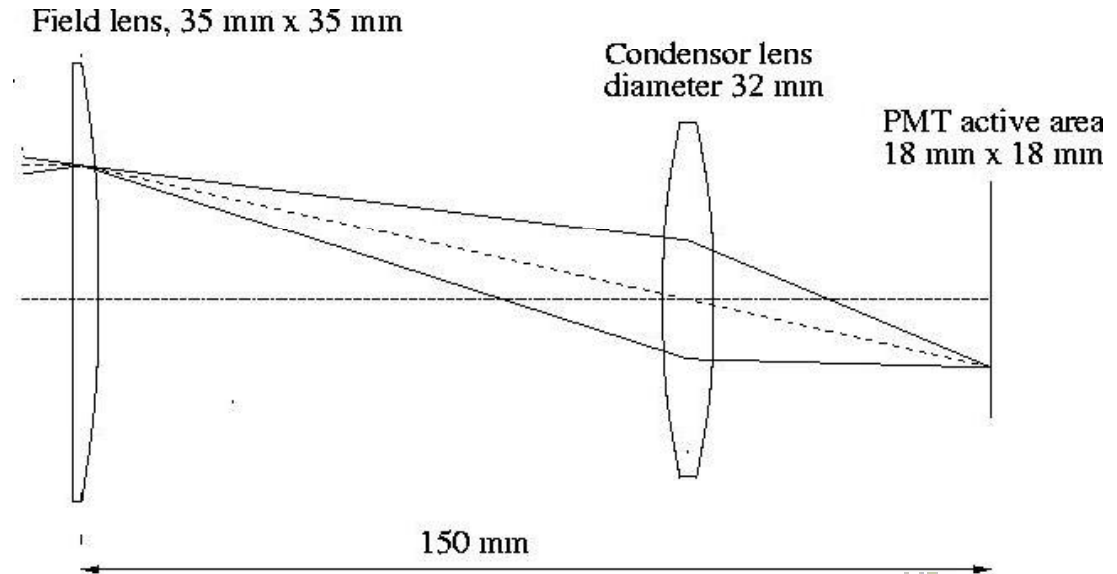
Multianode Hamamatsu R5900-M16 PMT



Light collectors

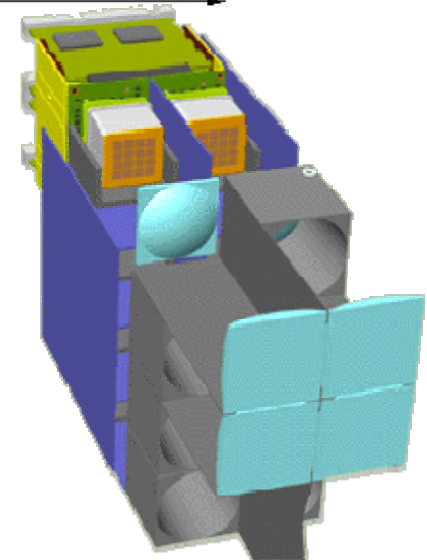
Used to reduce the dead area between photon detector segments (in particular in the case of PMTs), or to adapt the required photon detector granularity to the PMT (pad) size

- Winston cones
- Reflectors
- Lens combinations



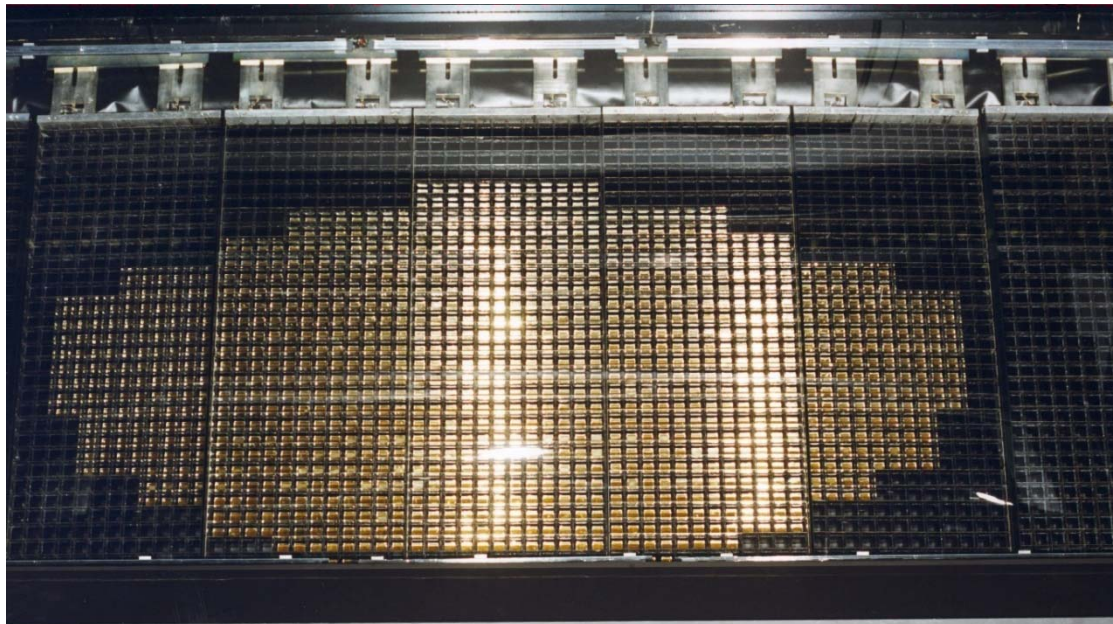
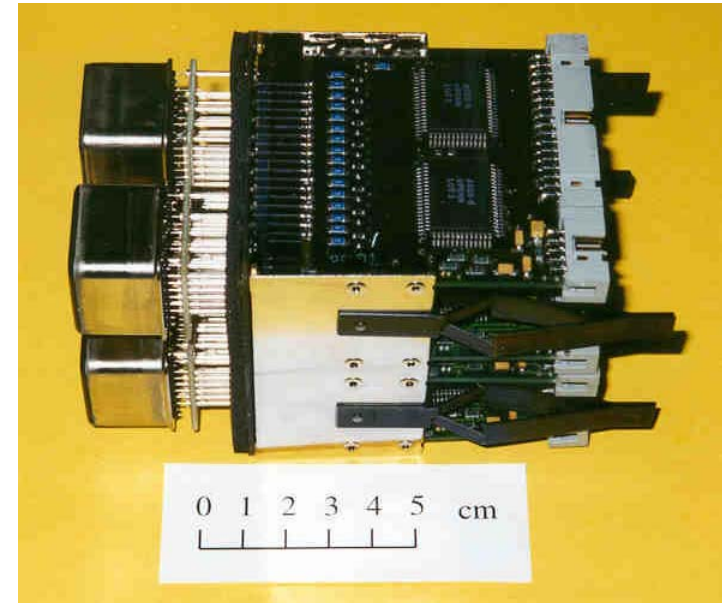
Example: Optical system for light collection and demagnification of the HERA-B RICH.

→ angular acceptance, material transmission



RICH photon detector with multi-anode PMTs: HERA-B

2300 PMT, 16 and 4 channel type, 30.000 electronic channels

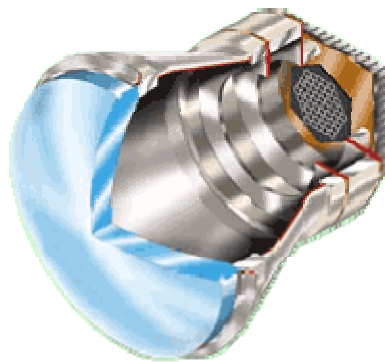
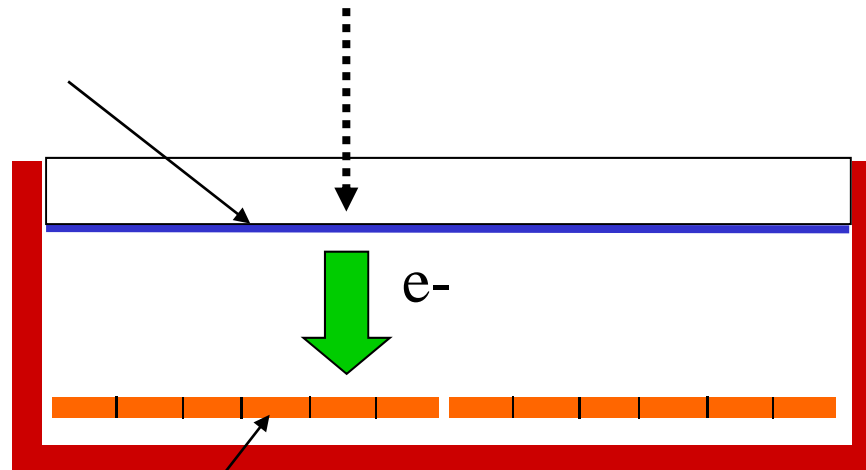


Detect photoelectron in silicon: hybrid photon detector - HPD

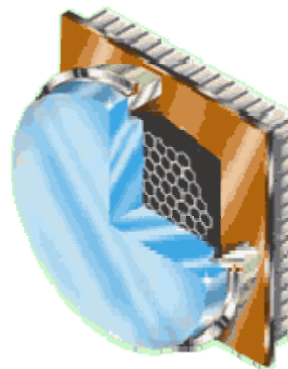
Multialkali photocathode

-10kV
15~25mm

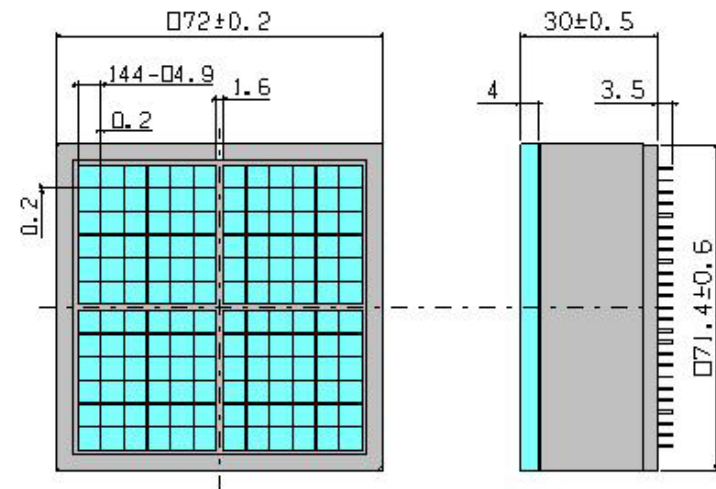
Pixel PD or APD



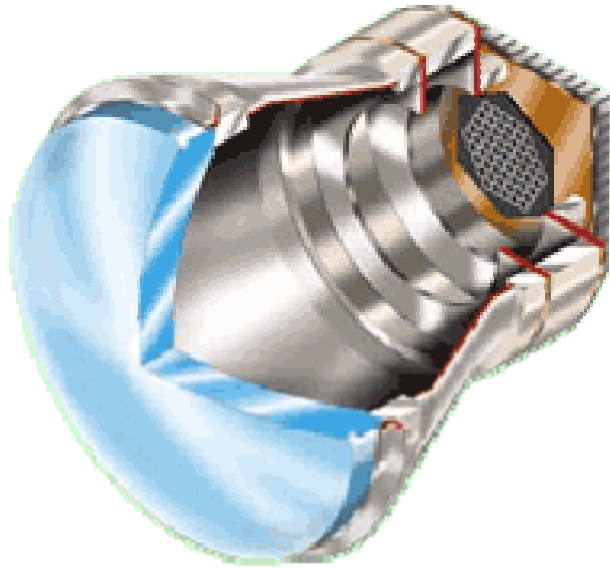
Focusing in E field
with additional
electrodes



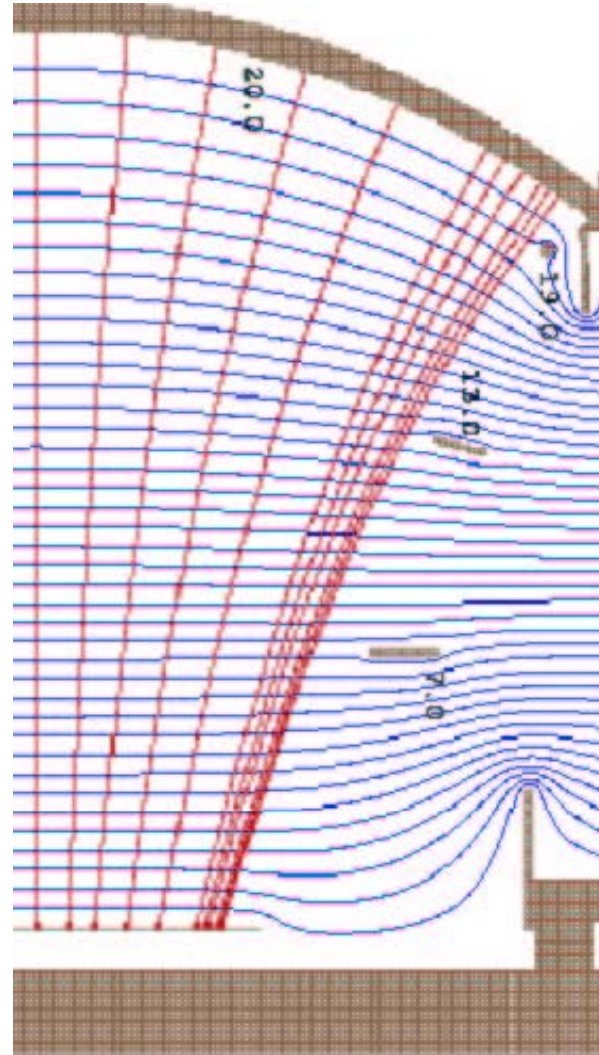
No focusing in E field



Hybrid photon detector – focusing type



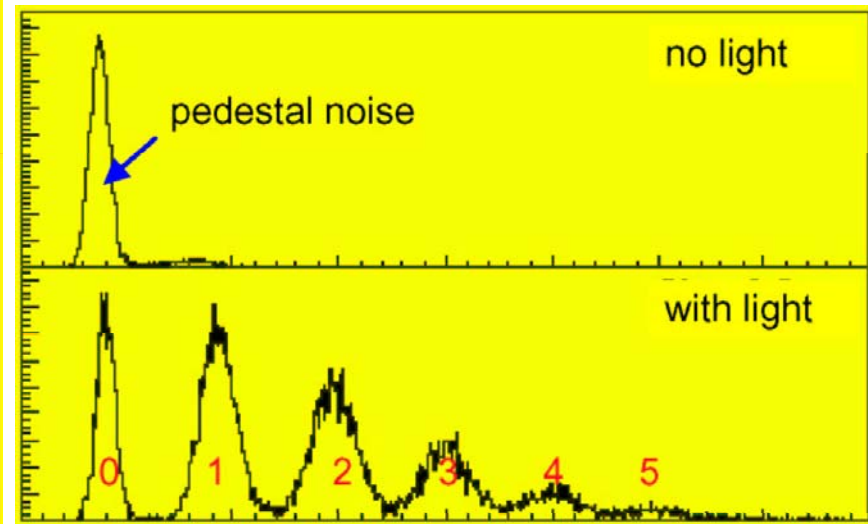
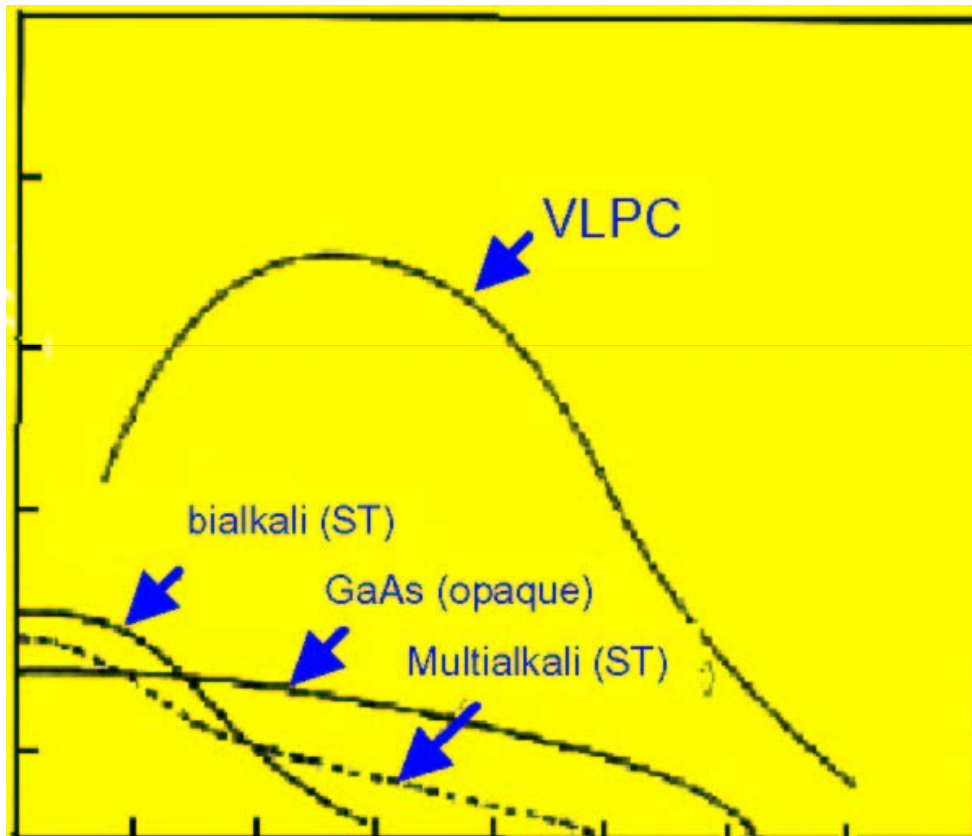
Focusing in E field
with additional
electrodes



Visual photon detection in silicon

Need a very low noise device – one possibility: run at liquid He temperatures

VLPC: visual light photon counter



Polprevodniški detektorji

Kako se polvod. detektorji kvalificirajo za detekcijo posameznih (ali malega števila) fotonov?

- **PIN diode:** standardne (n.pr. CsI kalorimetri: CLEO, Belle, BaBar), prag ~ 100 fotonov
- **APD:** prag ~ 20 fotonov, prvi večji eksperiment: elektromagnetni kalorimeter EMC v CMS.
- **SiPM (GAPD):** v principu lahko z njimi zaznavamo posamezne fotone

Avalanche Photodiodes or Geiger-mode Avalanche Photodiodes

Avalanche photodiodes (APDs) have internal gain which improves the signal to noise ratio compared to normal photodiodes but still some 20 photons are needed for a detectable signal. The excess noise, the fluctuations of the avalanche multiplication, limits the useful range of gain.

Geiger-mode Avalanche photodiodes (G-APDs or SiPMs) can detect single photons and the gain is in the range of 10^5 to 10^7 and no (or at most a simple) amplifier is needed. Pickup noise is no more a concern (no shielding). The excess noise factor can be close to 1 (APDs >2 , PMTs: 1.2).

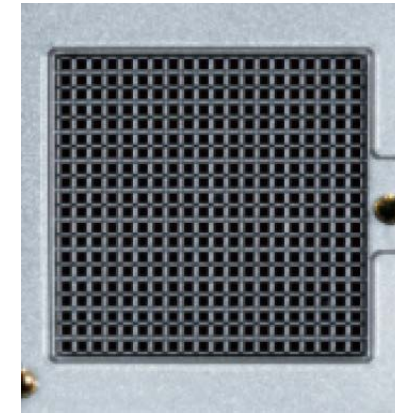
SiPMs as photon detectors for RICH

SiPM is an array of APDs operating in Geiger mode. Characteristics:

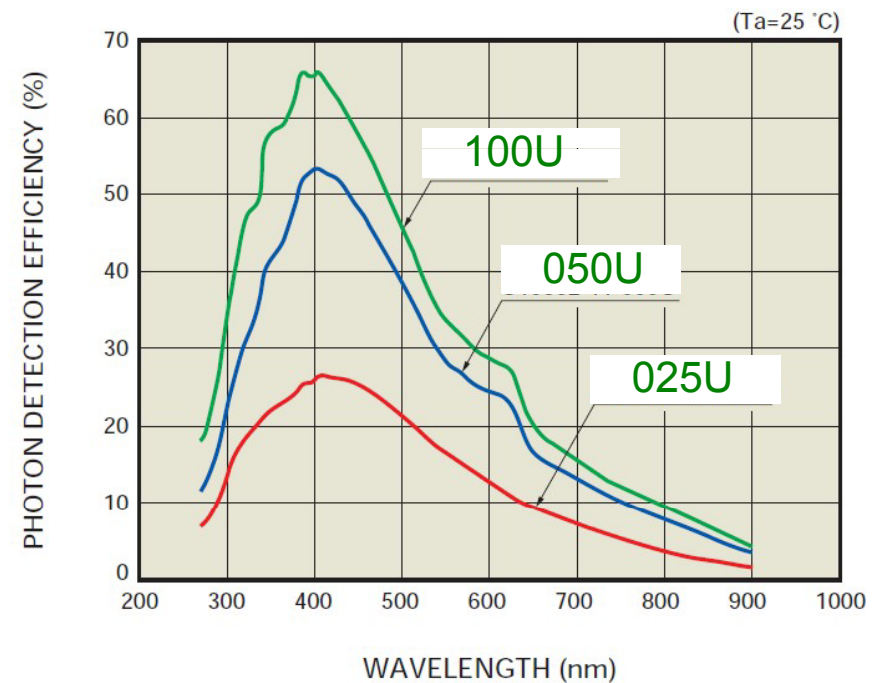
- low operation voltage $\sim 10\text{-}100\text{ V}$
- gain $\sim 10^6$
- peak PDE up to 65%(@400nm)

$$\text{PDE} = \text{QE} \times \epsilon_{\text{geiger}} \times \epsilon_{\text{geo}}$$

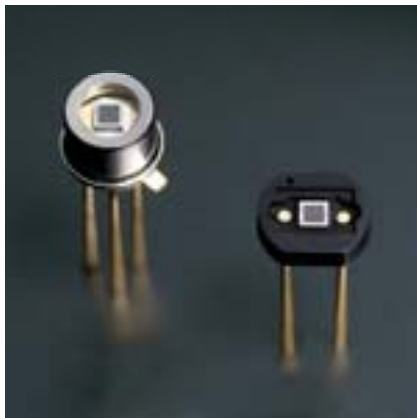
- ϵ_{geo} – dead space between the cells
- time resolution $\sim 100\text{ ps}$
- works in high magnetic field
- dark counts $\sim \text{few } 100\text{ kHz/mm}^2$
- radiation damage (p,n)



1 mm



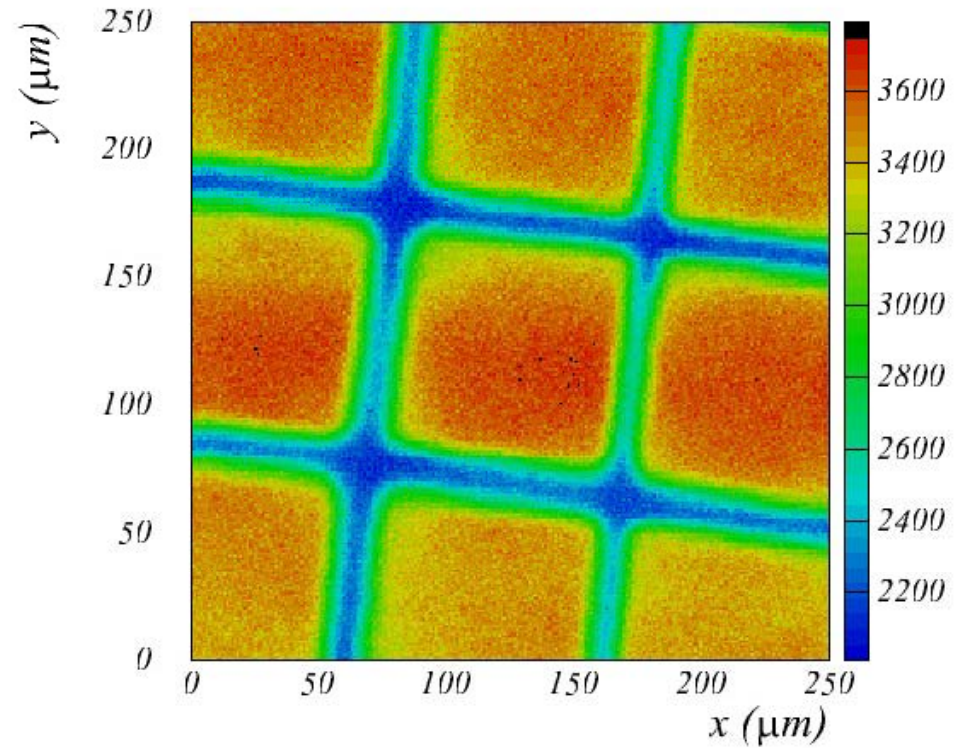
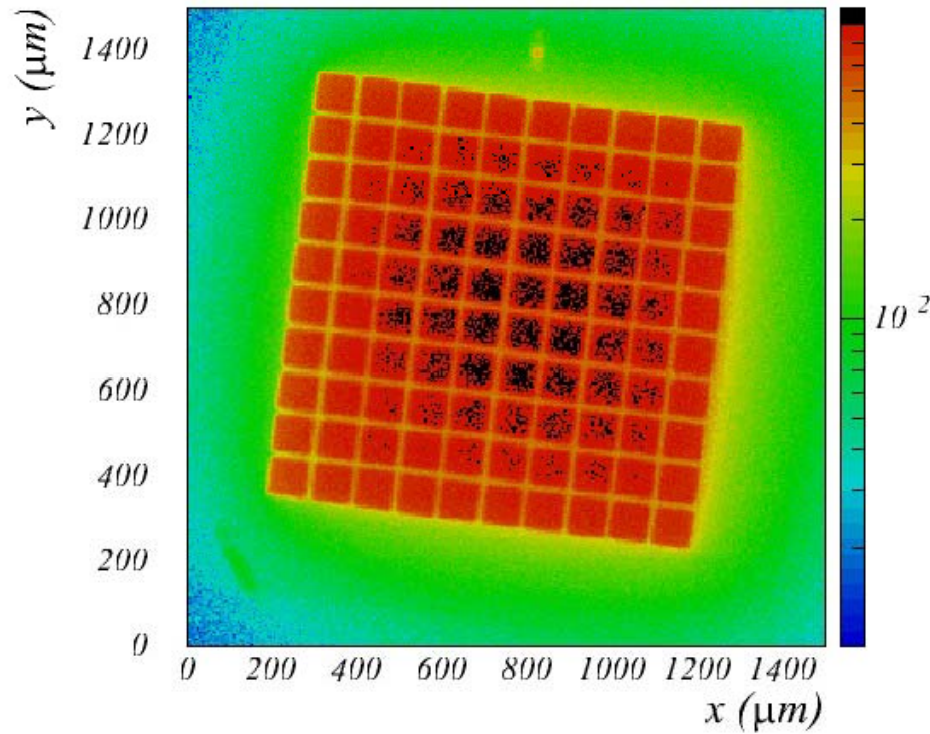
Hamamatsu MPPC: S10362-11



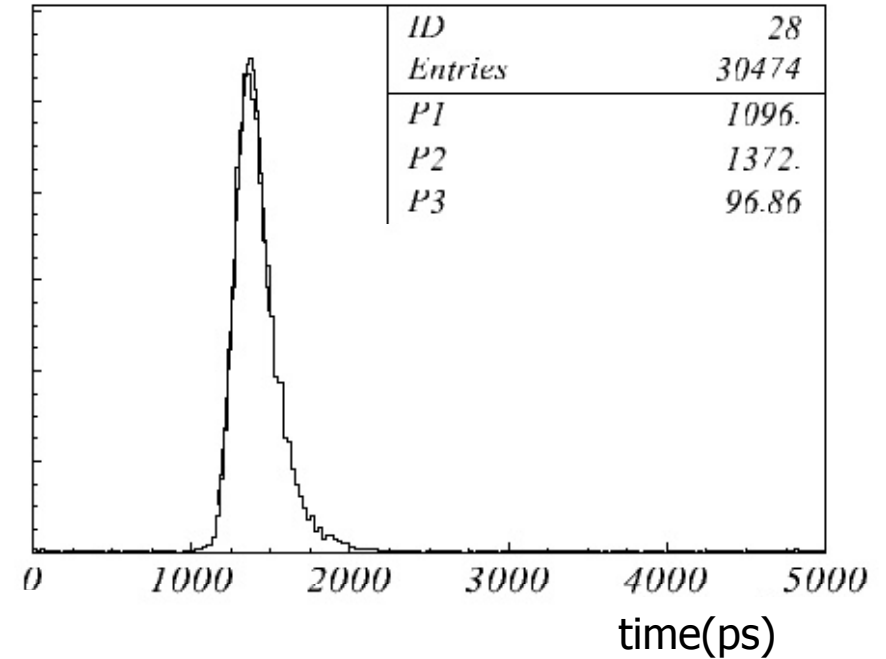
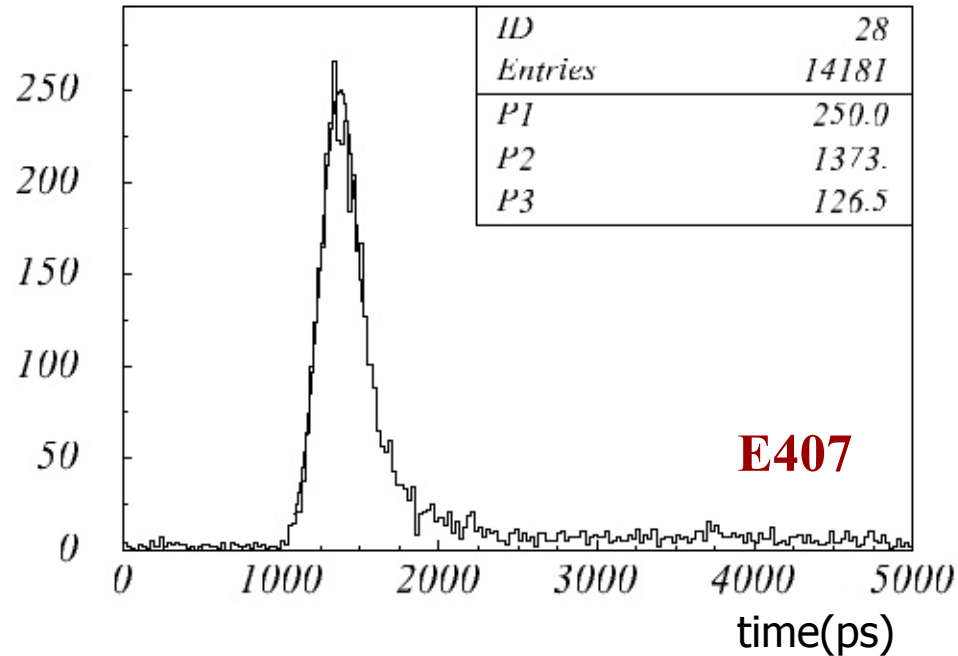
Surface sensitivity for single photons 4

Hamamatsu MPPCs

H100C



Time resolution: blue vs red



	E407	S137	H100C	H025C
σ_{red} (ps)	127	182	145	154
σ_{blue} (ps)	97	151	136	135

• $\sigma \approx 100$ ps

• $\sigma_{\text{red}} > \sigma_{\text{blue}}$

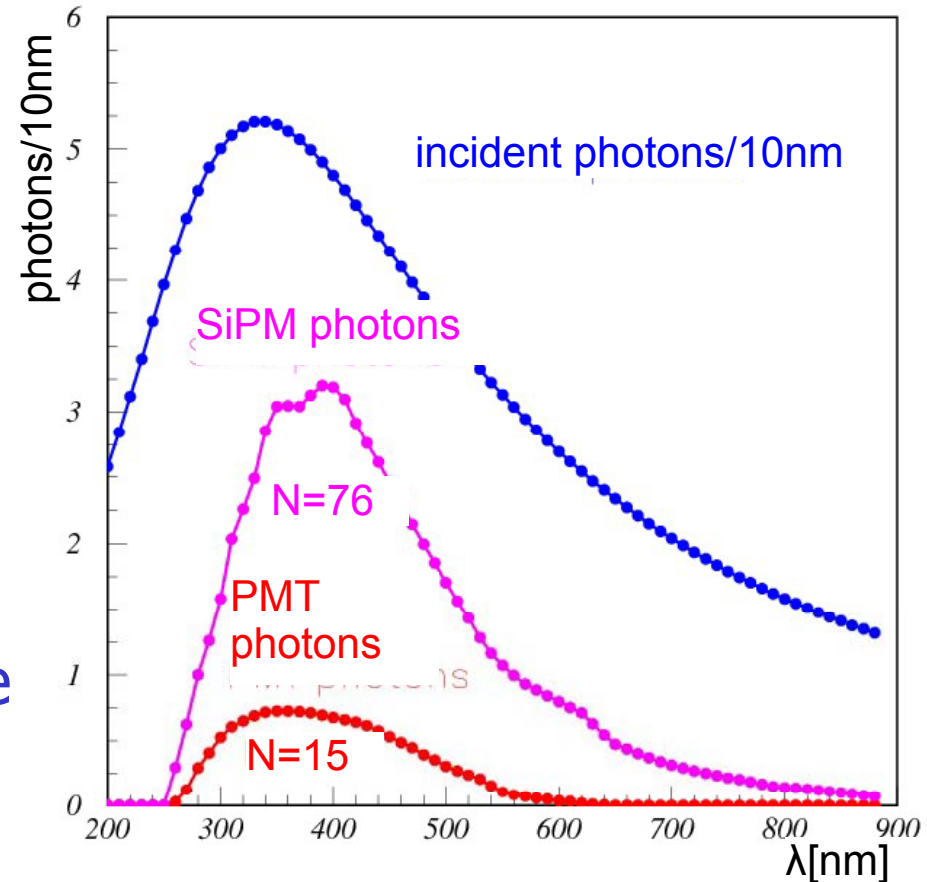
Expected number of photons for aerogel RICH

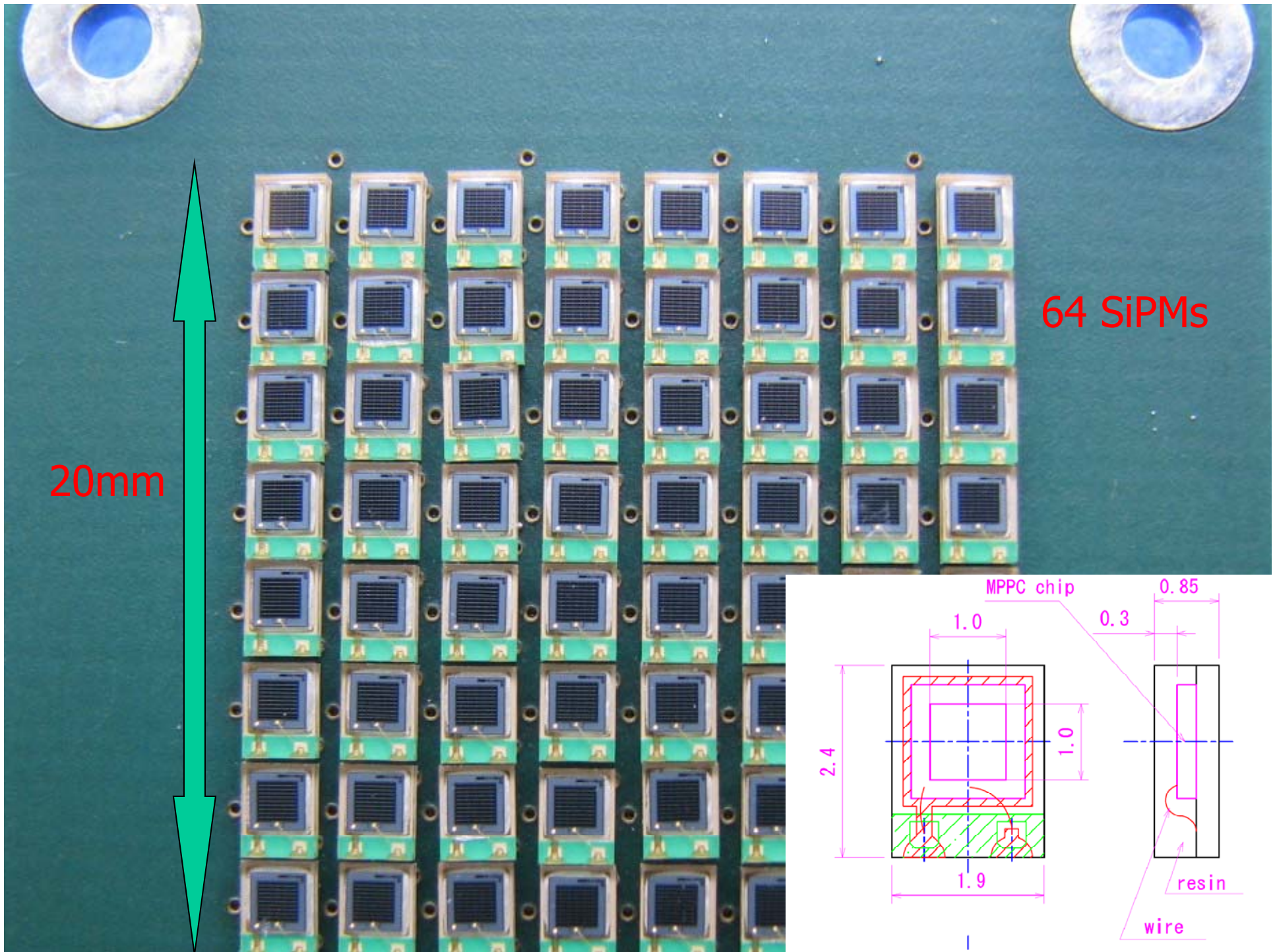
with multianode PMTs or SiPMs(100U), and
aerogel radiator: thickness 2.5 cm, $n = 1.045$
and transmission length (@400nm) 4 cm.

$$N_{\text{SiPM}}/N_{\text{PMT}} \sim 5$$

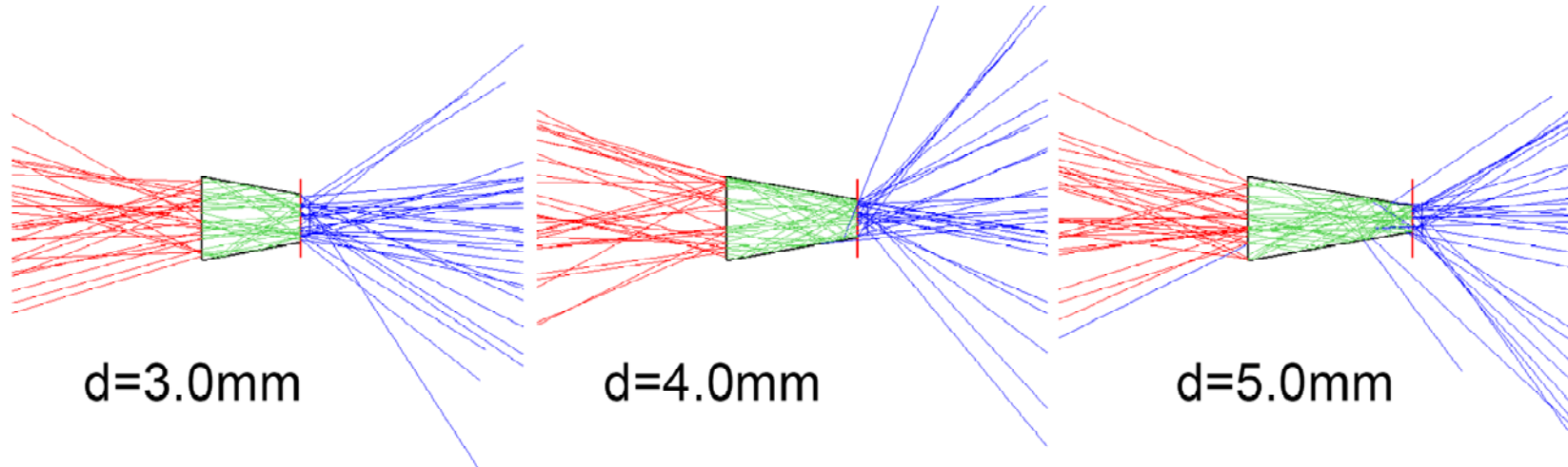
Assuming 100% detector
active area

Never before tested in a RICH
where we have to detect single
photons. ← Dark counts have
single photon pulse heights
(rate 0.1-1 MHz)



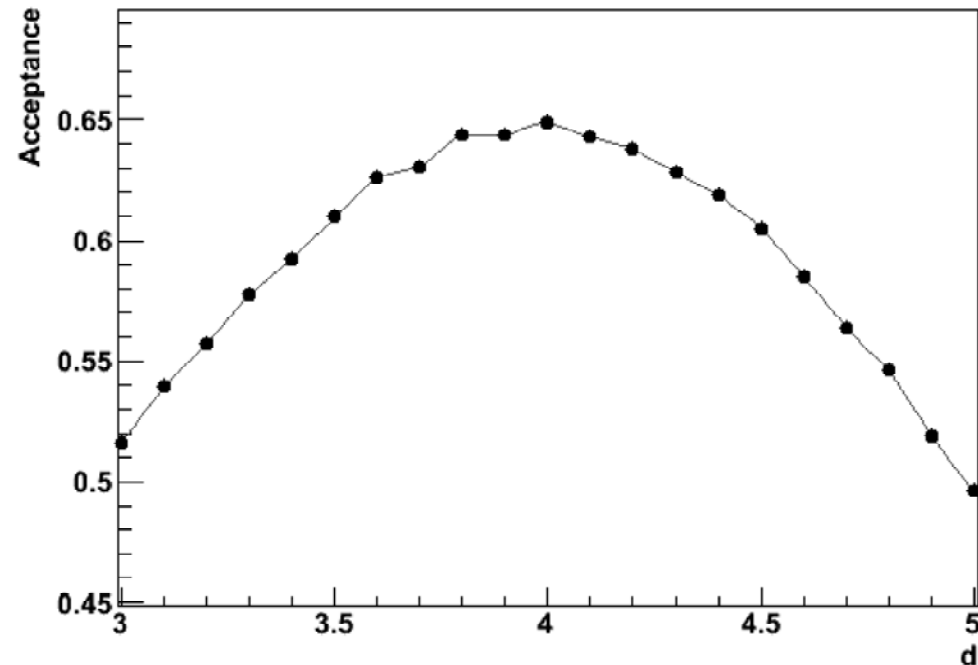


Increase the active area: use light guides



d (mm)	out (mm)	accept. (%)
3.0	1.48	51.6
3.1	1.45	54.0
3.2	1.41	55.7
3.3	1.38	57.8
3.4	1.34	59.2
3.5	1.31	61.0
3.6	1.27	62.6
3.7	1.24	63.1
3.8	1.20	64.4
3.9	1.16	64.4
4.0	1.13	64.9
4.1	1.09	64.3
4.2	1.06	63.8
4.3	1.02	62.8
4.4	0.99	61.8
4.5	0.95	60.5
4.6	0.92	58.5
4.7	0.88	56.4
4.8	0.85	54.6
4.9	0.81	51.9

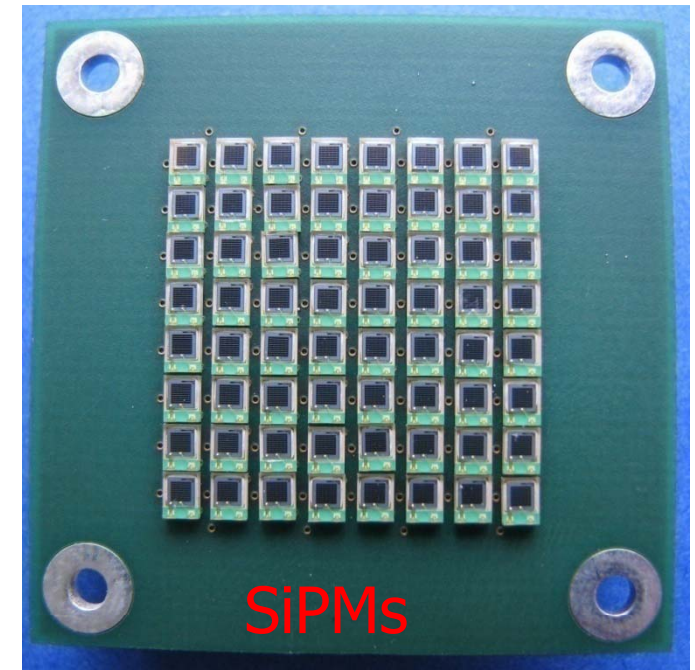
SiPM = 0.8, M = 3.3, d = 5.0 | gap(y,z) = (0.0, 0.0) | $\theta = 30.0$ Thu May 8 14:02:15 2008



Detector module for beam tests at KEK

SiPMs: array of 8x8 SMD mount
Hamamatsu S10362-11-100P
with 0.3mm protective layer

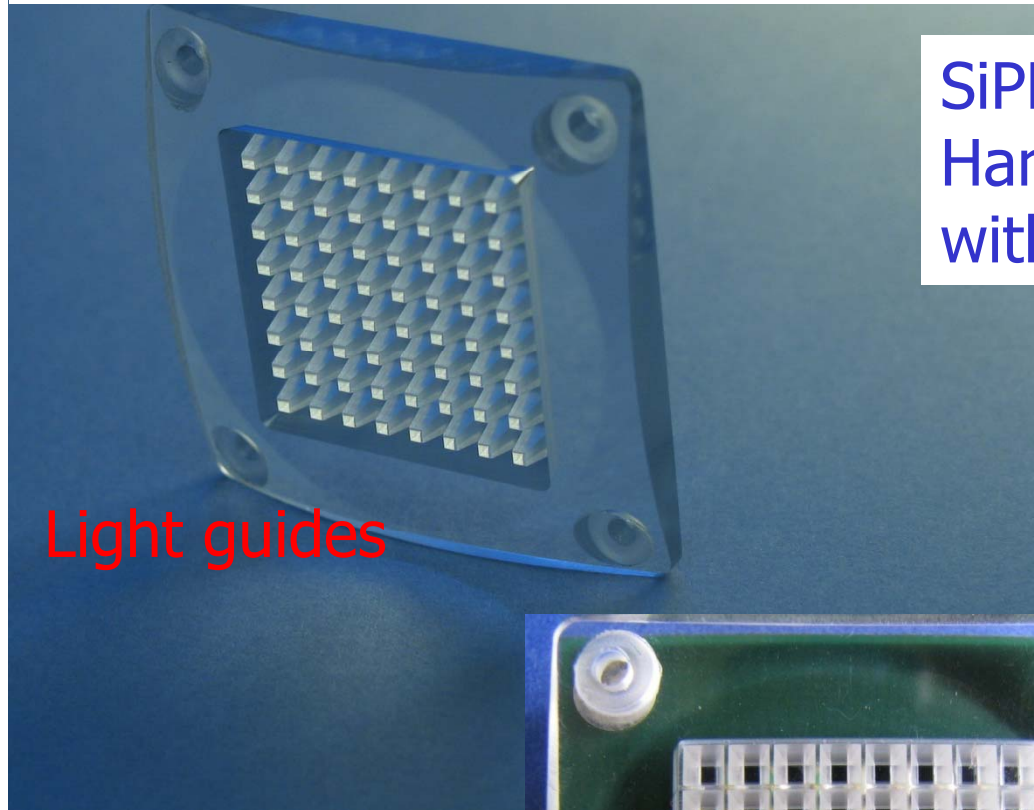
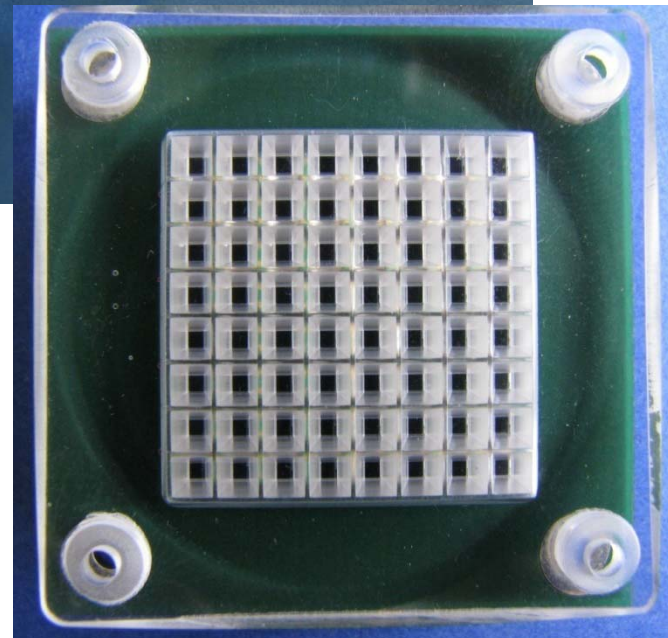
Light guides



SiPMs

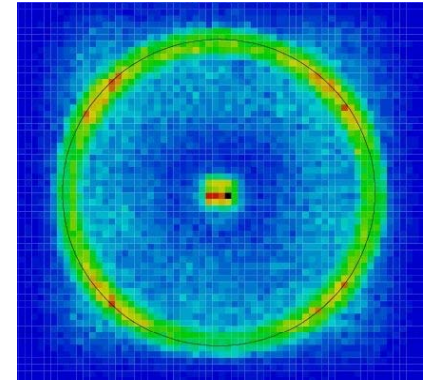
2cm

SiPMs + light guides

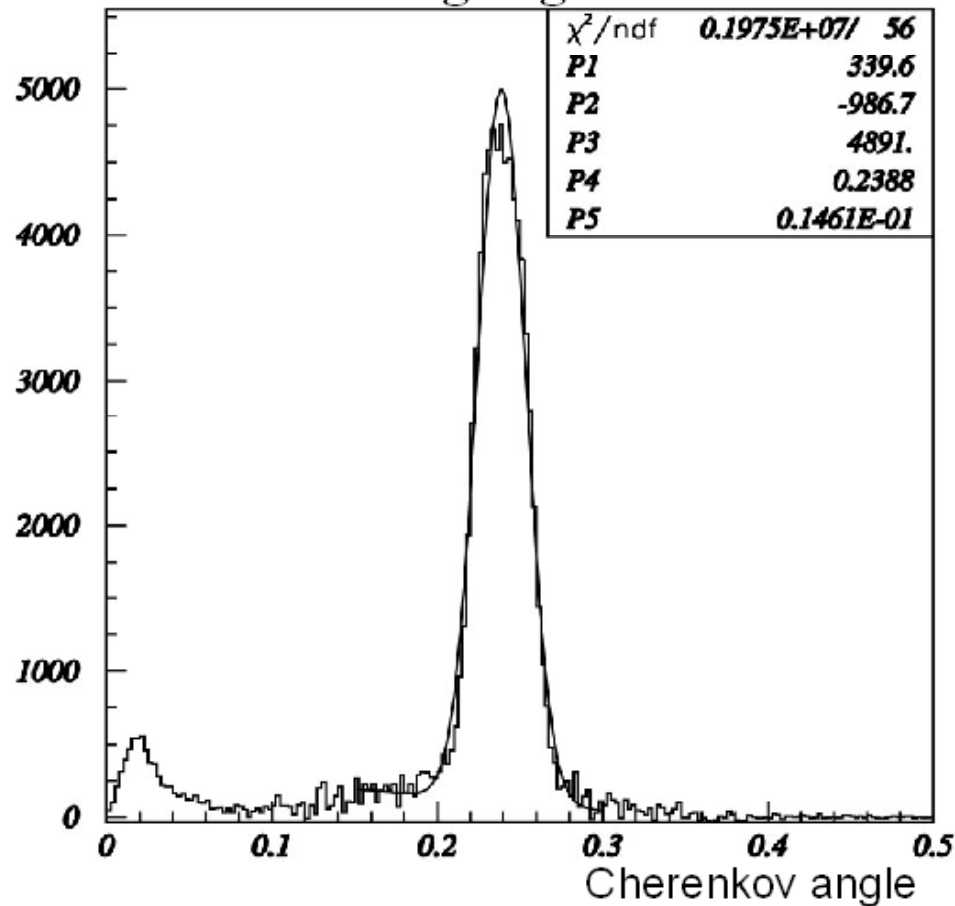


Cherenkov angle distributions

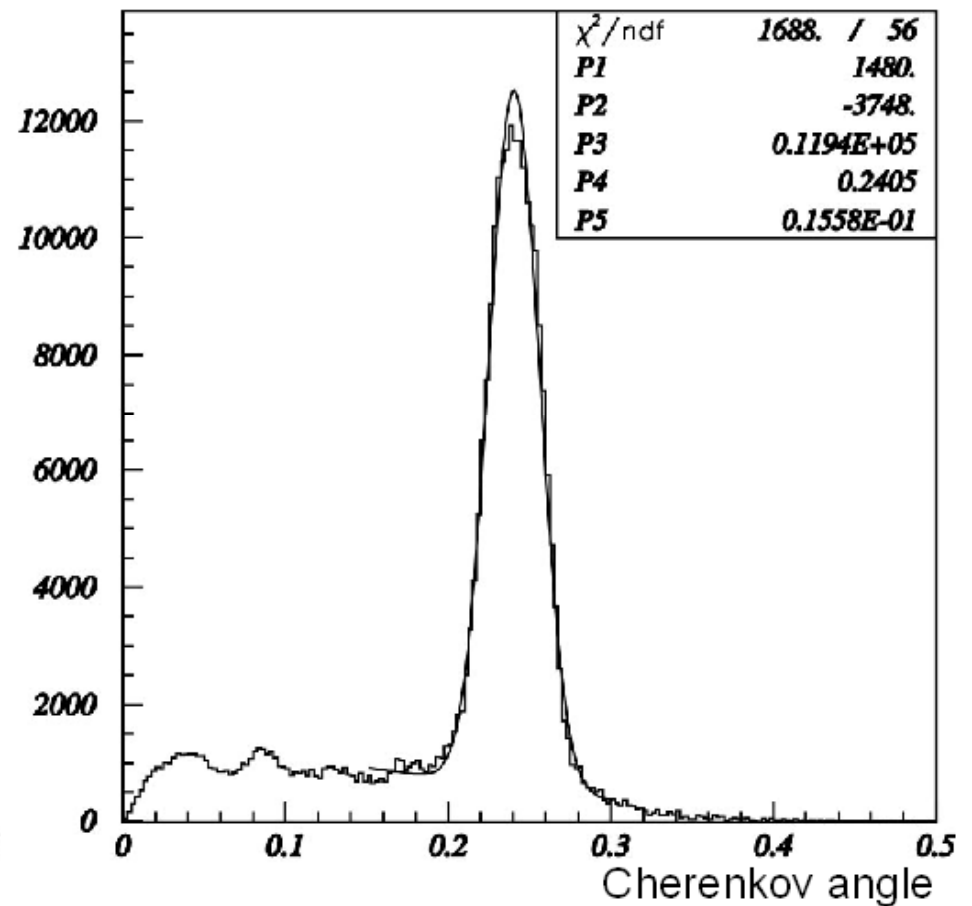
- background subtracted distributions
- ratio of detected photons w/ and w/o: ~ 2.3
- resolution within expectations (14.5mrad)



w/o light guides



w/ light guides



Transition radiation

X rays emitted at the boundary of two media with different refractive indices, emission angle $\sim 1/\gamma$

Emission rate depends on γ (Lorentz factor): becomes important at $\gamma \sim 1000$

- Electrons at 0.5 GeV
- Pions, muons above 100 GeV

In between: discrimination e vs pions, muons

Detection of X rays: high Z gas – Xe

Few photons per boundary can be detected

Need many boundaries

- Stacks of thin foils or
- Porous materials – foam with many boundaries of individual 'bubbles'

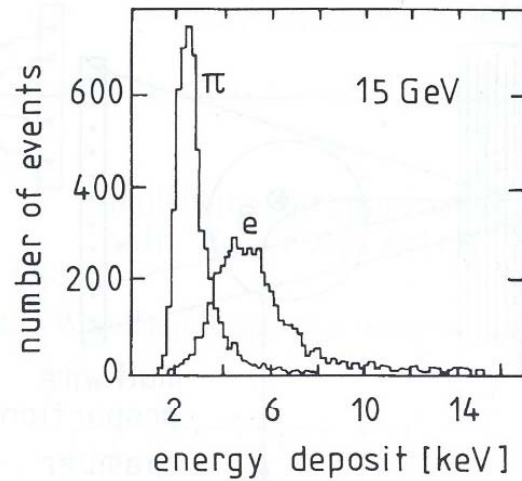


Fig. 6.25. Energy-loss distribution of 15 GeV electrons and pions in a transition radiation detector [467].

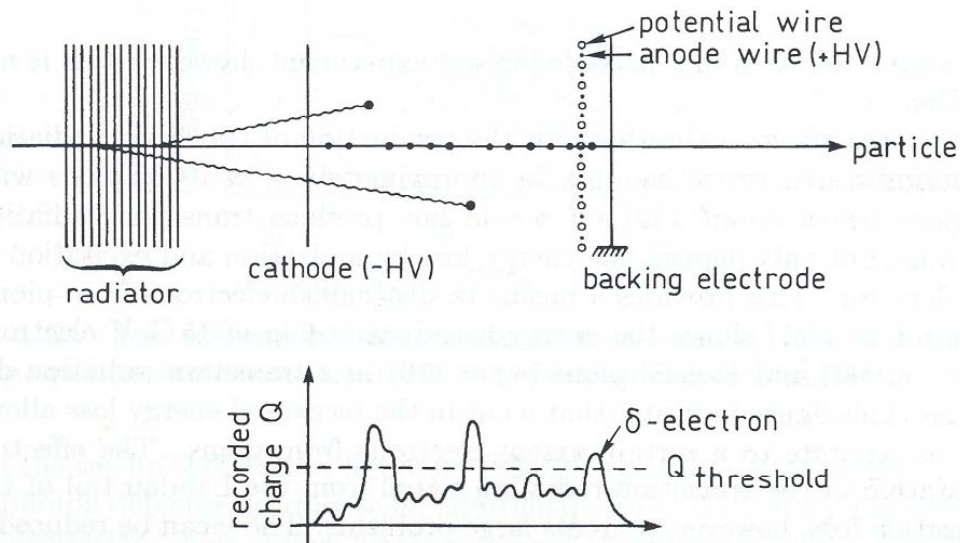
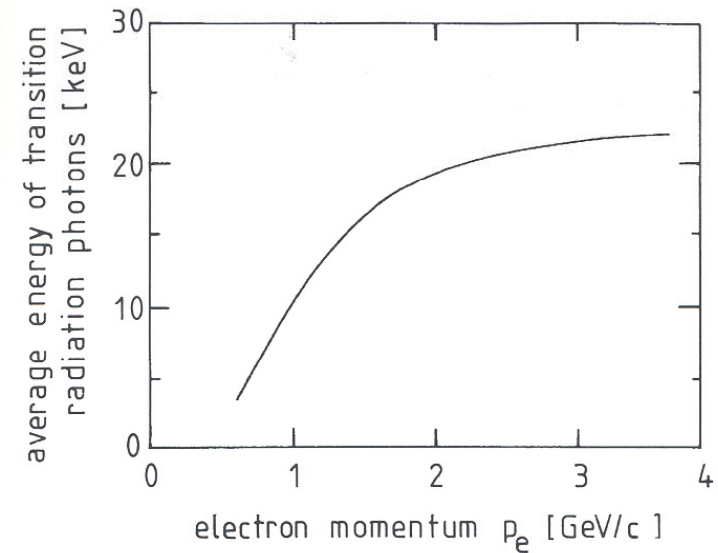


Fig. 6.26. Principle of separating ionization energy loss from the energy loss from emission of transition radiation photons.

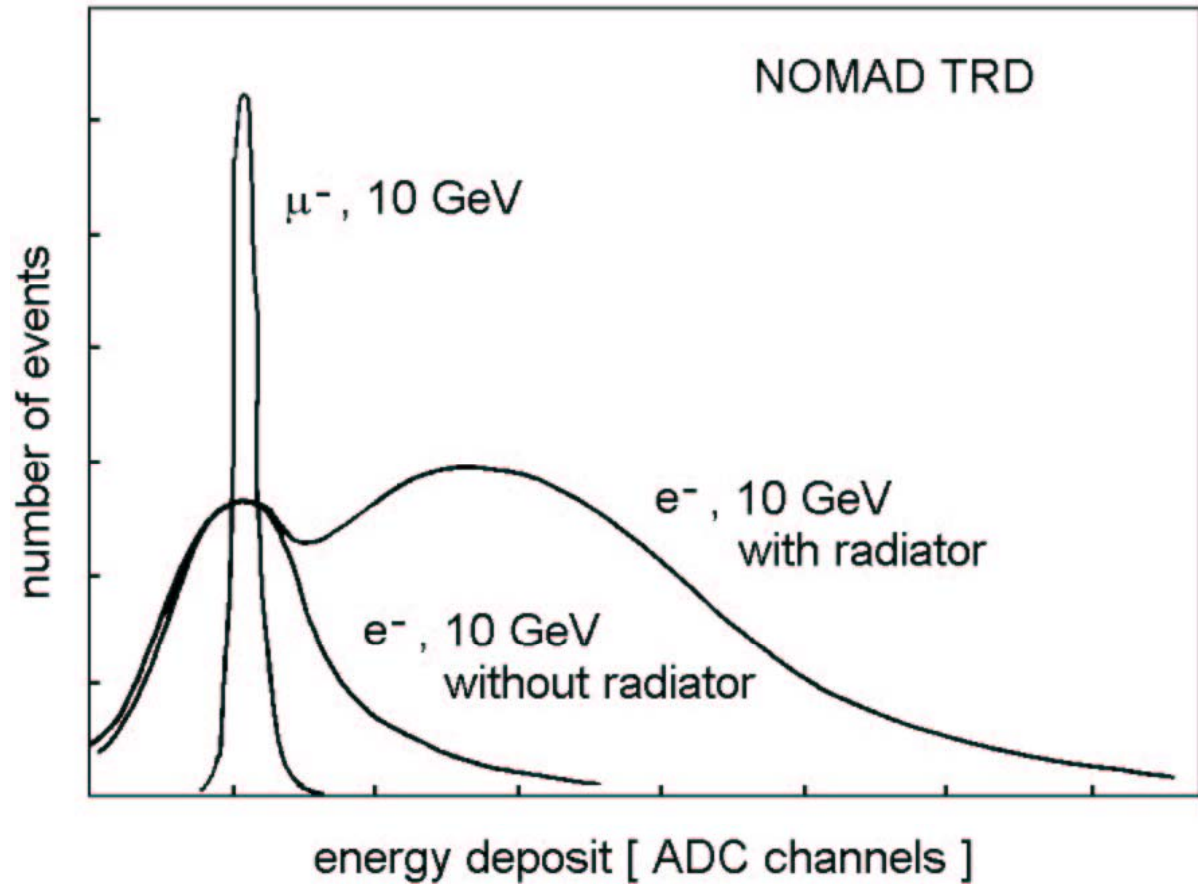
Transition radiation - 1

Separation of X ray detection –
high energy deposit on one
place – against ionisation
losses



Transition radiation - 2

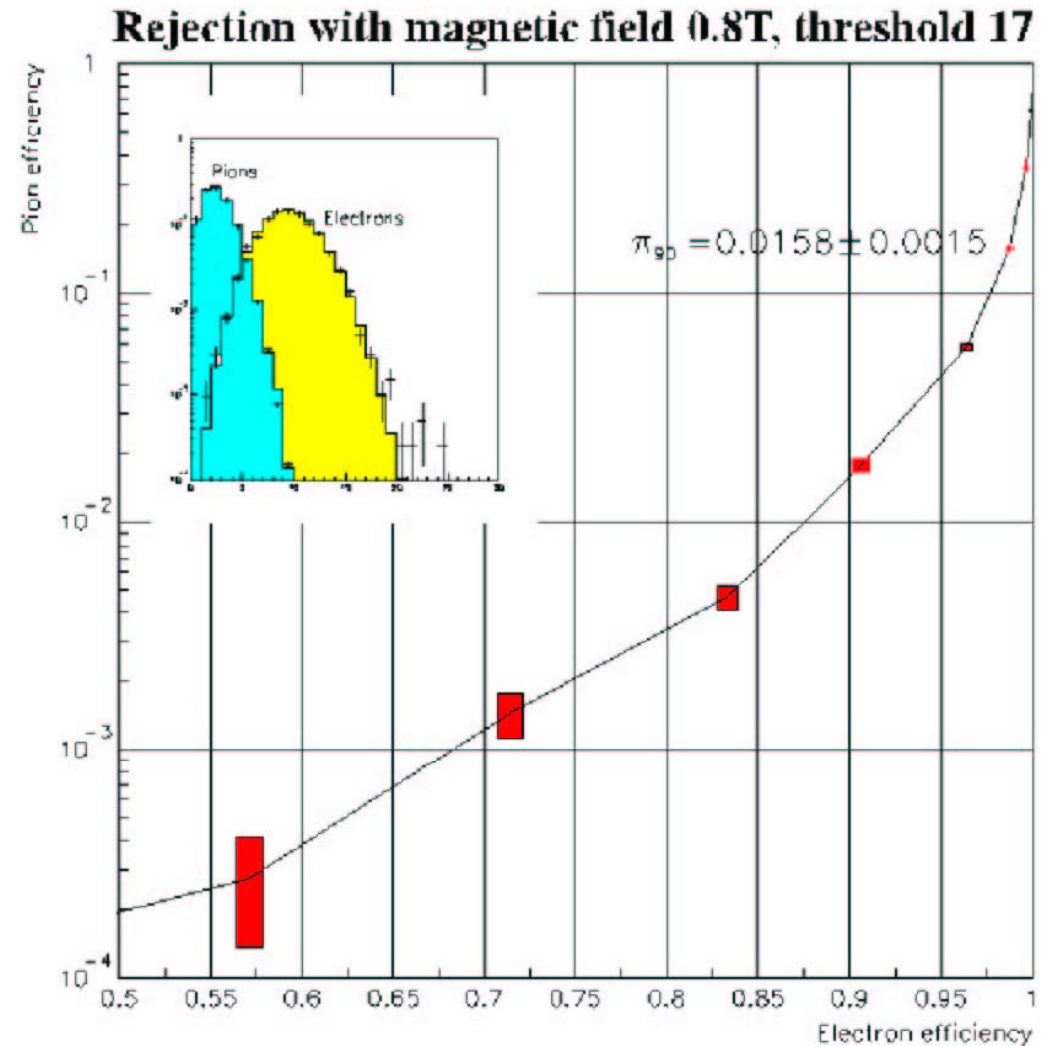
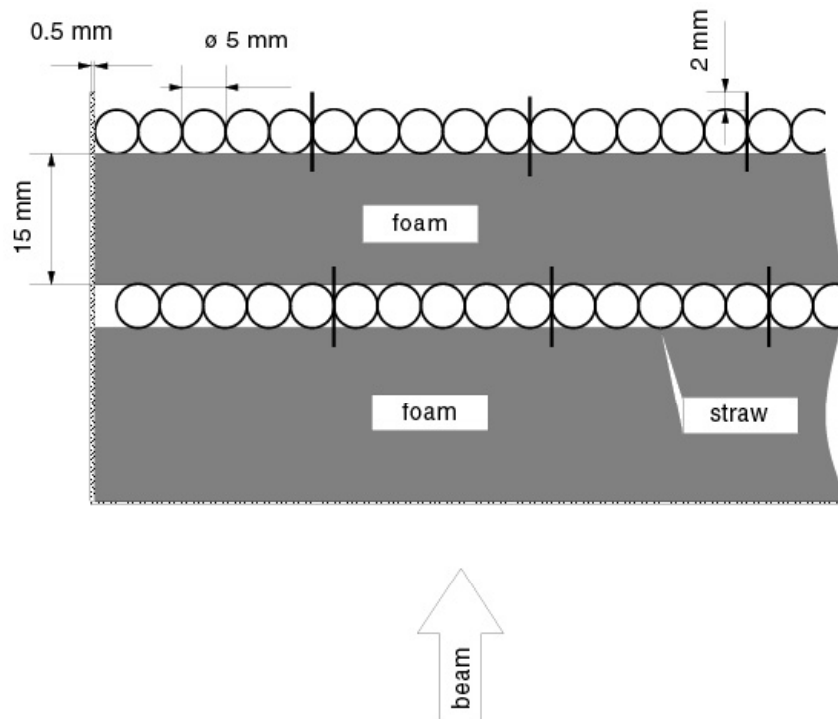
Example of performance –
NOMAD experiment



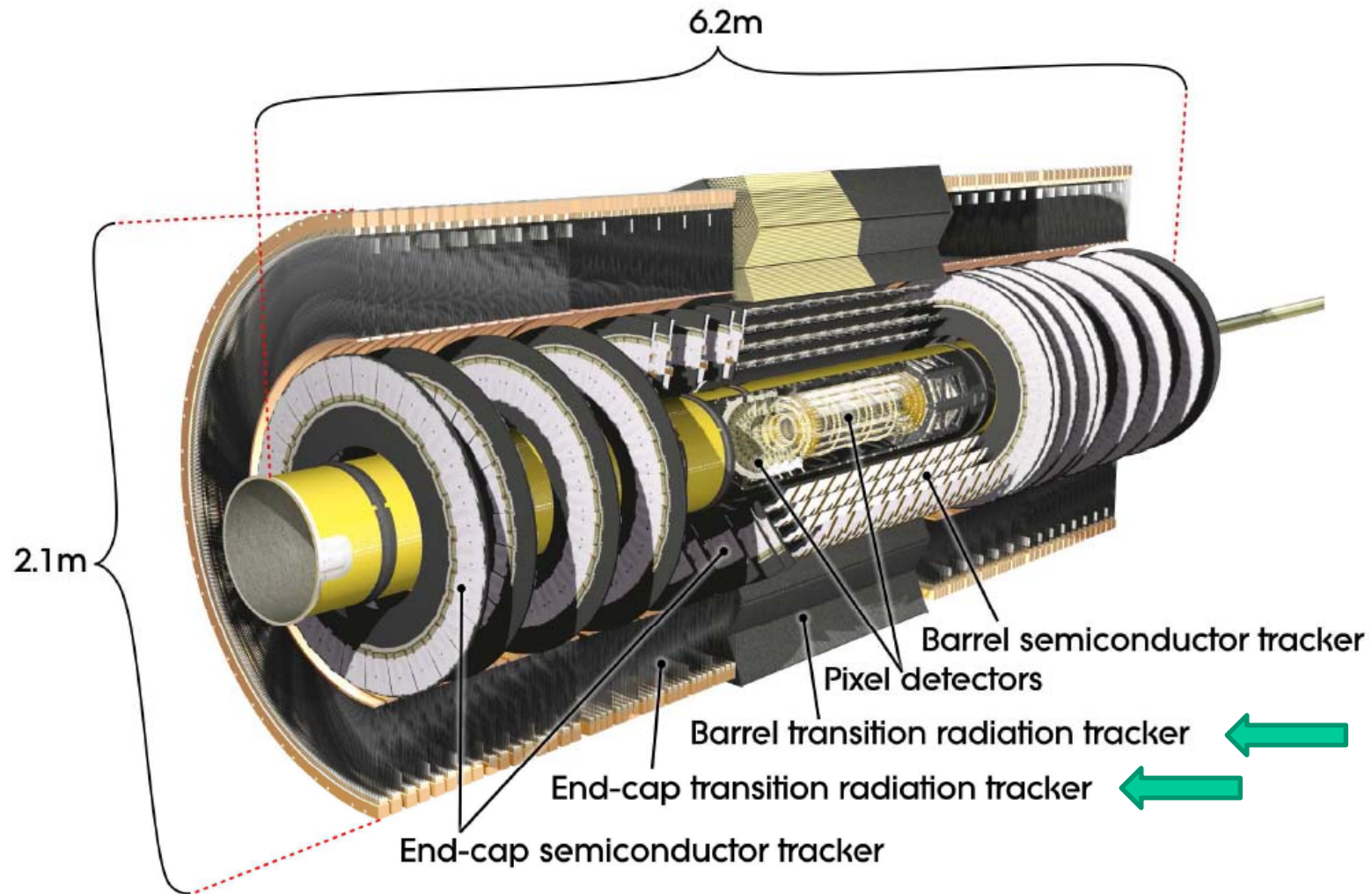
Transition radiation - 3

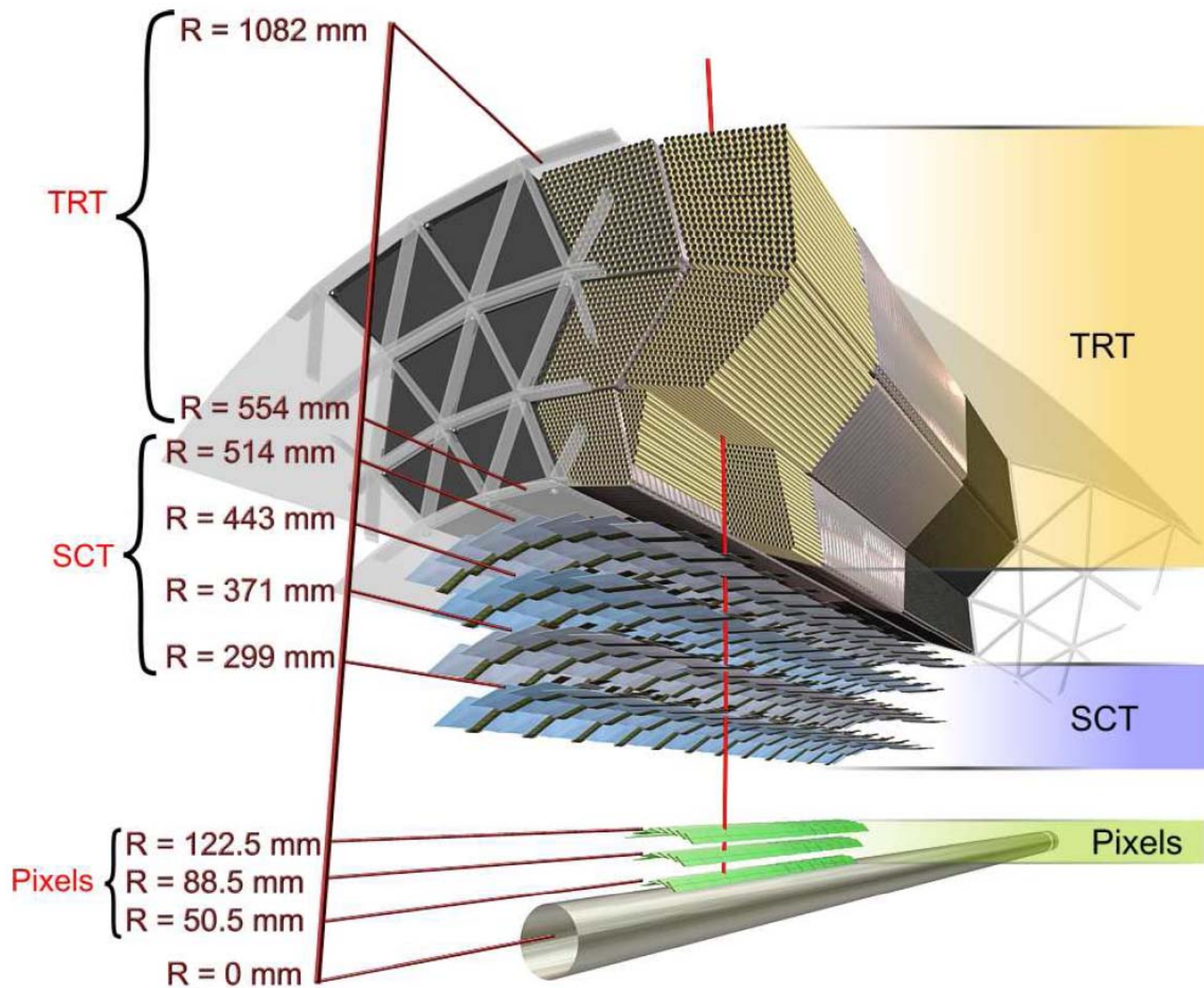
Example of performance 2

Radiator: organic foam
between the detector tubes
(straws made of capton
foil)



Transition radiation detector in ATLAS: combination of a tracker and a transition radiation detector

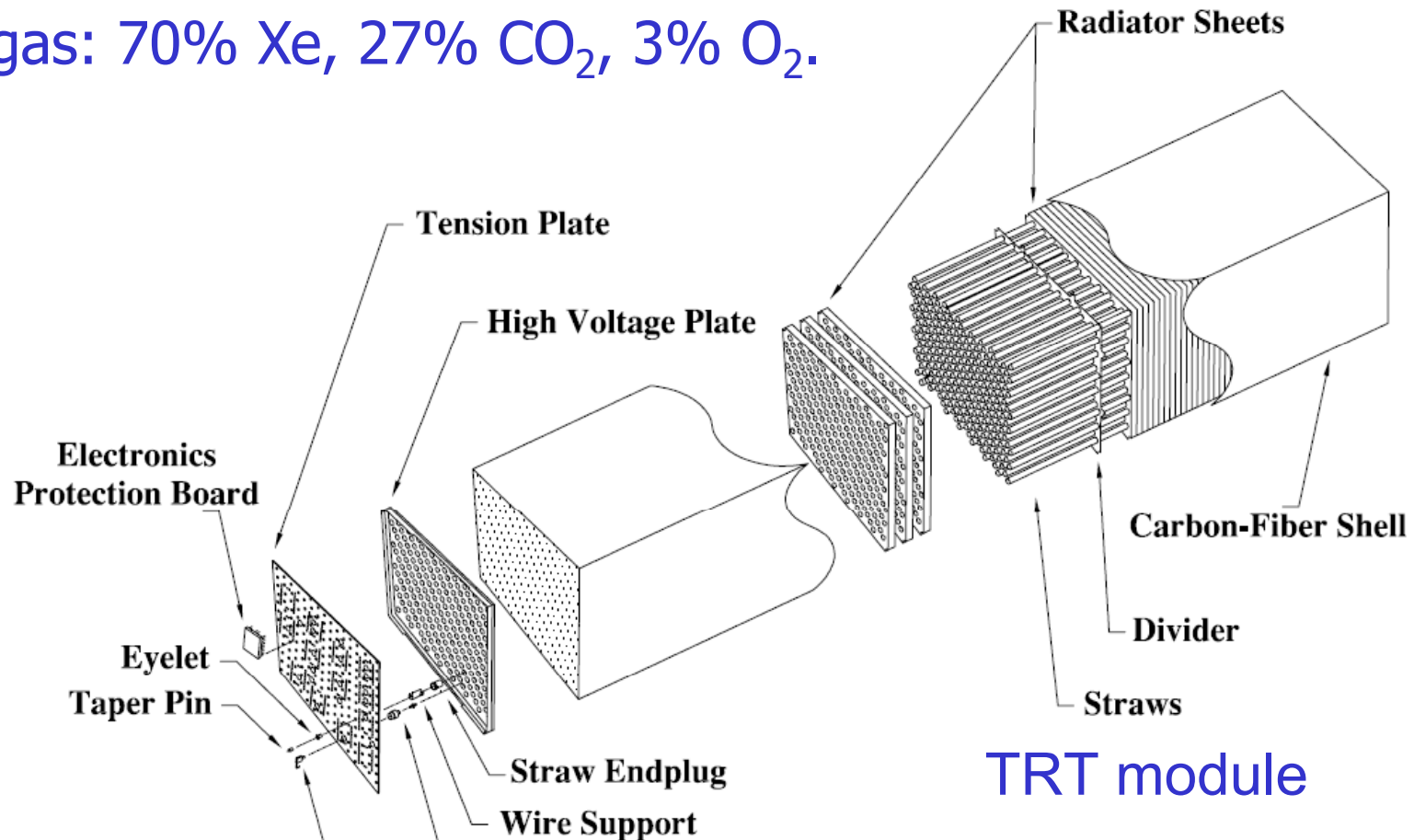




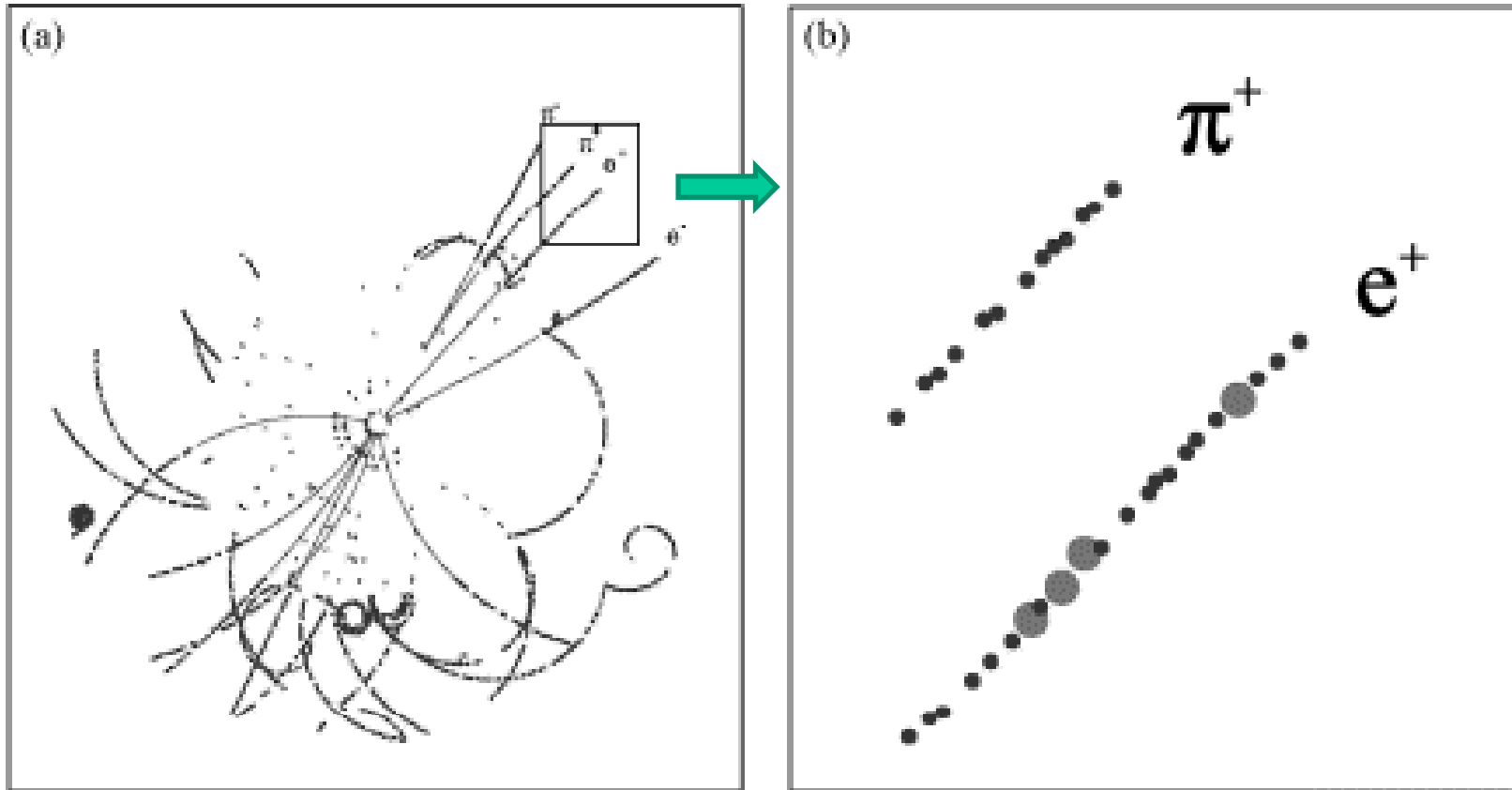
ATLAS TRT: combination of Transition Radiation detector and a Tracker

Radiator: 3mm thick layers made of polypropylene-polyethylene fibers with ~ 19 micron diameter, density: 0.06 g/cm^3

Straw tubes: 4mm diameter with 31 micron diameter anode wires, gas: 70% Xe, 27% CO₂, 3% O₂.

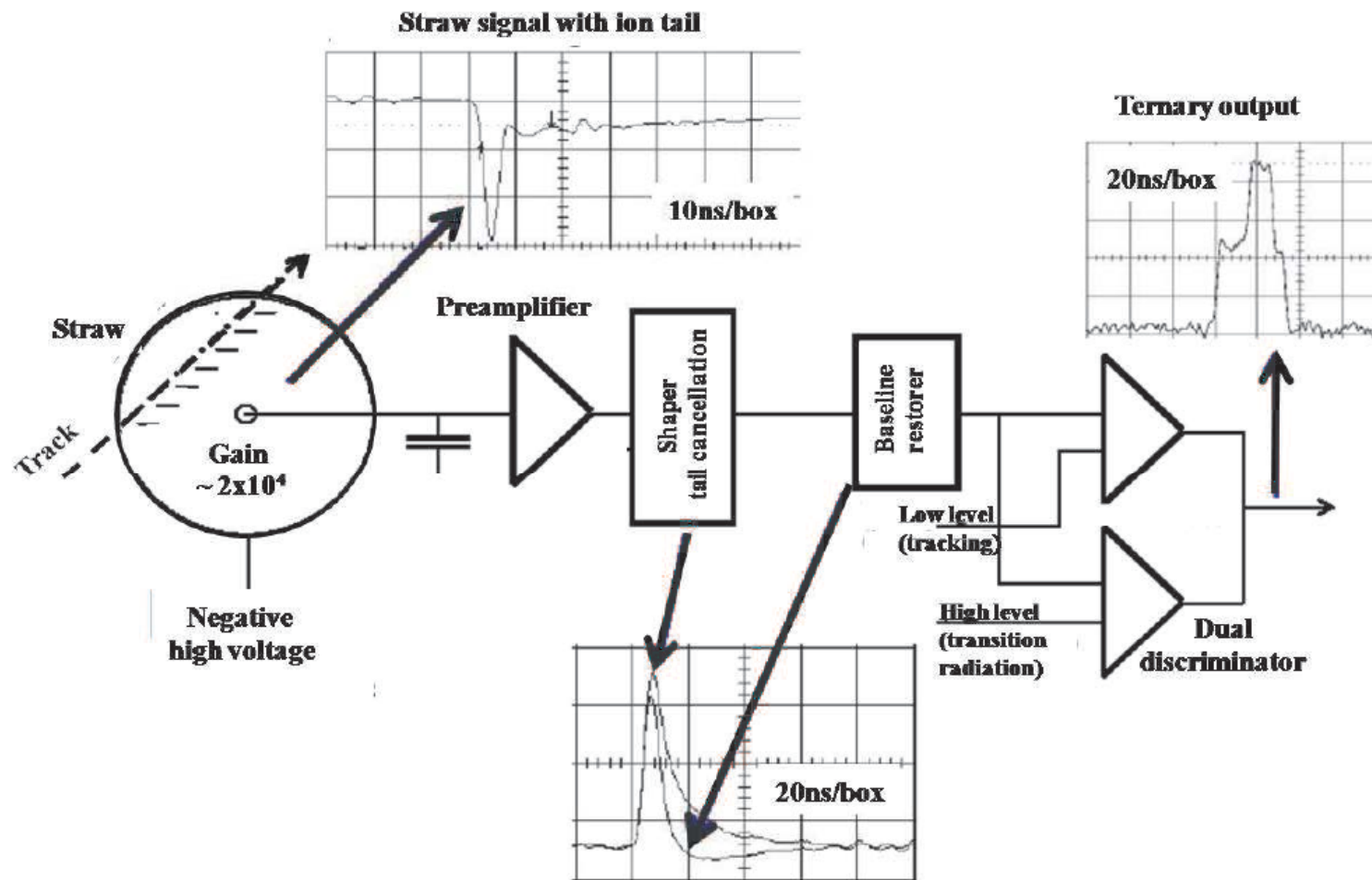


TRT: pion-electron separation



Small circles: low threshold (ionisation), big circles: high threshold (X ray detection)

TRT read-out electronics: two thresholds (low for tracking, high for X rays)



TRT performance

at 90% electron efficiency

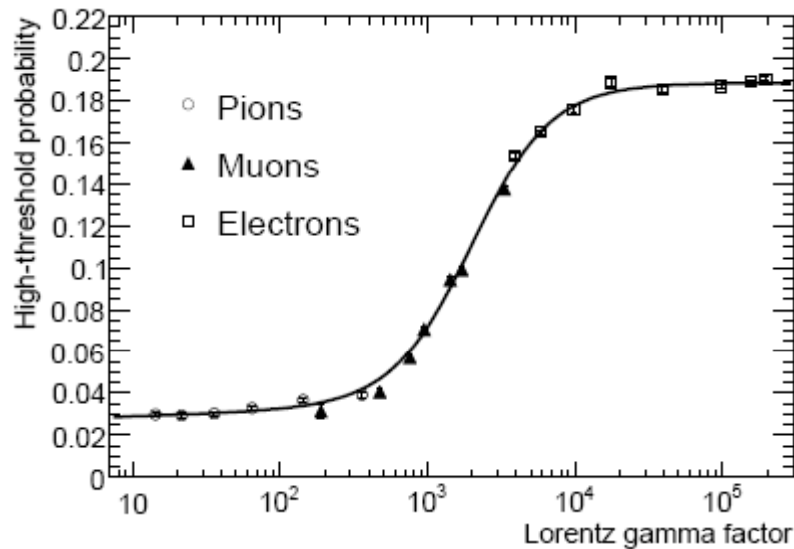


Figure 10.25: Average probability of a high-threshold hit in the barrel TRT as a function of the Lorentz γ -factor for electrons (open squares), muons (full triangles) and pions (open circles) in the energy range 2–350 GeV, as measured in the combined test-beam.

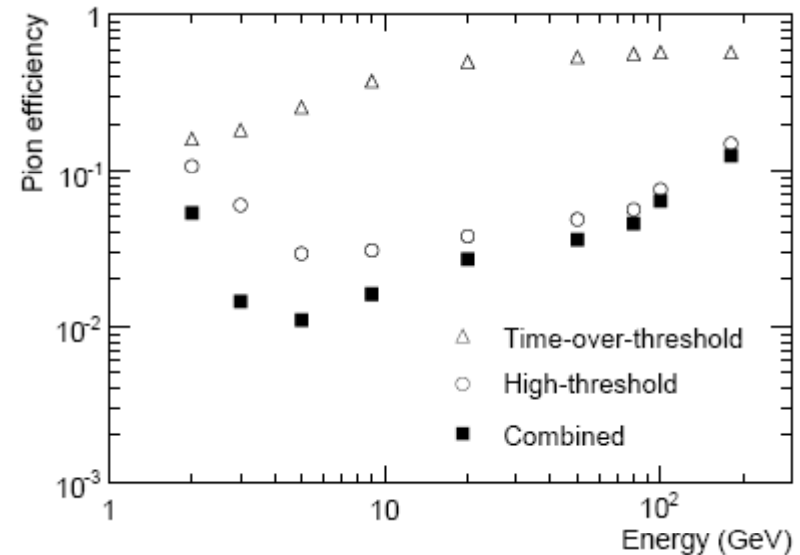
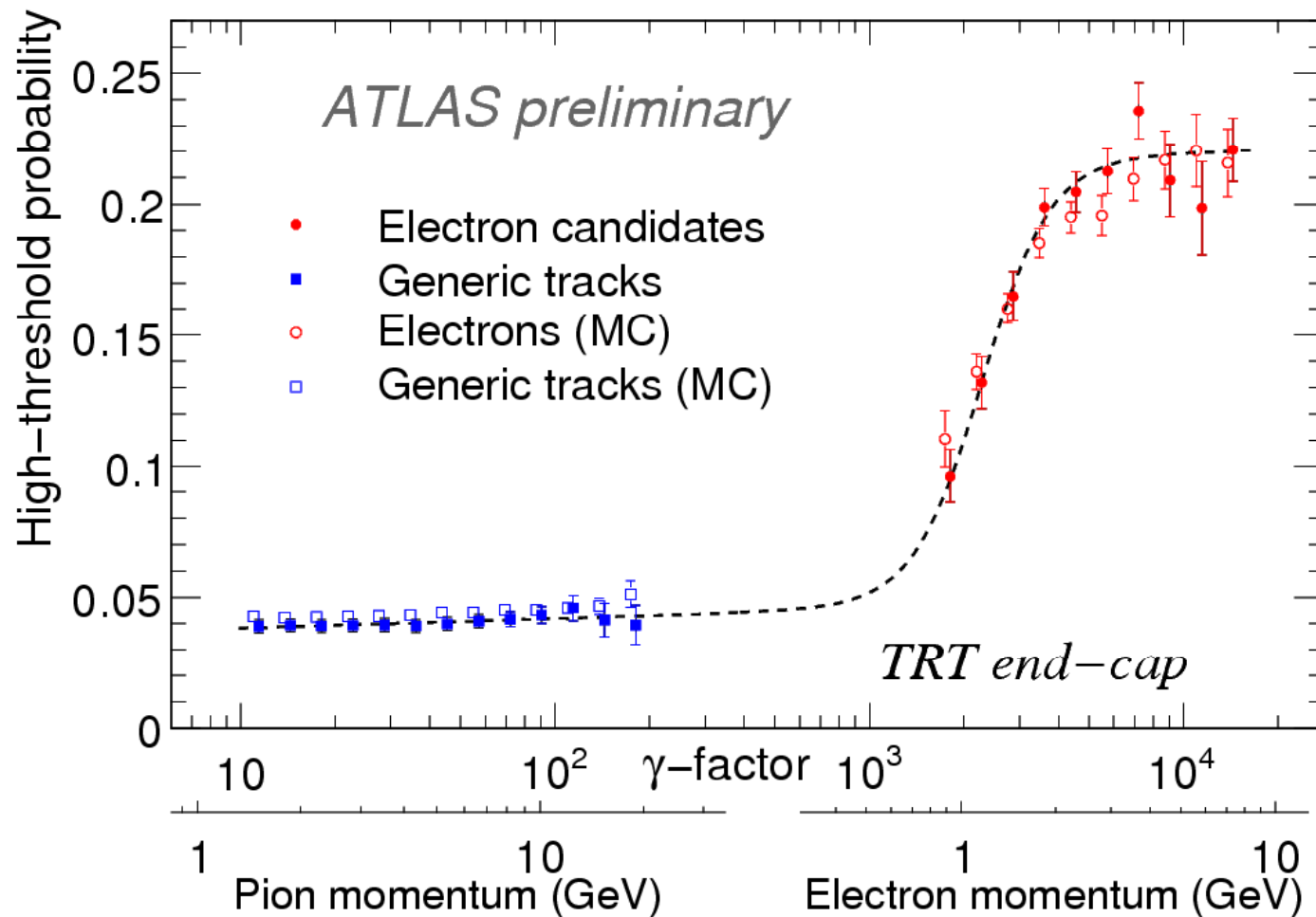


Figure 10.26: Pion efficiency shown as a function of the pion energy for 90% electron efficiency, using high-threshold hits (open circles), time-over-threshold (open triangles) and their combination (full squares), as measured in the combined test-beam.

TRT performance

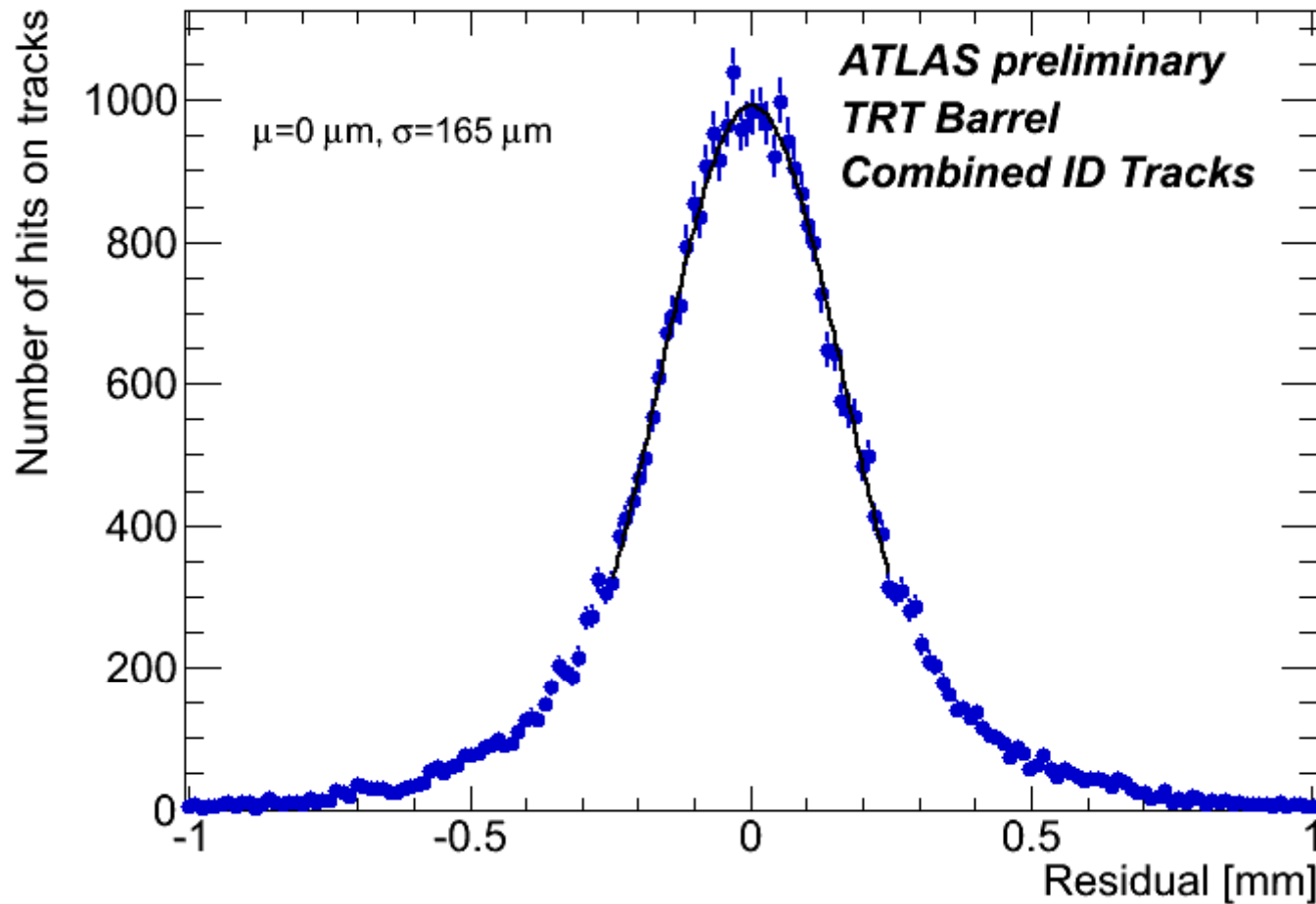
Data from the first LHC collisions in Nov/Dec 2009



e/pion
separation:
high threshold
hit probability
per straw

TRT performance

Data from the first LHC collisions in Nov/Dec 2009



Resolution in
coordinate for
individual
straws

Particle identification

Comparison of methods

Time-of-flight

dE/dx measurement

Čerenkov counters

Transition radiation counters

Compare by calculating the length of detector needed for a given separation (3σ)

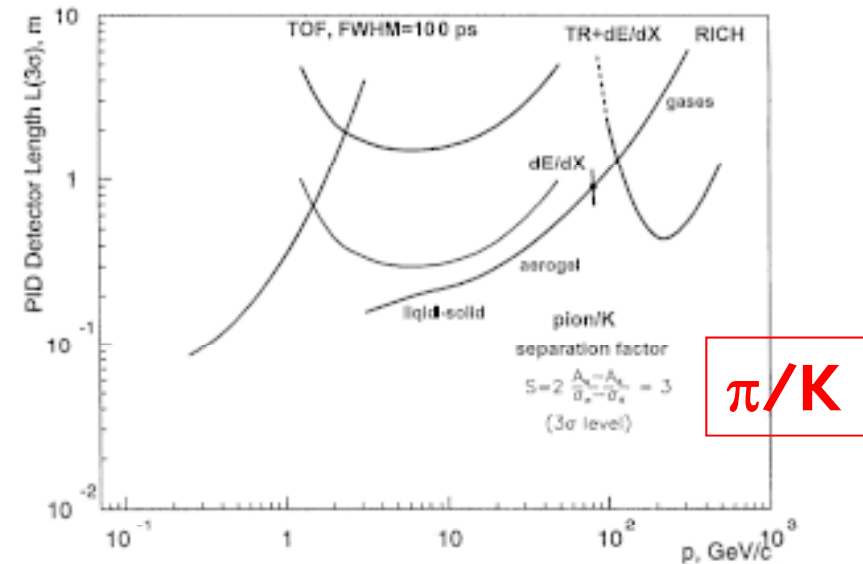


Fig. 14. Pion-kaon separation by different PID methods: the length of the detectors needed for 3 sigma separation.

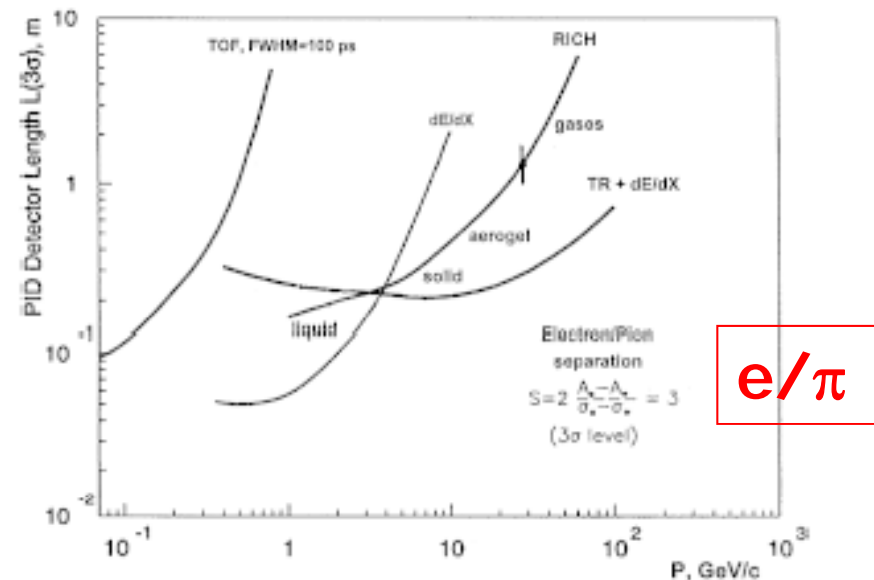


Fig. 15. The same as Fig. 14 for electron-pion separation.

Muon and K_L detector at B factories

Separate muons from hadrons (pions and kaons): exploit the fact that muons interact only electromag., while hadrons interact strongly \rightarrow need a few interaction lengths to stop hadrons (interaction lengths = about 10x radiation length in iron, 20x in CsI). A particle is identified as muon if it penetrates the material.



Detect K_L interaction (cluster): again need a few interaction lengths.

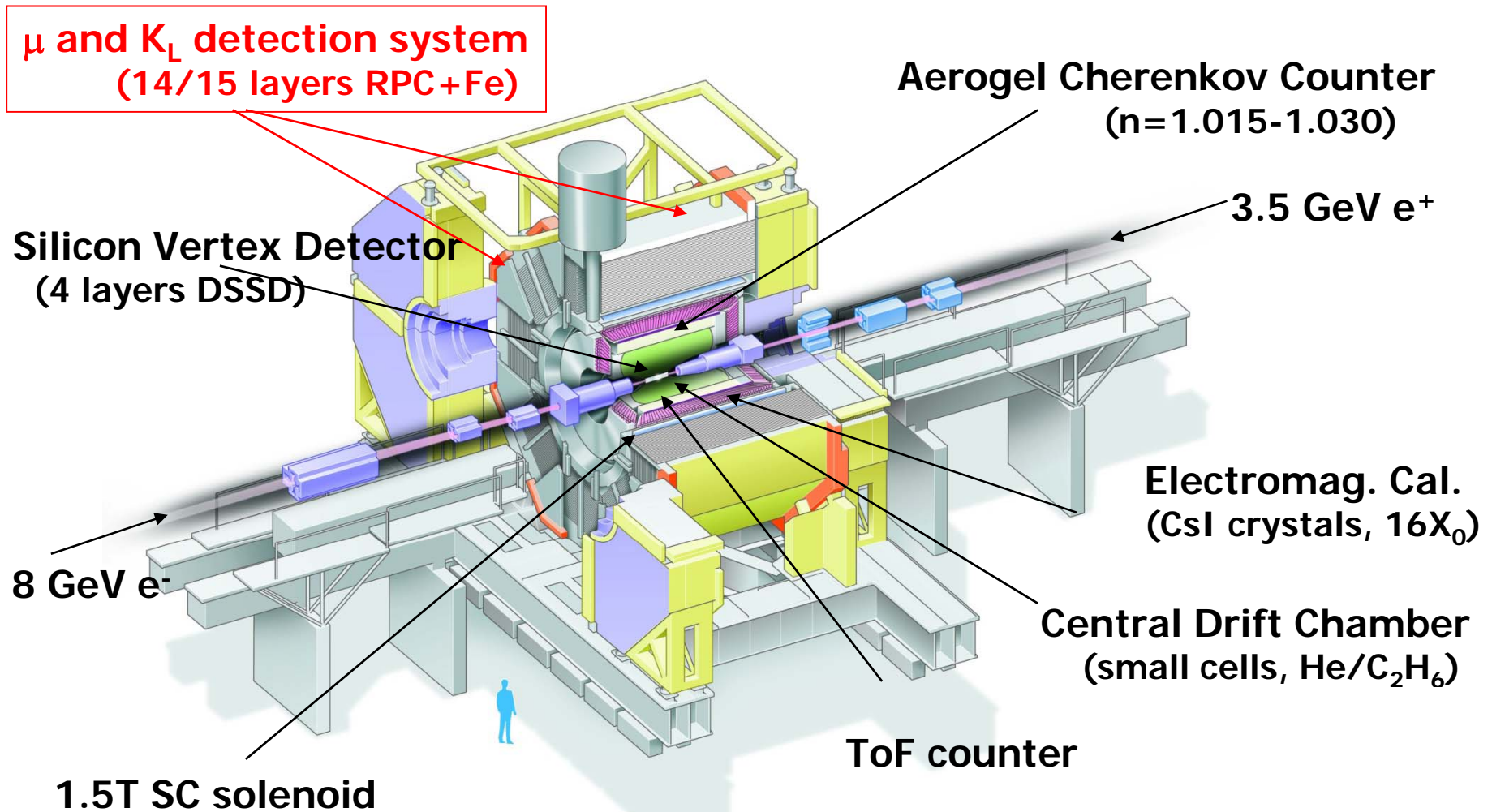
Some numbers: 0.8 interaction length (CsI) + 3.9 interaction lengths (iron)

Interaction length: iron 132 g/cm², CsI 167 g/cm²

$(dE/dx)_{\min}$: iron 1.45 MeV/(g/cm²), CsI 1.24 MeV/(g/cm²)

$\rightarrow \Delta E_{\min} = (0.36+0.11) \text{ GeV} = 0.47 \text{ GeV} \rightarrow$ reliable identification of muons possible above $\sim 600 \text{ MeV}$

Example: Muon and K_L detection at Belle



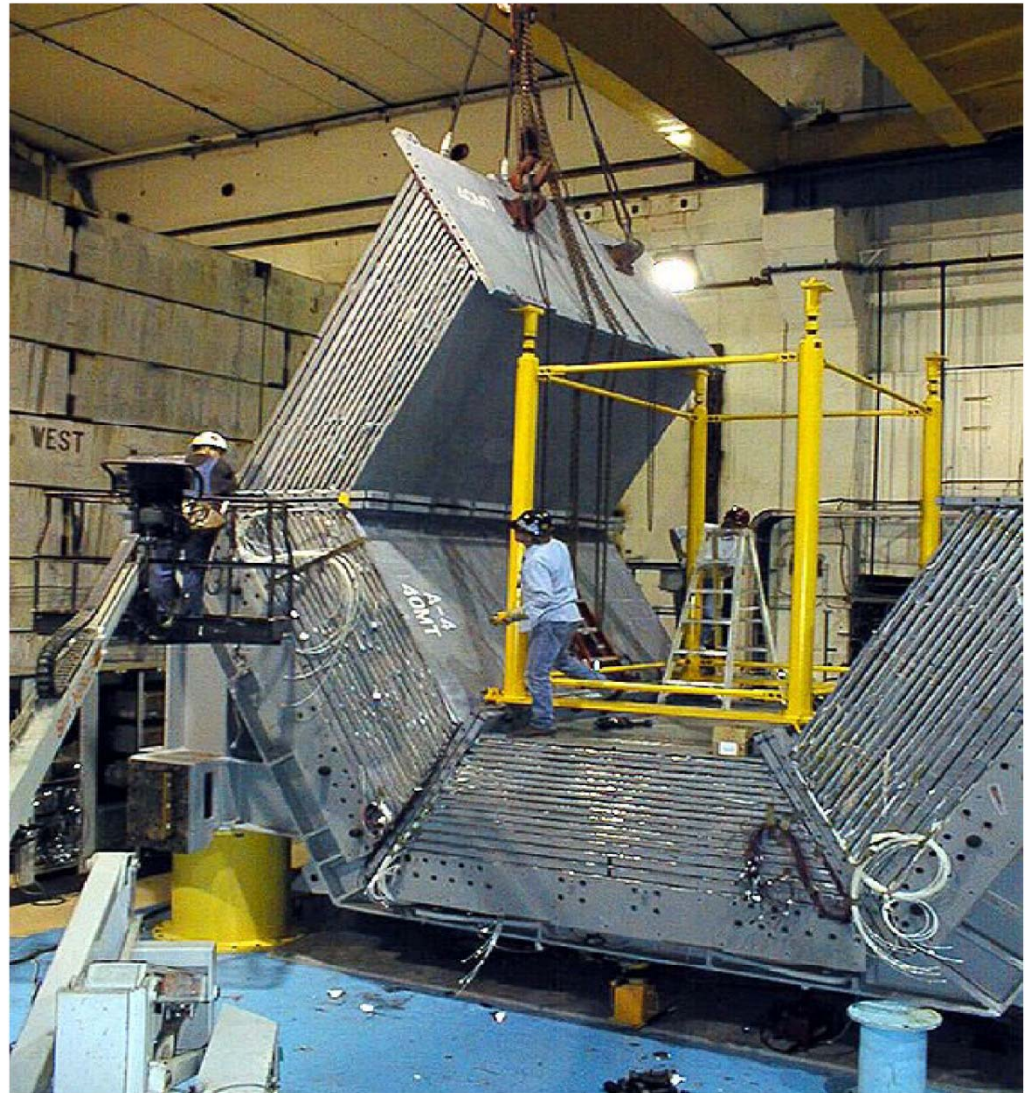
Muon and K_L detector

Up to 21 layers of resistive-plate chambers (RPCs) between iron plates of flux return

Bakelite RPCs at BABAR

Glass RPCs at Belle

(better choice)



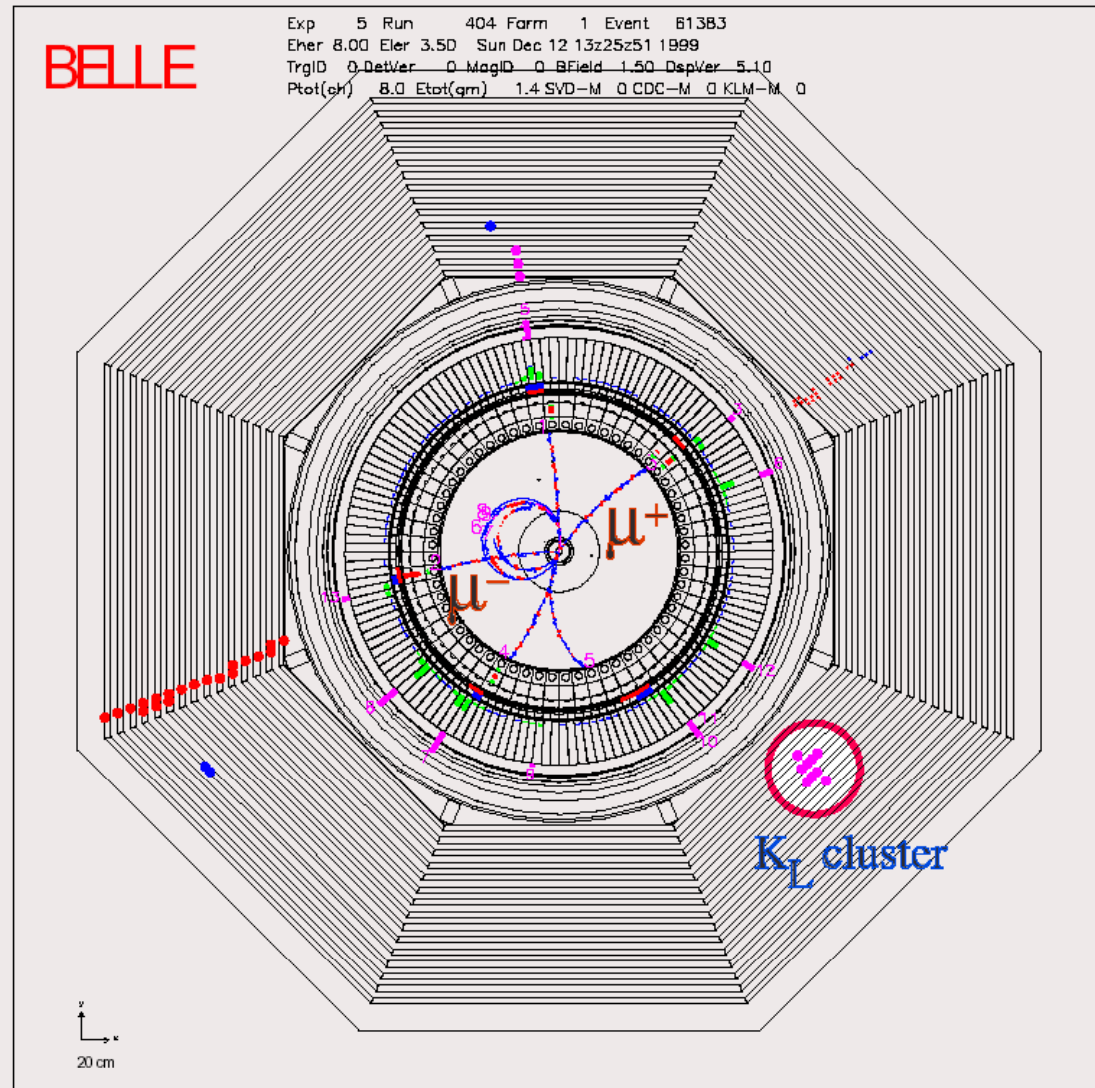
Muon and K_L detector

Example:

event with

- two muons and a
- K_L

and a pion that
partly penetrated



Muon and K_L detector performance

Muon identification: efficient for $p > 800$ MeV/c

efficiency

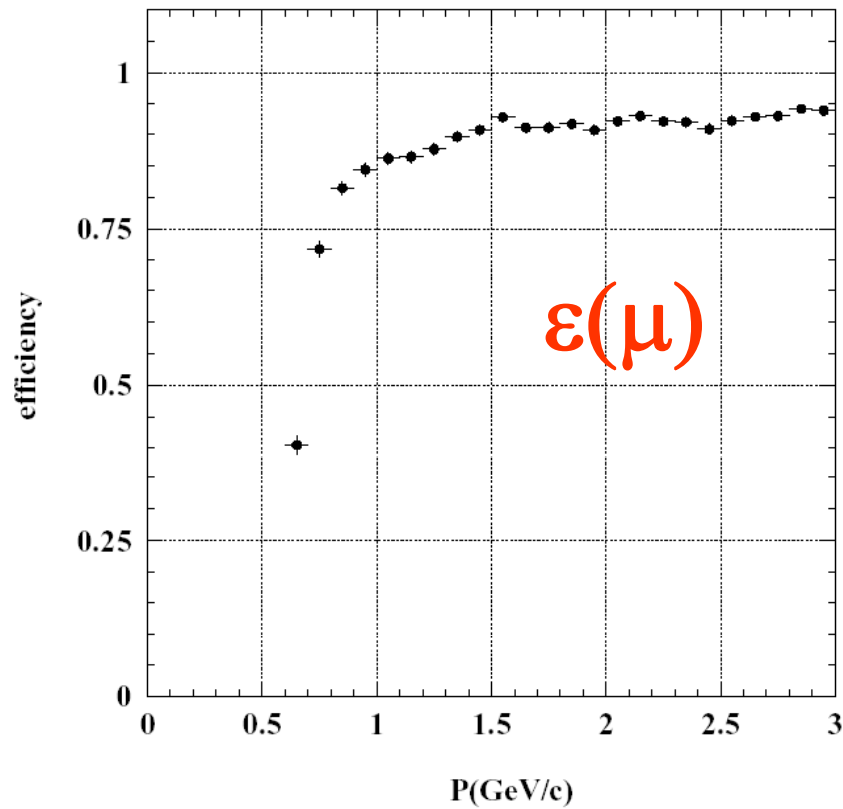


Fig. 109. Muon detection efficiency vs. momentum in KLM.

fake probability

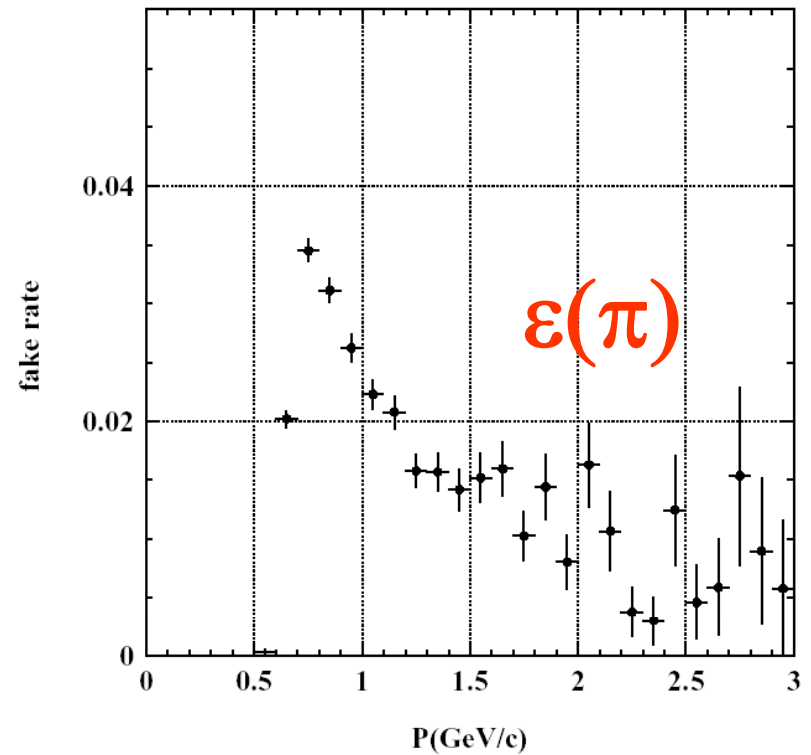


Fig. 110. Fake rate vs. momentum in KLM.

Muon and K_L detector performance

K_L detection: resolution in direction →

K_L detection: also with possible with electromagnetic calorimeter (0.8 interaction lengths)

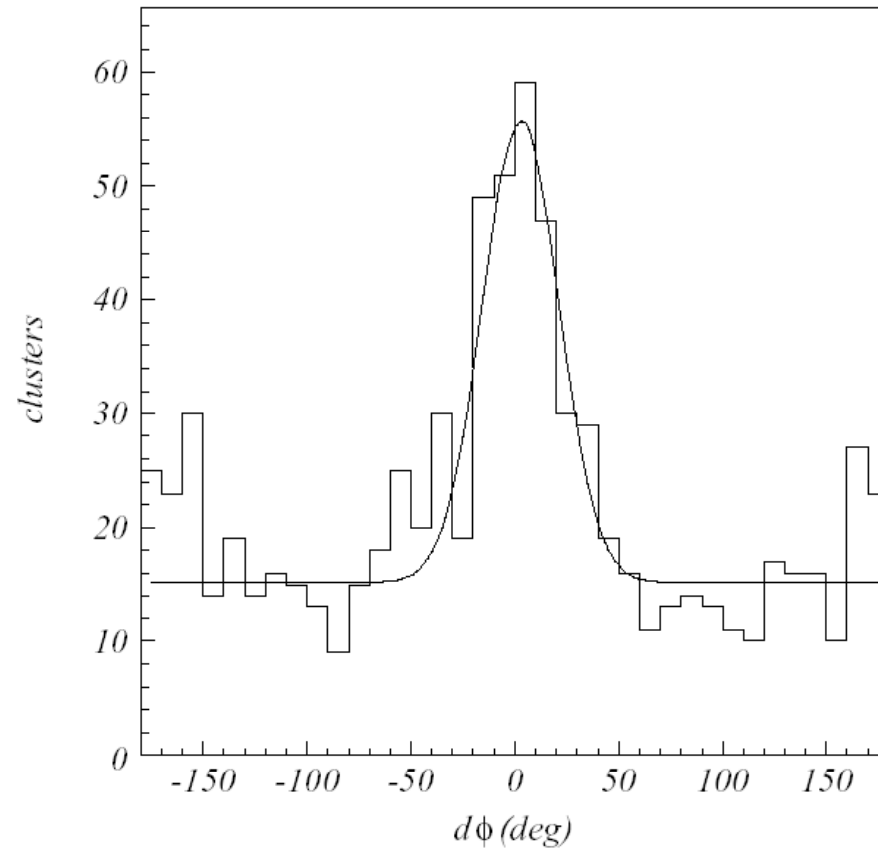
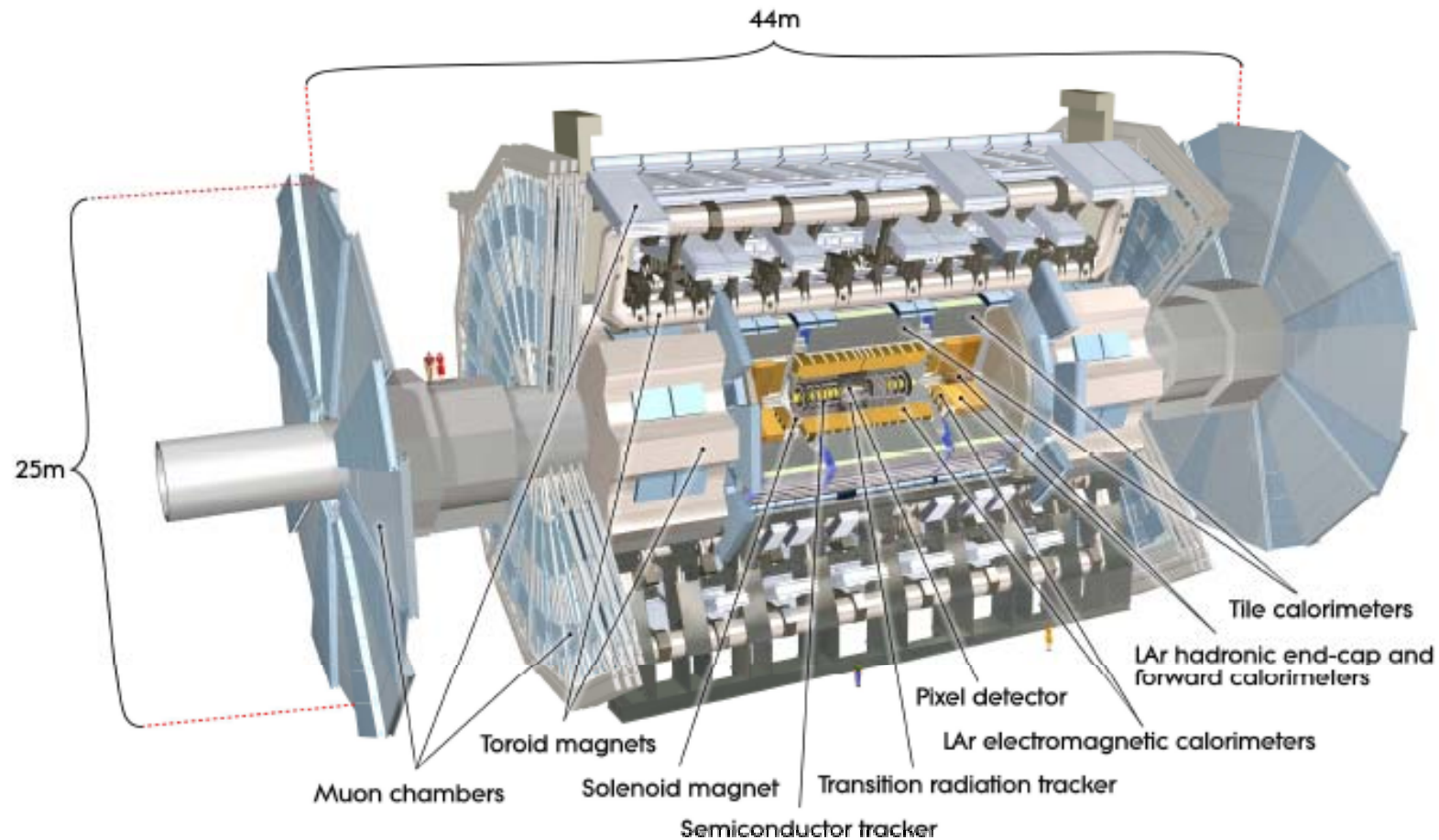
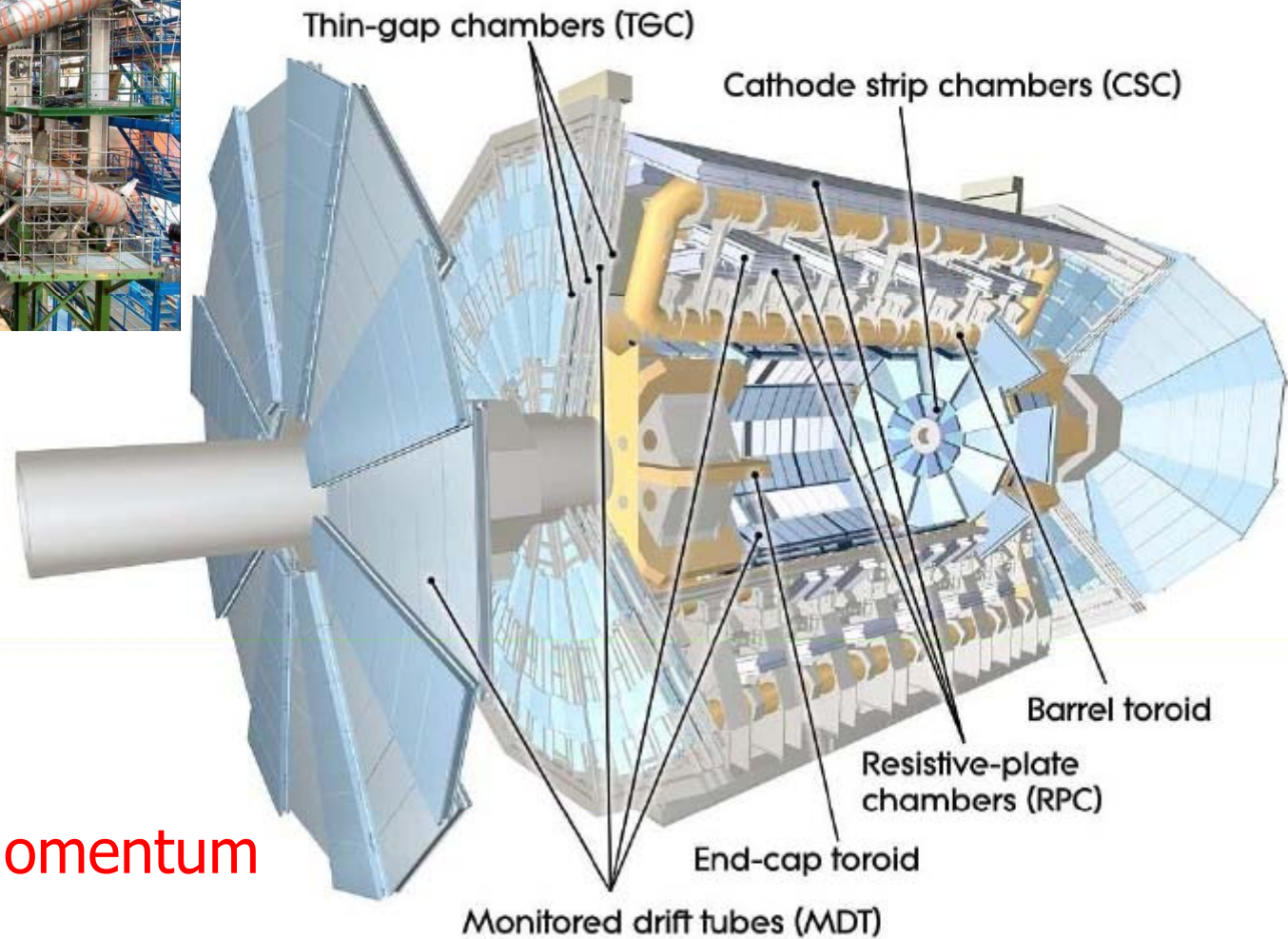
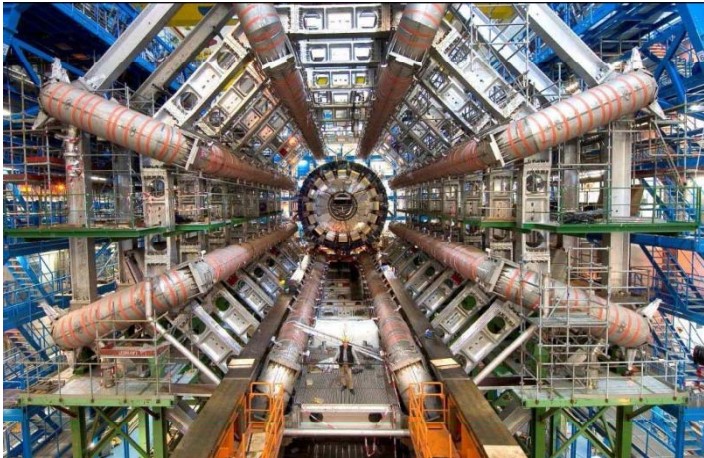


Fig. 107. Difference between the neutral cluster and the direction of missing momentum in KLM.

Identification of muons at LHC - example ATLAS

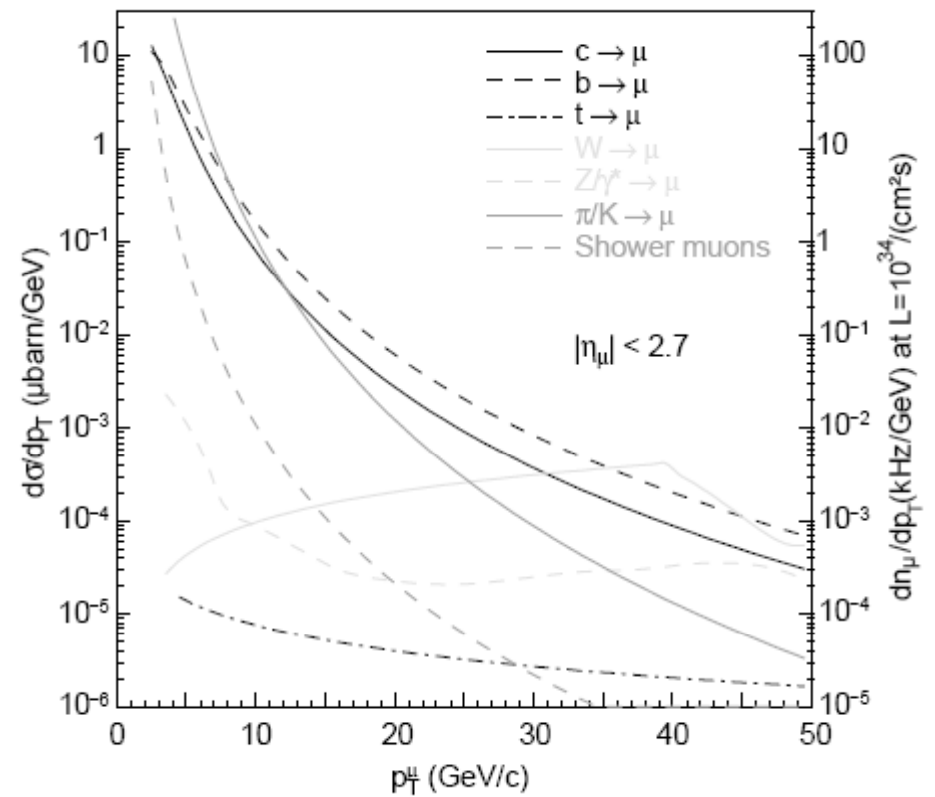
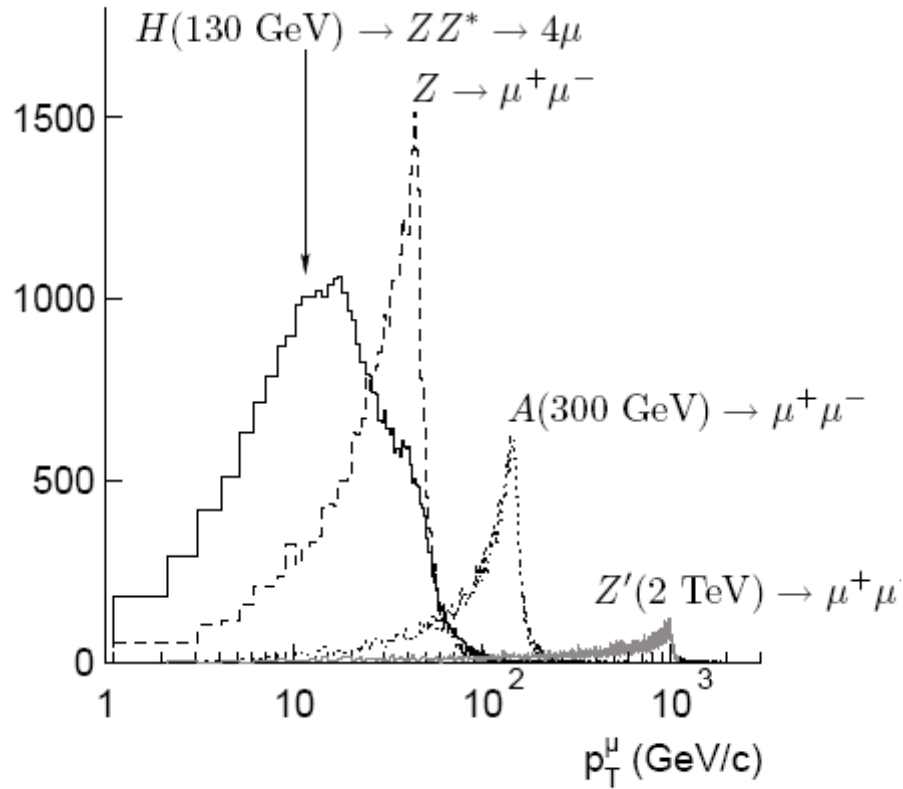


Identification of muons in ATLAS



- Identify muons
- Measure their momentum

Muon spectrum



Muon identification in ATLAS

Material in front of the muon system

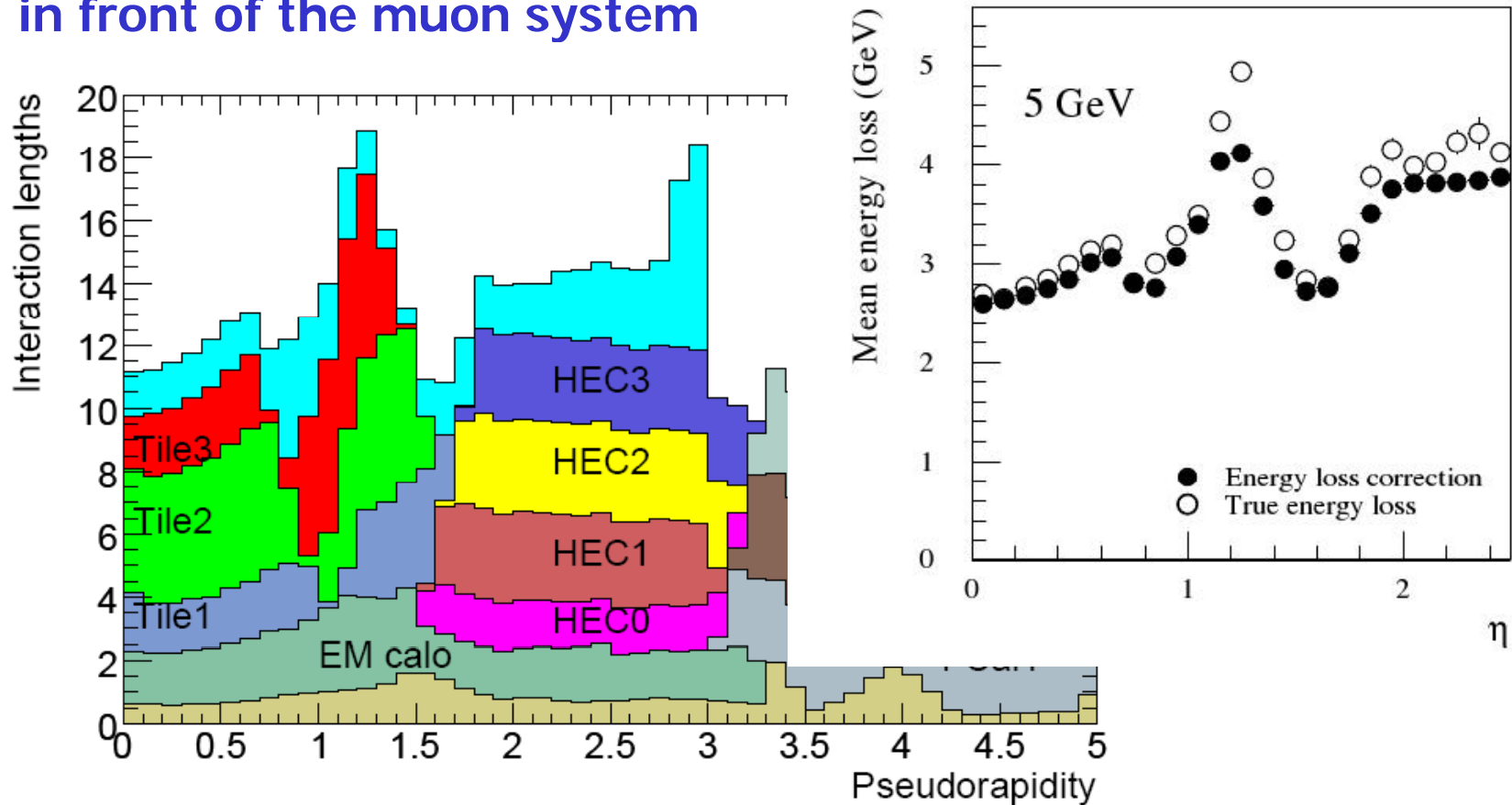


Figure 5.2: Cumulative amount of material, in units of interaction length, as a function of $|\eta|$, in front of the electromagnetic calorimeters, in the electromagnetic calorimeters themselves, in each hadronic layer, and the total amount at the end of the active calorimetry. Also shown for completeness is the total amount of material in front of the first active layer of the muon spectrometer (up to $|\eta| < 3.0$).

Muon identification efficiency

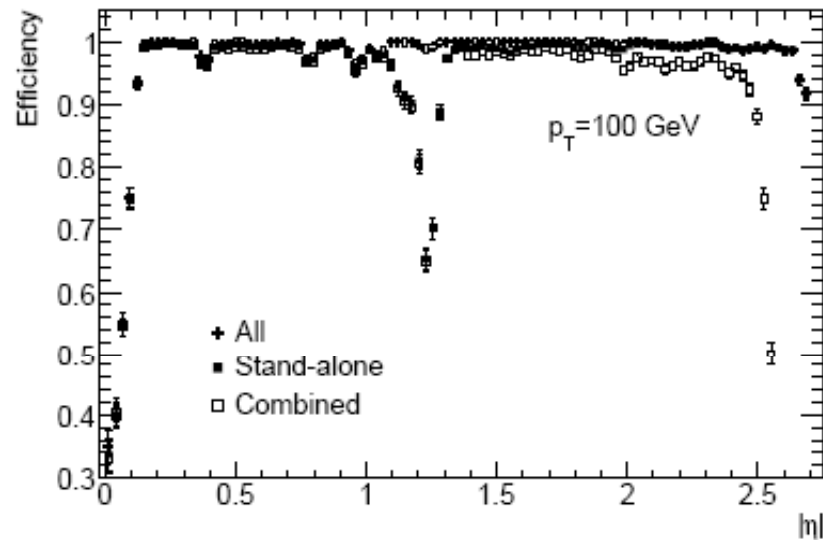


Figure 10.37: Efficiency for reconstructing muons with $p_T = 100$ GeV as a function of $|\eta|$. The results are shown for stand-alone reconstruction, combined reconstruction and for the combination of these with the segment tags discussed in the text.

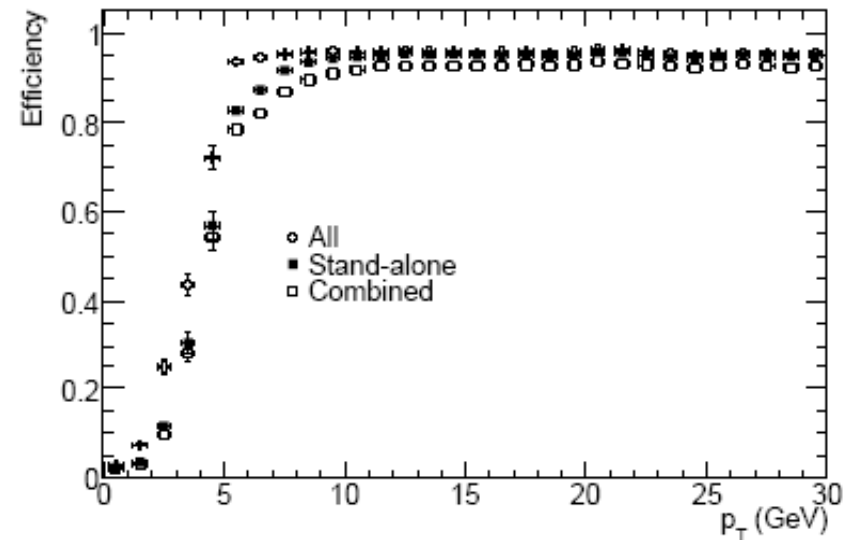


Figure 10.38: Efficiency for reconstructing muons as a function of p_T . The results are shown for stand-alone reconstruction, combined reconstruction and for the combination of these with the segment tags discussed in the text.

Muon fake probability

Sources of fakes:

-Hadrons: punch through negligible, >10 interaction lengths of material in front of the muon system (remain: muons from pion and kaon decays)

-Electromagnetic showers triggered by energetic muons traversing the calorimeters and support structures lead to low-momentum electron and positron tracks, an irreducible source of fake stand-alone muons. Most of them can be rejected by a cut on their transverse momentum ($p_T > 5$ GeV reduces the fake rate to a few percent per triggered event); can be almost entirely rejected by requiring a match of the muon-spectrometer track with an inner-detector track.

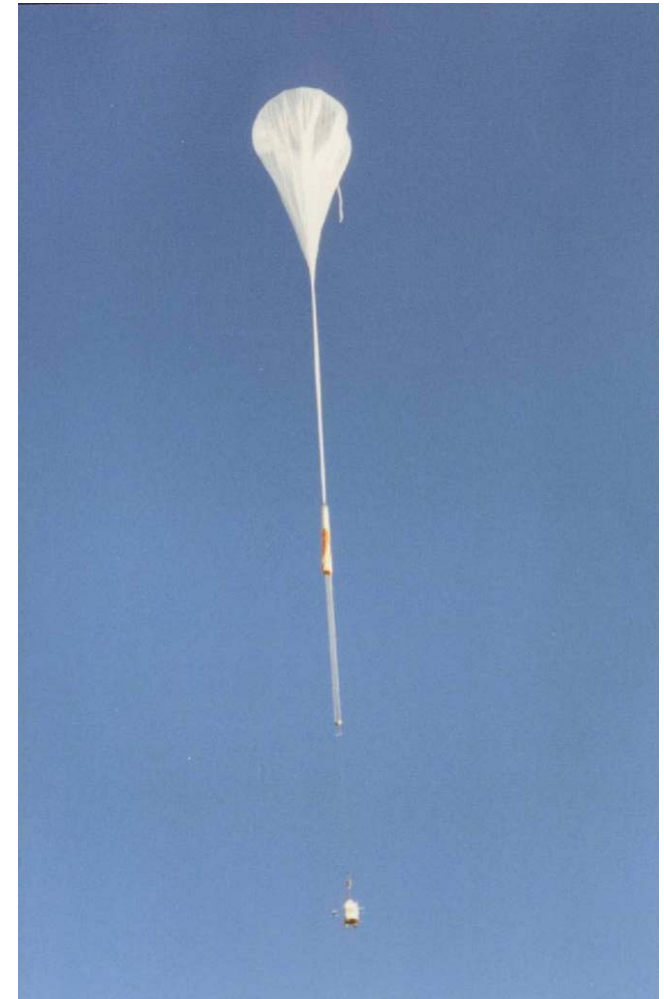
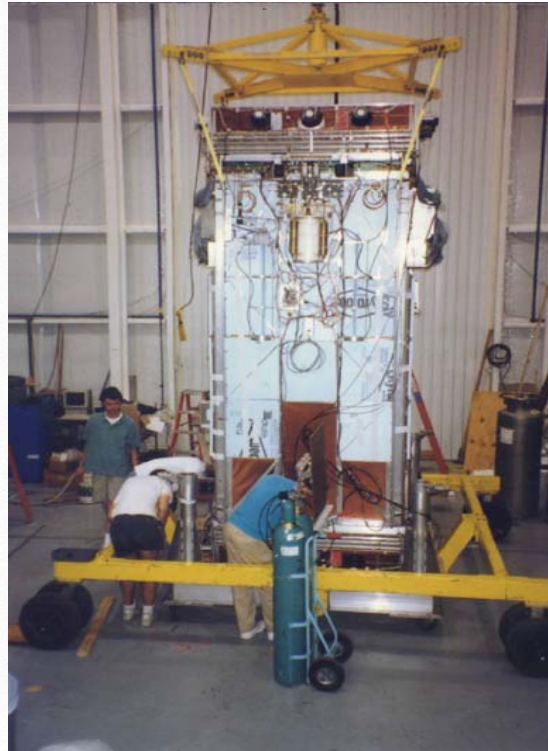
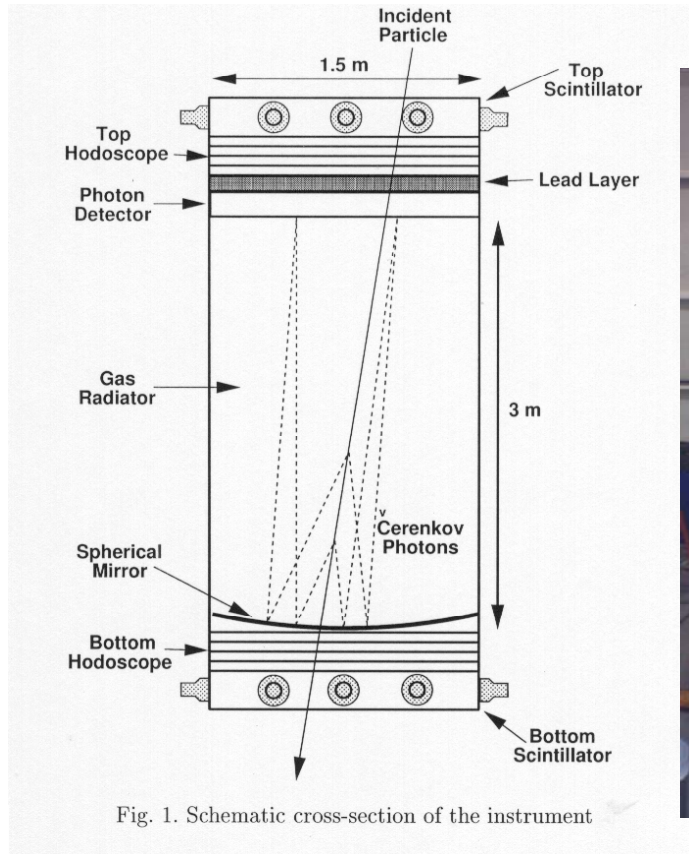
- Fake stand-alone muons from the background of thermal neutrons and low energy γ -rays in the muon spectrometer ("cavern background"). Again: $p_T > 5$ GeV reduces this below 2% per triggered event at 10^{33} cm⁻² s⁻¹. Can be reduced by almost an order of magnitude by requiring a match of the muon-spectrometer track with an inner-detector track.

Identification in astro-physics/astroparticle physics - 1

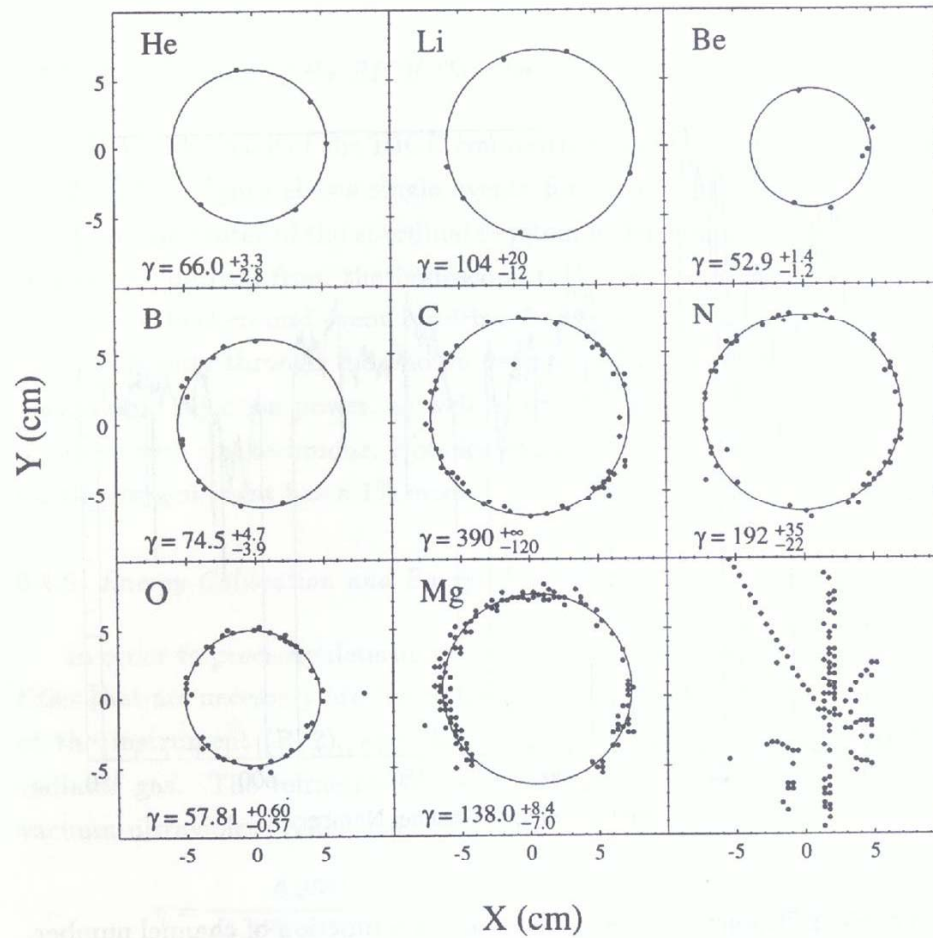
- Study composition of cosmic rays in balloon or satellite flights
- Identify (very) high energy cosmic rays and photons with detectors on the ground

Short flight small area detectors (Balloons)

Examples of Balloon-flown RICH detectors



3-metre N_2 radiator, TMAE/ CH_4 : $\gamma_{th}=40$
p + He at high energy:
3-metre C_2F_6 radiator, TMAE/ C_2H_6 : $\gamma_{th}=25$



**Heavy nucleus rings from 1991 flight –
 Note that carbon here has total energy
 $\sim 12 * 390 \text{ GeV} = 4.6 \text{ TeV}$**

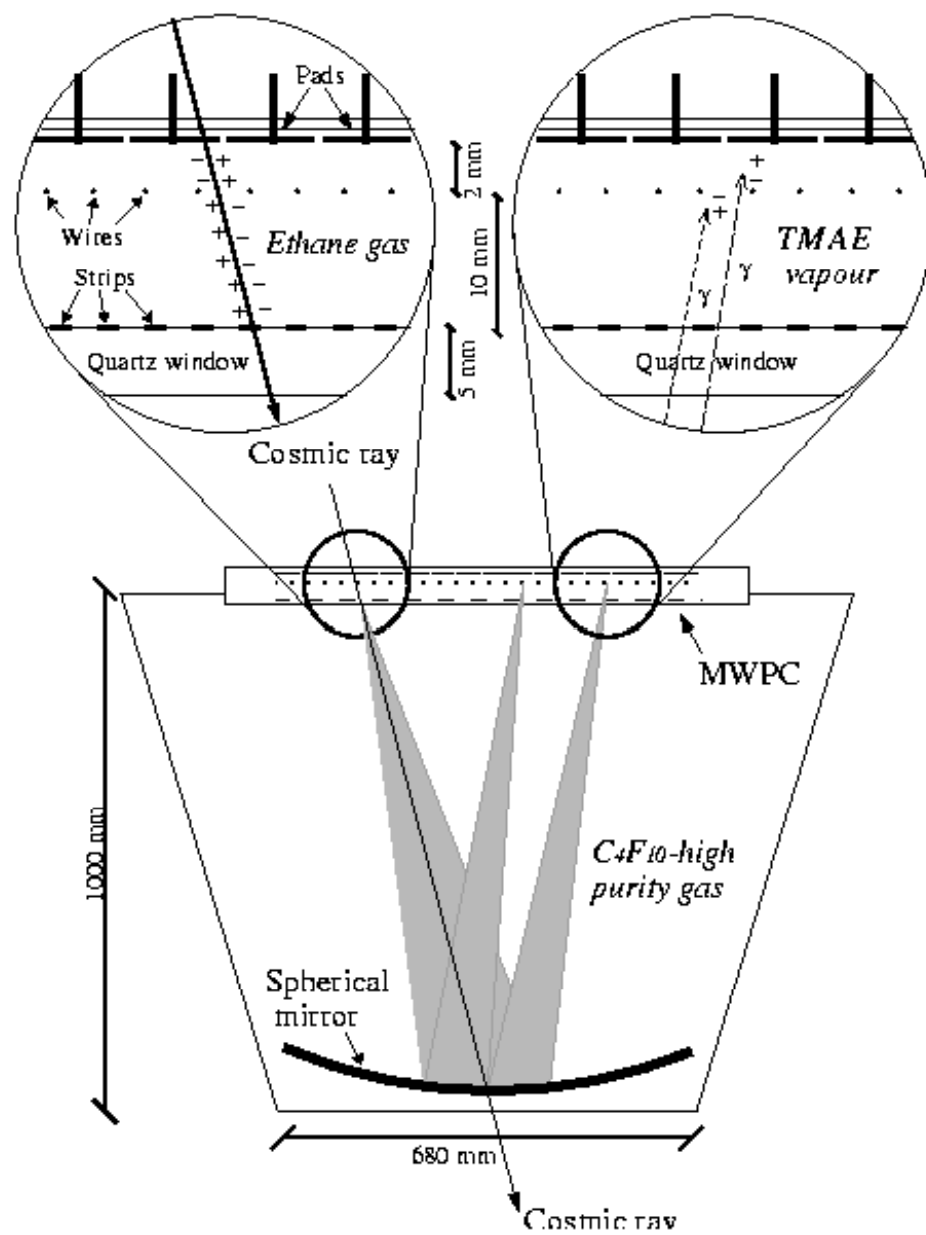


Figure 1.4: Schematic view of the CAPRICE98 RICH detector.

HESS 1 UHE Gamma Ray Telescope Stereoscopic Quartet

Khomas Highland, Namibia, (23°16'S, 16°30'E, elev. 1800m)

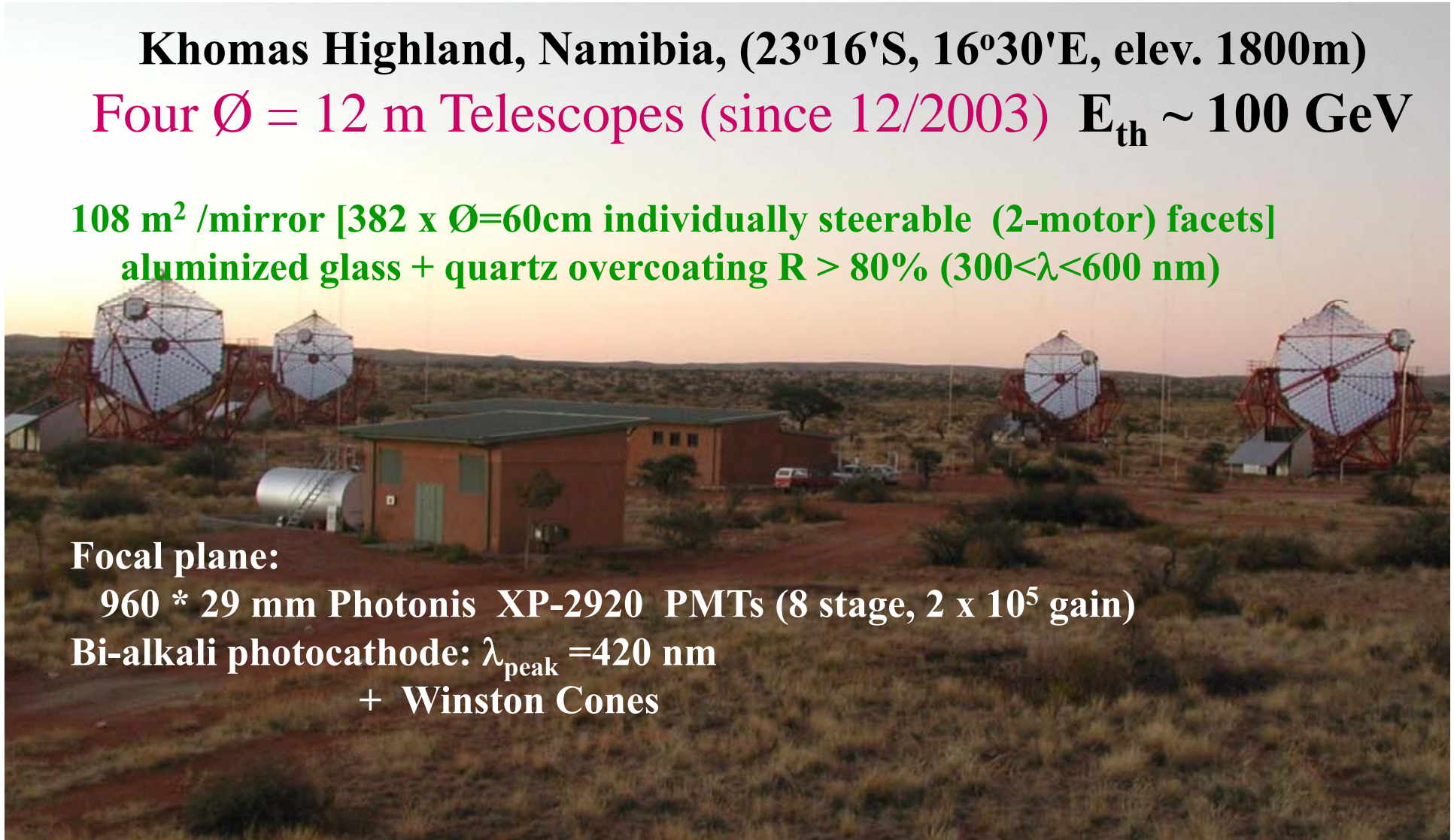
Four $\varnothing = 12$ m Telescopes (since 12/2003) $E_{th} \sim 100$ GeV

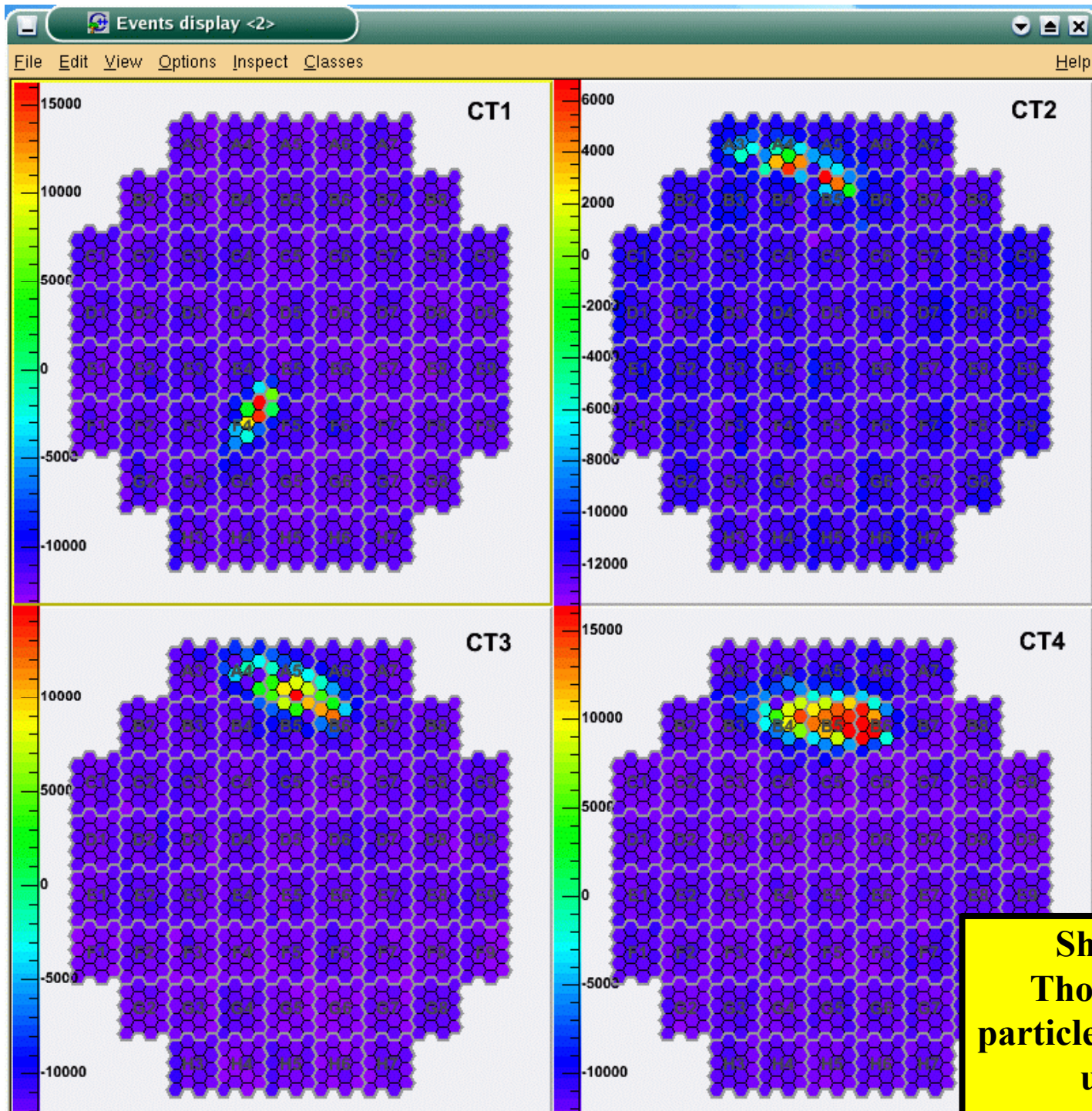
**108 m² /mirror [382 x $\varnothing=60$ cm individually steerable (2-motor) facets]
aluminized glass + quartz overcoating $R > 80\%$ ($300 < \lambda < 600$ nm)**

Focal plane:

960 * 29 mm Photonis XP-2920 PMTs (8 stage, 2×10^5 gain)

**Bi-alkali photocathode: $\lambda_{peak} = 420$ nm
+ Winston Cones**





Detection of high-energy gamma rays

using Cherenkov telescopes

The HESS 1
Concept

Shower mainly E-M.
Thousands of relativistic particles give Čerenkov light in upper atmosphere

Appendix A: derivation of $d\sigma/d(h\omega)$ a la Allison, Cobb

- Solve Maxwell's Equations with charge density

$$\rho = e_0 \delta^3(\vec{r} - \vec{\beta}ct) \text{ and current density } \vec{j} = \vec{\beta}c\rho \rightarrow$$

$$\phi(\vec{k}, \omega) = \frac{e_0}{2\pi\epsilon_0 k^2} \delta(\omega - \vec{k} \cdot \vec{\beta}c)$$

$$\vec{A}(\vec{k}, \omega) = \frac{e_0}{2\pi\epsilon_0 c^2} \frac{(\omega\vec{k}/k^2 - \vec{\beta}c)}{(\epsilon\omega^2/c^2 - k^2)} \delta(\omega - \vec{k} \cdot \vec{\beta}c)$$

$$\rightarrow \vec{E}(\vec{r}, t) =$$

$$\frac{1}{(2\pi)^2} \int \int [i\omega\vec{A}(\vec{k}, \omega) - i\vec{k}\phi(\vec{k}, \omega)] e^{i(\vec{k}\cdot\vec{r} - \omega t)} d^3k d\omega \quad (\text{A1})$$

- The energy loss is due to the component of this electric field in the direction β doing work on the particle at the point $\vec{r} = \vec{\beta}ct$:

$$\langle \frac{dE}{dx} \rangle = \frac{e_0}{\beta} \vec{E}(\vec{\beta}ct, t) \cdot \vec{\beta} \quad (\text{A2})$$

- The energy loss is re-written as a probability of energy transfers

$$\langle \frac{dE}{dx} \rangle =$$

$$- \int_0^\infty d(h\omega) \int_{\frac{h\omega}{\beta c}}^\infty d(hk) n_e h\omega \frac{d^2\sigma}{d(h\omega)d(hk)} \quad (\text{A3})$$

where n_e is the electron density and $\frac{d^2\sigma}{d(h\omega)d(hk)}$ is the double differential cross section per electron.

Appendix A: derivation of $d\sigma/d(h\omega)$ a la Allison, Cobb - 2

Θ is the phase of $1 - \epsilon_1\beta^2 + i\epsilon_2\beta^2$.

Comments:

First term has a factor $\log(1 - \beta^2\epsilon_1)$, responsible for the relativistic rise of the energy-loss cross section and its saturation.

Third term: in the optical region σ_γ vanishes and only the second term contributes \rightarrow Čerenkov radiation.

Phase: $\Theta = 0$ below, and π above threshold. We get the familiar formula for the number of emitted photons per unit track path length and photon energy interval $\frac{d^2N}{d(h\omega)dx} = \frac{\alpha}{hc} \left[1 - \frac{1}{\beta^2\epsilon}\right]$

When ϵ_2 and σ_γ do not vanish, the separate interpretation of this term in the cross section dissolves and it may even be negative.

Last term: constituent scattering from electrons. It is a Rutherford scattering term, shows no relativistic behaviour and is the only non-zero term for energy transfers $\hbar\omega$ in the far X-ray region, describes δ -ray production.