Prehod nabitih delcev in fotonov skozi snov

I Prehod nabitih delcev

- Interakcija delec - snov
- Energijske izgube težkih nabitih delcev
- Energijske izgube elektronov in pozitronov
- Večkratno sipanje
- Energijsko stresanje
- Energijske izgube visokoenergijskih mionov

II. Prehod fotonov

- Fotoefekt
- Comptonsko sipanje
- Tvorba parov

III EM plaz

Literatura:

W. Leo : Techniques for Nuclear and Particle .....
K. Kleinknecht : Detectors for Particle Radiation
T. Ferbel : Experimental Techniques in HEP
W. Heitler : The Quantum Theory of Radiation
Particle Data Group : Review of Particle Properties, 1996
+ MX2 → 2008

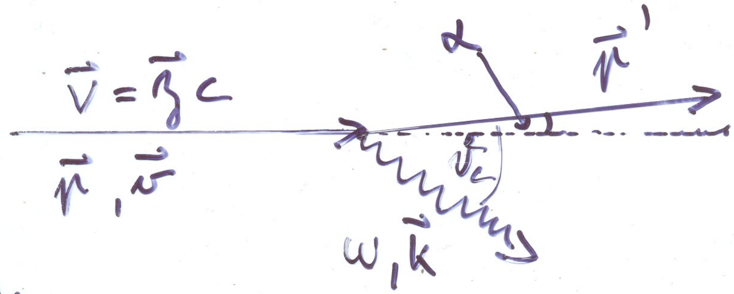
I Prehod nabitih delcev

a) interakcija delec - snov

delec : $M, \; p, \; \vec\beta = \frac{\vec v}{c}$

snov : $\varepsilon(\omega) = \varepsilon_1(\omega) + i\,\varepsilon_2(\omega)$

$$\omega^2 = \frac{k^2 c^2}{\varepsilon} \qquad (1)$$



$\vec p = \vec p\,' + \hbar\vec k$

$E = E' + \hbar\omega$

$\hbar k \ll p$

$\hbar\omega \ll E$: $\hbar\omega = \Delta E = \frac{p\,\Delta p}{E} = v\,\Delta p$

$\vec p - \vec p\,' = \Delta\vec p = \hbar\vec k \;|\cdot\vec v \Rightarrow (\vec p - \vec p\,')\cdot\vec v = pv - p'v\cos\alpha$

$\approx v\,\Delta p$ $(\alpha \ll 1)$

$\Rightarrow \hbar\vec k\cdot\vec v = \hbar\omega$

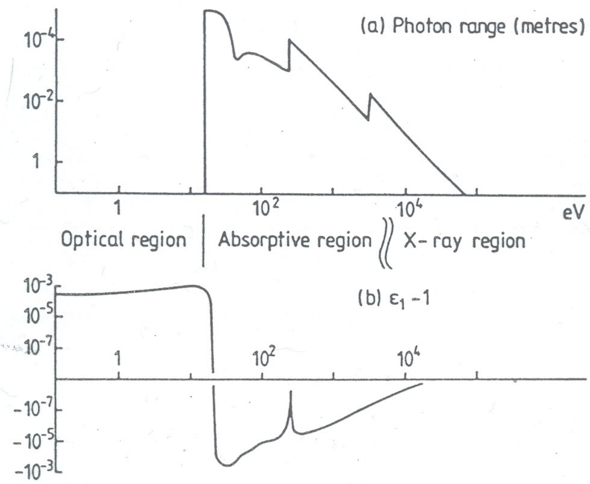
$$\omega = k\,v\cos\vartheta_c \qquad (2)$$

iz (1) in (2)

$$\frac{c^2}{\varepsilon} = v^2\cos^2\vartheta_c$$

$$\underline{\sqrt{\varepsilon}\,\frac{v}{c}\cos\vartheta_c = 1} \qquad (3)$$

Odvisnost $\varepsilon(\omega)$



ε_2

$\varepsilon_1 - 1$

Fig.3. The dependence of ε for argon at normal density on photon energy,
a) imaginary part expressed as a range and
b) real part - 1 on a split log scale.

tri območja

a) optično $\quad \varepsilon_2 = 0 \quad \varepsilon_1 > 1$

$\hbar\omega < 2\,\mathrm{eV}$

$v \geq \frac{c}{\sqrt{\varepsilon}} \Rightarrow \vartheta_c$ realen $\rightarrow$ sevanje Čerenkova

b) absorbtivno $\quad \varepsilon_2 > 0 \quad \varepsilon_1 < 1$

$2\,\mathrm{eV} < \hbar\omega < 5\,\mathrm{keV}$

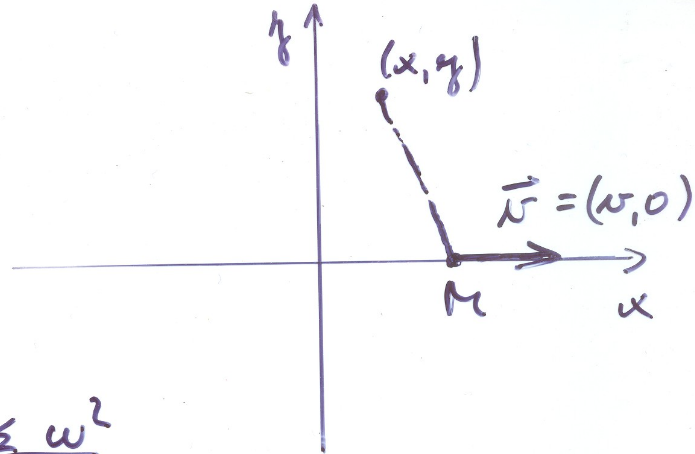
absorbcija virtualnih fotonov $\rightarrow$ ionizacija vzbujanje

c) rentgensko $\quad \varepsilon_2 \ll 1 \quad \varepsilon_1 \lessapprox 1$

$\hbar\omega > 5\,\mathrm{keV}$

malo ionizacije (δ-elektroni)

sevanje na prehodih snovi $\rightarrow$ prehodno sevanje



iz (2)

$$v k_x = \omega$$

iz (1)

$$k^2 = k_x^2 + k_y^2 = \frac{\varepsilon \omega^2}{c^2}$$

$$\Rightarrow k_y = \frac{\omega}{v}\sqrt{\frac{\varepsilon v^2}{c^2} - 1}$$

vpeljimo: $c_m = \frac{c}{\sqrt{\varepsilon}}$; $\beta' = \frac{v}{c_m}$; $\gamma' = \frac{1}{\sqrt{1-\beta'^2}}$

$$k_y = \frac{\omega}{v}\sqrt{\beta'^2 - 1} \qquad (4)$$

območji – $\beta' > 1$ k_x, k_y realna

→ sevanje $e^{i(\vec{k}\vec{r} - \omega t)}$

– $\beta' < 1$ k_y imaginaren, polje dušeno v transverzalni smeri

$$e^{i(\vec{k}\vec{r} - \omega t)} = e^{i\omega\left(\frac{x}{v} - t\right)} e^{-\frac{y}{y_0}}$$

$$y_0 = \frac{v}{\omega}\frac{1}{\sqrt{1-\beta'^2}} = \frac{\beta'\gamma'}{k} \qquad (5)$$

doseg narašča z $\beta'\gamma'$ → relativistični dvig z β, γ

$$y_0 = \frac{\beta}{k}\frac{1}{\sqrt{\frac{1}{\gamma^2} + (1-\varepsilon)\beta^2}} \qquad (6)$$

optično : $\varepsilon > 1$ $\quad \gamma' \to 1 \Rightarrow y_0 \to \infty$ Čerenkov

$\varepsilon < 1$ $\quad y_0$ narašča z γ

$$y_{0,max} = \frac{1}{k\sqrt{1-\varepsilon}} \qquad (7)$$

nasičenje od $\frac{1}{\gamma^2} \sim (1-\varepsilon)\gamma^2 \Rightarrow \gamma\gamma \sim \frac{1}{\sqrt{1-\varepsilon}}$

$1-\varepsilon$ susceptibilnost $\propto \rho$

nasičenje $\gamma\gamma \sim \frac{1}{\sqrt{\rho}}$

večja gostota → prej do nasičenja
→ manjši relativistični dvig

plini 1,3 → 1,7 (Xe)

emulzije 1,15

polvodniki 1,1

plastični scint. 1,01 - 1,02

interakcija (virtualnih) fotonov z atomi – fotoabsorbcijski ionizacijski model

dif. presek na posamezen elektron v atomih v snovi

$$\frac{d\sigma}{dE} = \frac{\alpha}{\pi\beta^2}\,\frac{\sigma_\gamma(E)}{E\,Z}\,\ln\frac{1}{\sqrt{(1-\beta^2\varepsilon_1)^2+\beta^4\varepsilon_2^2}} \quad + (a)$$

$$+\frac{\alpha}{\pi\beta^2}\,\frac{\sigma_\gamma(E)}{E\,Z}\,\ln\left(\frac{2mc^2\beta^2}{E}\right) \quad + (b)$$

$$+\frac{\alpha}{\pi\beta^2}\,\frac{1}{E^2}\int_0^E\frac{\sigma_\gamma(E')}{Z}\,dE' \quad + (c)$$

$$+\frac{\alpha}{\pi\beta^2}\,\frac{1}{N\hbar c}\left(\beta^2-\frac{\varepsilon_1}{|\varepsilon|^2}\right)\Theta \quad (d)$$

(8)

$\sigma_\gamma(E)$ – fotoabsorbcijski presek – glej II. poglavje

Θ – faza izraza $1-\varepsilon_1\beta^2+i\varepsilon_2\beta^2$

(a), (b) – vzbujanje, ionizacija

(c) – Rutherfordovo sipanje → δ-elektroni

(d) – Čerenkov + prehodno sevanje ($\Theta: 0 \xrightarrow[\text{prag}]{} \pi$)

Allison, Wright v Fernel str. 376–382

b) energijske izgube težkih nabitih delcev

težki : vsi razen $e^{\pm}$

trki z elektroni – majhna sprememba energije
– veliko trkov ($\sigma \sim 10^{-16}\,\mathrm{cm}^2$)

maksimalni prenos energije

$M \quad e$

$$T_{max} = \frac{2\gamma^2 m_e c^2 \beta^2}{1 + 2\gamma \frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2} \qquad (9)$$

za visoke E!

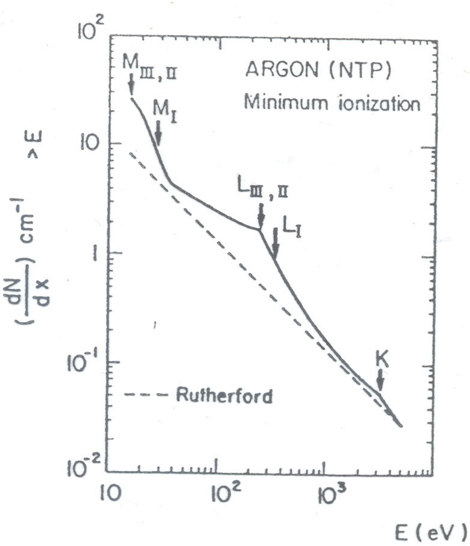


Figure 1 Collisions per unit length, in argon at NTP, with an energy transfer $\geq E$ as a function of the energy (1).

minimalni prenos energije

$$T_{min} \sim E_{eksc}$$

Bethe-Blochova enačba

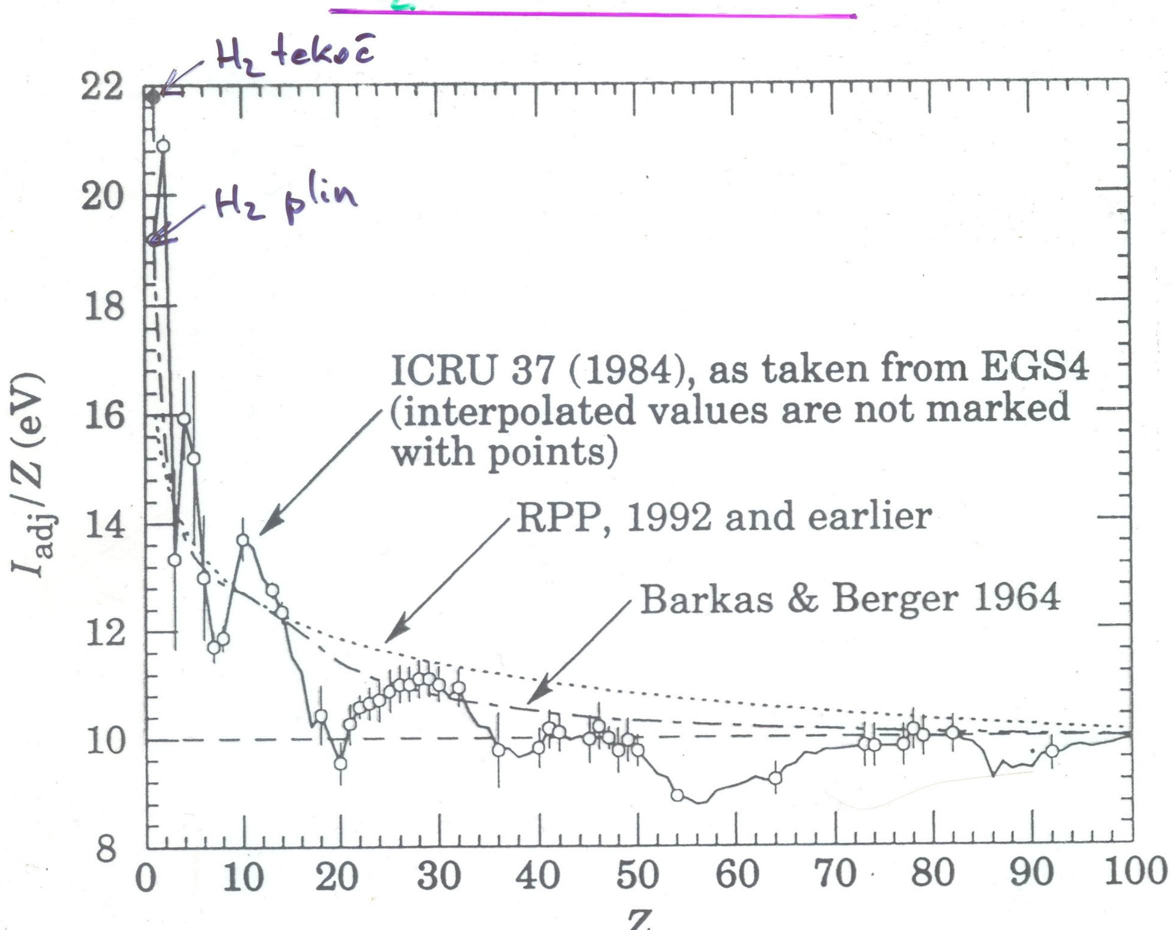
- smer M konstantna
- elektron miruje

dobimo jo tudi z integracijo enačbe (8) po prenosih energij med I in T_{max}

opisuje povprečne izgube energije težkih delcev v snovi

I - povprečni ionizacijski potencial

$$\frac{I}{Z} \sim 10\,\mathrm{eV} \qquad (10)$$

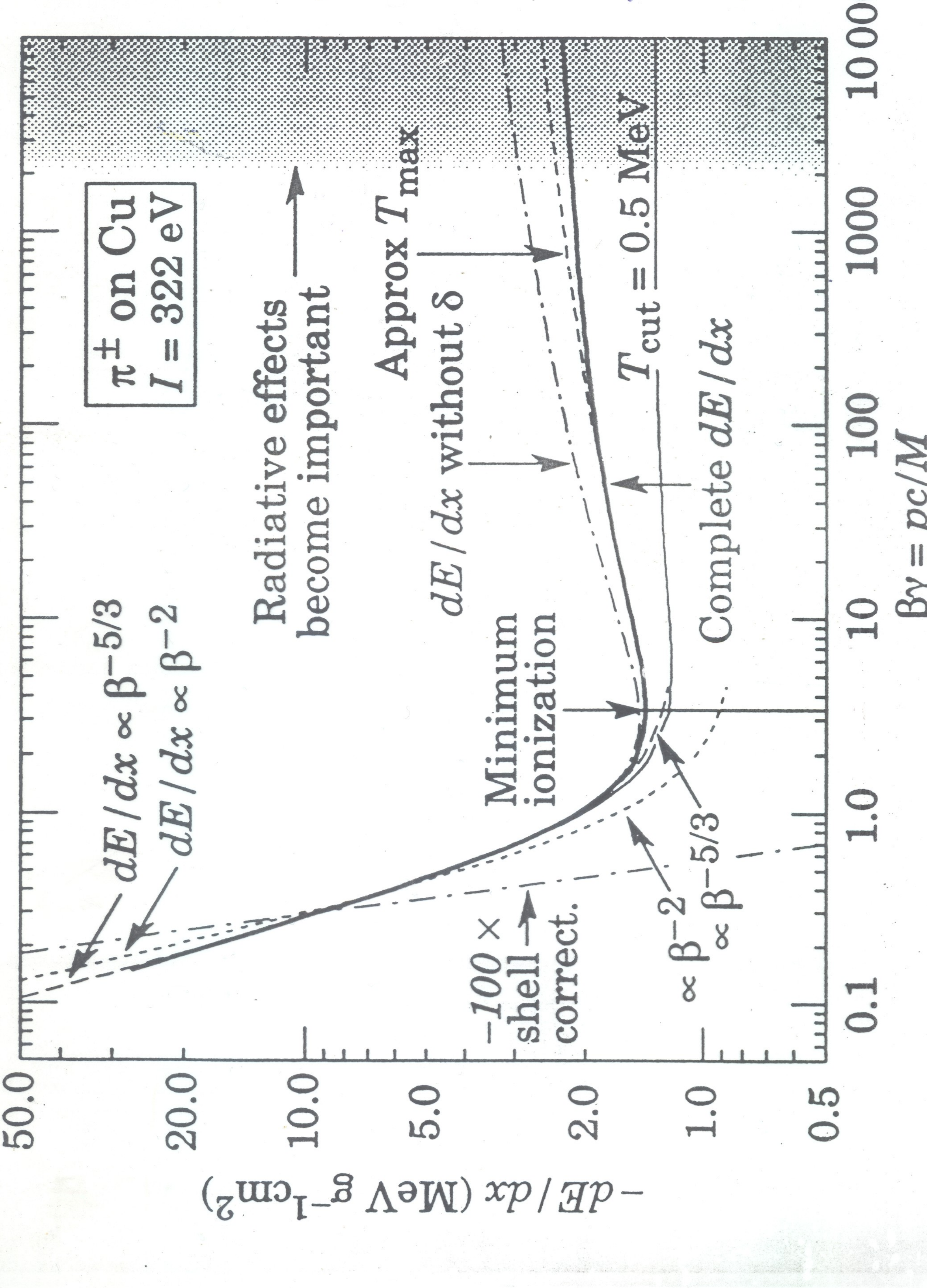


bolj natančno → tabele

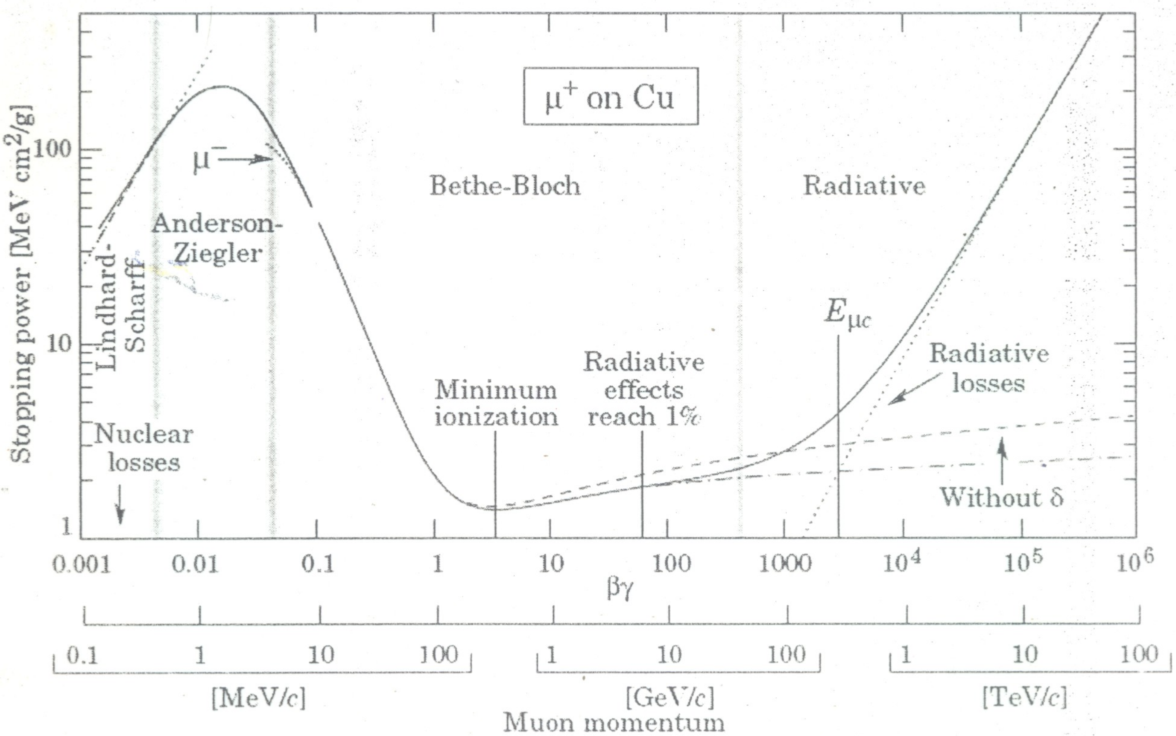
$$-\frac{dE}{dx} = K z_p^2 \frac{\rho Z}{A} \frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta}{2}\right] \quad g \quad (M)$$

$$K = 4\pi N_A r_e^2 m_e c^2 = 0{\cdot}307\ \mathrm{MeV}\ \frac{\mathrm{cm}^2}{\mathrm{g}}$$

δ - efekt gostote - senčenje polja (ε!)



9'



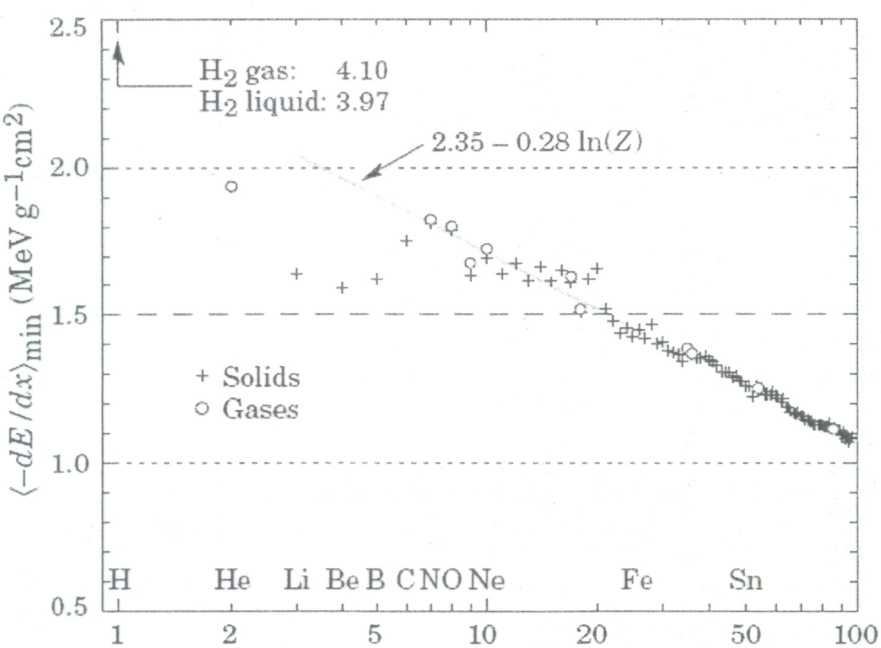


Figure 27.2: Stopping power at minimum ionization for the chemical elements. The straight line is fitted for $Z > 6$. A simple functional dependence on Z is not to be expected, since $\langle -dE/dx \rangle$ also depends on other variables.

9a

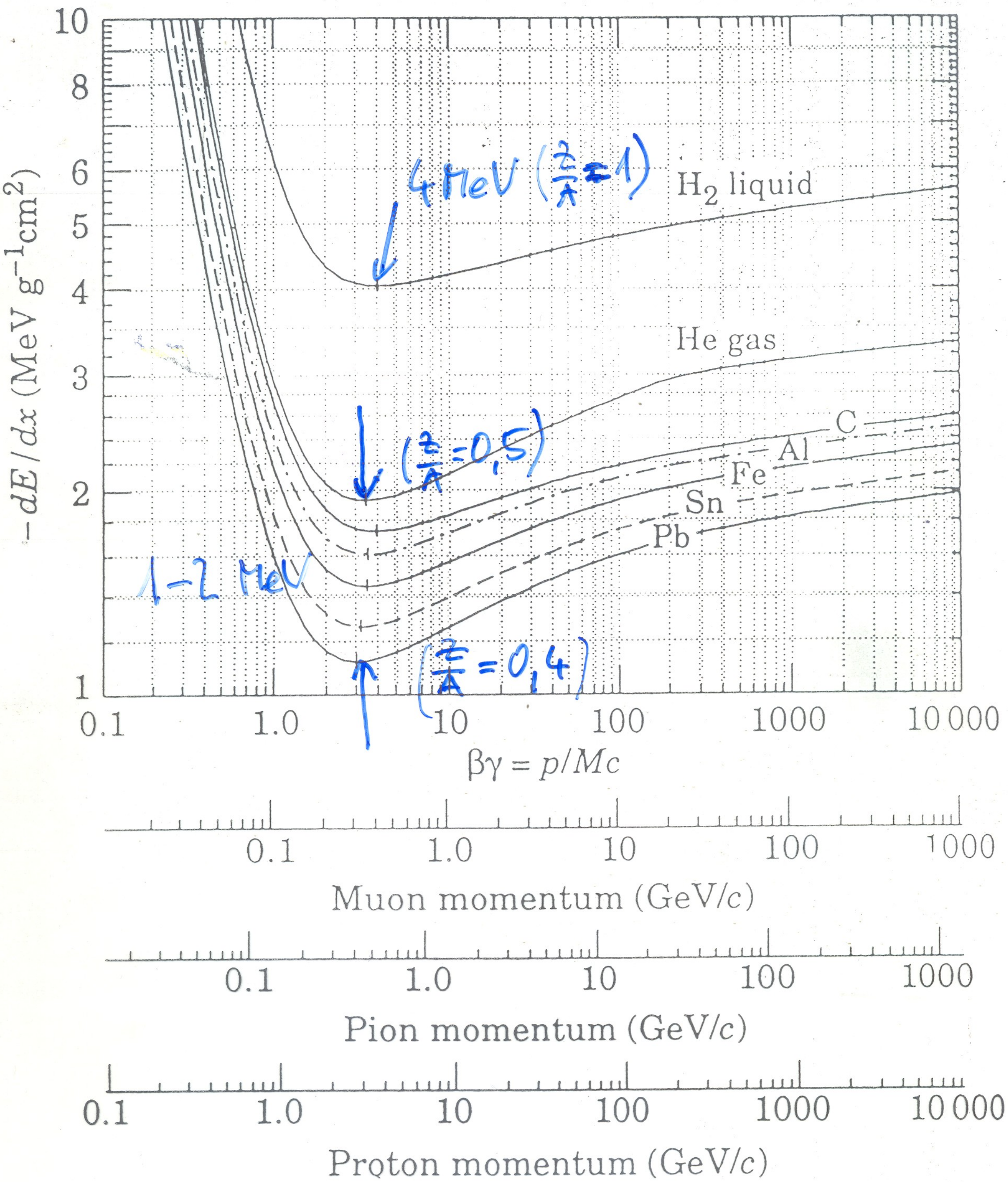


Figure 22.2: Energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, tin, and lead.

za dan medij $\rightarrow \frac{dE}{dx} = f(w) = g\left(\frac{p}{M}\right) = h\left(\frac{T}{M}\right)$

energijska odvisnost

majhni $\beta > \frac{z}{137} \rightarrow \frac{dE}{dx} \propto \frac{1}{\beta^2}$ $(\beta^{-5/3}$!$)$

$\beta\gamma \sim 3{,}5$ širok
$\beta \sim 0{,}96$ minimum

$1-2$ MeV $\frac{cm^2}{g}$

visok: $\gamma \gg 4$ relativistični dvig

rel. dvig - počasen $\sim 2 \ln \beta\gamma \xrightarrow{\gamma \gg 1} \ln \beta\gamma$

- zmanjšan zaradi gostote

$\gamma \gg 1: \; \delta/2 = \ln(\hbar\omega_p/I) + \ln\beta\gamma - 1/2$

- visokoenergijski elektroni uidejo

$\rightarrow$ Fermijev plato

omejene energijske izgube : $T < T_{rez}$

$$-\frac{dE}{dx}\bigg|_{T<T_{rez}} = K z_T^2 \frac{\rho Z}{A} \frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2 T_u}{I^2} - \frac{\beta^2}{2}\left(1+\frac{T_u}{T_{max}}\right) - \frac{\delta}{2}\right] \qquad (12)$$

$T_u = \min(T_{rez}, T_{max})$ ~ deponirana energija

delež visokoenergijskih elektronov (δ-el.)

$$\frac{d^2N}{dT\,dx} = \frac{1}{2} K z_p^2 \frac{\rho Z}{A} \frac{1}{\beta^2} \frac{F(T)}{T^2} \qquad (13)$$

$$I \ll T < T_{max}$$

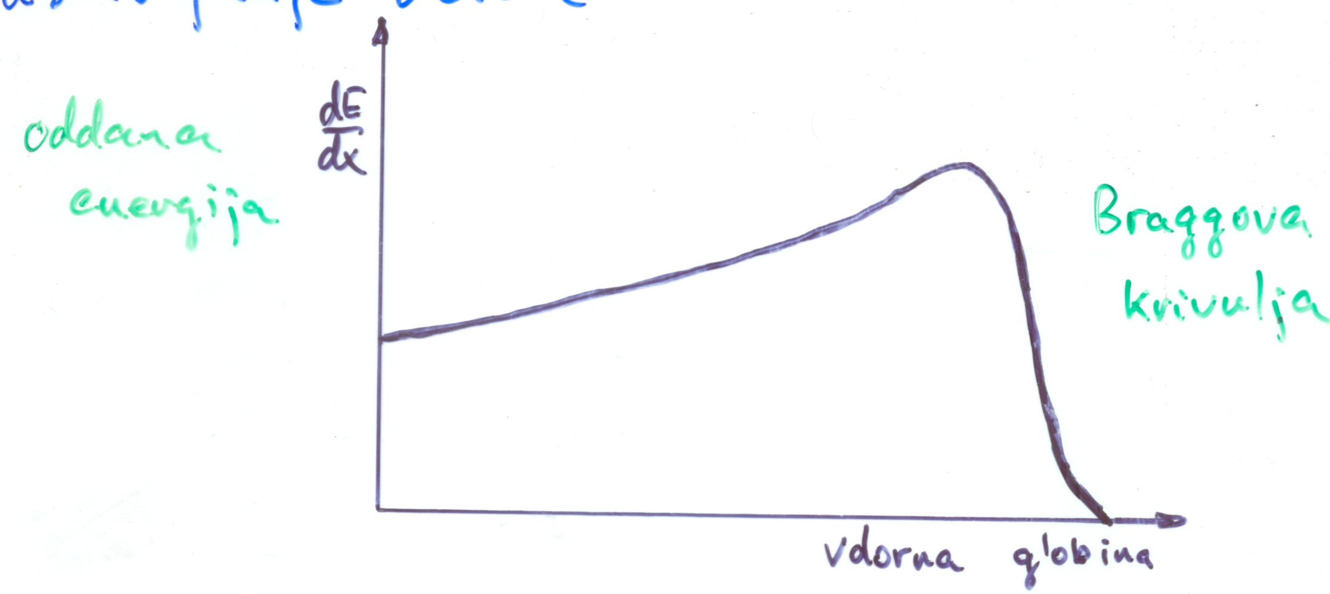
$F(T)$ odvisen od spina; $F \sim 1$ za $T \ll T_{max}$

$$\Rightarrow \left.\frac{dN}{dx}\right|_{E>E_0} \sim C/E_0 \qquad (14)$$

za Ar, 1GeV p

1e/m z $E > 10$ keV

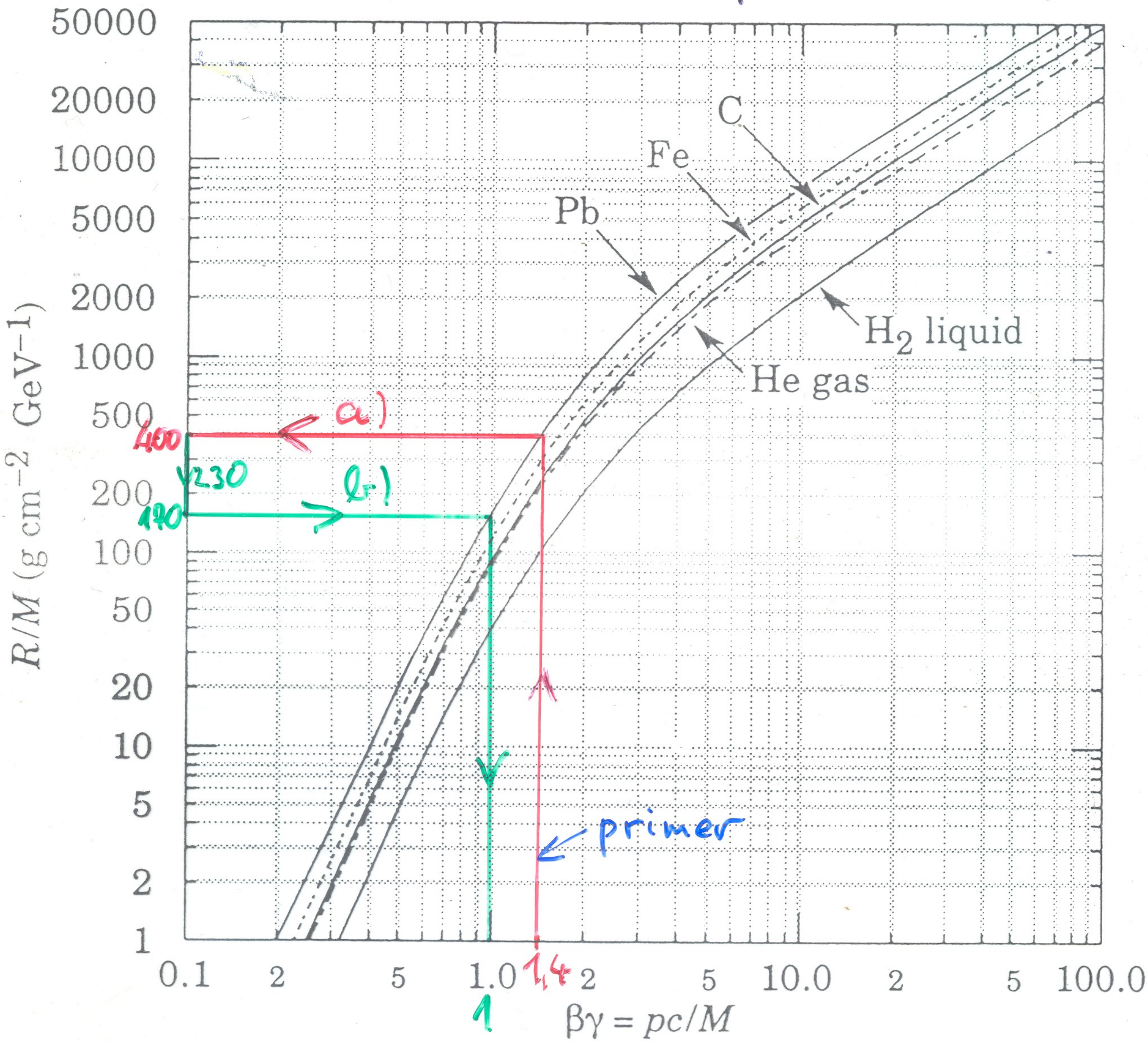
ustavljanje delca



doseg

$$R(E) = \int_E^0 \frac{dE}{\left(\frac{dE}{dx}\right)} \qquad (15)$$

v resnici je to pot delca, toda sipanje le malo pokvari relacijo pot = doseg



zveza (16)

$$R_a\left(M_a, z_a, \begin{Bmatrix} p_a \\ T_a \end{Bmatrix}\right) = \frac{M_a\, z_b^2}{M_b\, z_a^2} R_b\left(M_b, z_b, \begin{Bmatrix} p_b \\ T_b \end{Bmatrix} = \begin{Bmatrix} p_a \\ T_a \end{Bmatrix} \frac{M_b}{M_a}\right)$$

primer: K^+, $p = 700$ MeV/c, Pb : $p \to \beta\gamma = 1{,}42$; Pb: $R/M = 400$

$m = 494$ MeV ; $\rho = 11{,}35$ g/cm^3

a) $\Rightarrow R = 195$ g/cm^2 $\Rightarrow R = 17{,}4$ cm

b) $d = 10$ cm $\Rightarrow \frac{\rho d}{M} = 230 \Rightarrow R/M = 170 \Rightarrow \beta\gamma = 1 \Rightarrow p = 500$ MeV/c

12a

Doseg

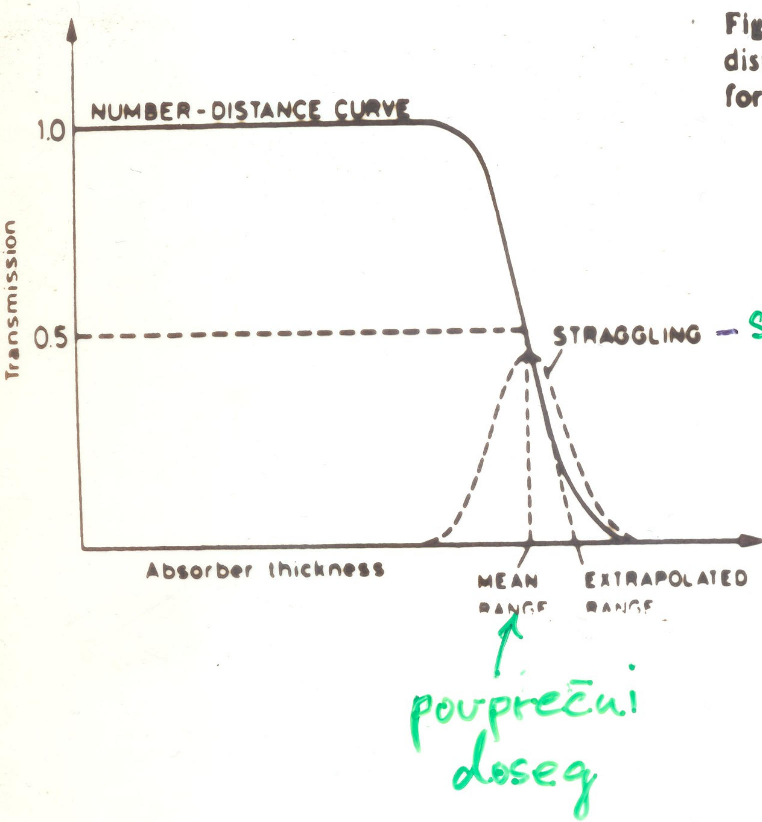


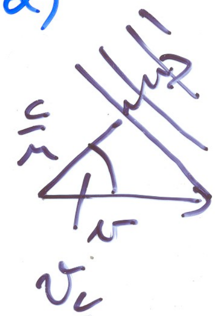
Fig. 2.7. Typical range number-distance curve. The distribution of ranges is approximately Gaussian in form

sevanje Čerenkova

optično področje $\quad v > \frac{c_0}{n}$

opis z enačbo (8 d)

$$\cos\vartheta_c = \frac{1}{\beta n} \qquad (17)$$



$\beta n \gtrsim 1 \Rightarrow \vartheta_c \to 0$

$\beta \sim 1 \quad \cos\vartheta_c = \frac{1}{n}$

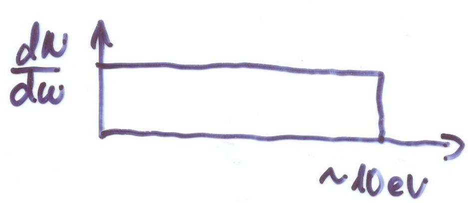
$H_2O \quad \vartheta_c = 41^\circ$

$SiO_2 \quad \vartheta_c = 47^\circ$

↳ totalni odboj $\sin\alpha_m = \frac{1}{n}$!

iz (8): $$\frac{dN}{dE_\gamma} = \frac{\alpha}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2}\right) L \propto \sin^2\vartheta_c \cdot L \qquad (18)$$

za optično področje $\quad n(\omega) \sim$ konst.



$$\frac{dE}{dx} = \frac{1}{L}\int \hbar\omega \frac{dN}{dE_\gamma}\, dE_\gamma \qquad (19)$$

majhen, za trdno snov $\sim 10^{-3}$ MeV $\frac{cm^2}{g}$

ali $\sim 10^3$ fotonov (večina UV)

c) Energijske izgube $e^{\pm}$

trki z elektroni ~ kot težki delci

toda
- velik odklon iz smeri
- identični delci (e)
- $W_{max} = \frac{T_e}{2}$

~ Bethe-Bloch

$$\ln \frac{2 m_e c^2 \gamma^2 \beta^2 T_{max}}{I^2} \to \ln \frac{\tau^2 (\tau + 2)}{2 (I/m_e c^2)^2}$$

$$\tau = \frac{T}{m_e c^2}$$

$-\beta^2 \to F(\tau)$ (Leo str. 35)

lahki delci zavorno sevanje

poma- $\left(w \propto \ddot{p}^2 \propto a^2 \propto \left(\frac{z}{M}\right)^2 \right)$ za $\mu \sim 40000$ manjša

gali $\left(w \propto |\langle f|H|i\rangle|^2 \propto \left(\frac{1}{M}\right)^2 \right)$

$w \propto \frac{1}{q^4}$

senčenje jedra z atomskimi elektroni

najugodnejši q pada z energijo

→ dlje stran → večje senčenje

za $E_0 > 137\, m_e c^2\, Z^{1/3}$ popolno senčenje

$$E = E_0\, e^{-\frac{x}{X_0}} \tag{20}$$

X_0 - radiacijska dolžina

$$\frac{1}{X_0} \sim \frac{4 Z^2 \rho N_A}{A}\, r_e^2\, \alpha\, \ln\left(183\, Z^{-1/3}\right) \tag{21}$$

$$\frac{1}{X_0} \propto \frac{\rho Z^2}{A} \tag{22}$$

primeri

	ρX_0	X_0
C	42,7 g/cm²	18,8 cm
Pb	6,37 g/cm²	0,56 cm
W	6,76 g/cm²	0,35 cm

efekt zavornega sevanja na elektronih

$Z^2 \to Z(Z+1)$

RPP '96:

$$\rho X_0 = \frac{716{,}4\ \mathrm{g/cm^2}\ A}{Z(Z+1)\,\ln\left(287/\sqrt{Z}\right)} \tag{23}$$

napaka $\pm 2{,}5\%$

kritična energija

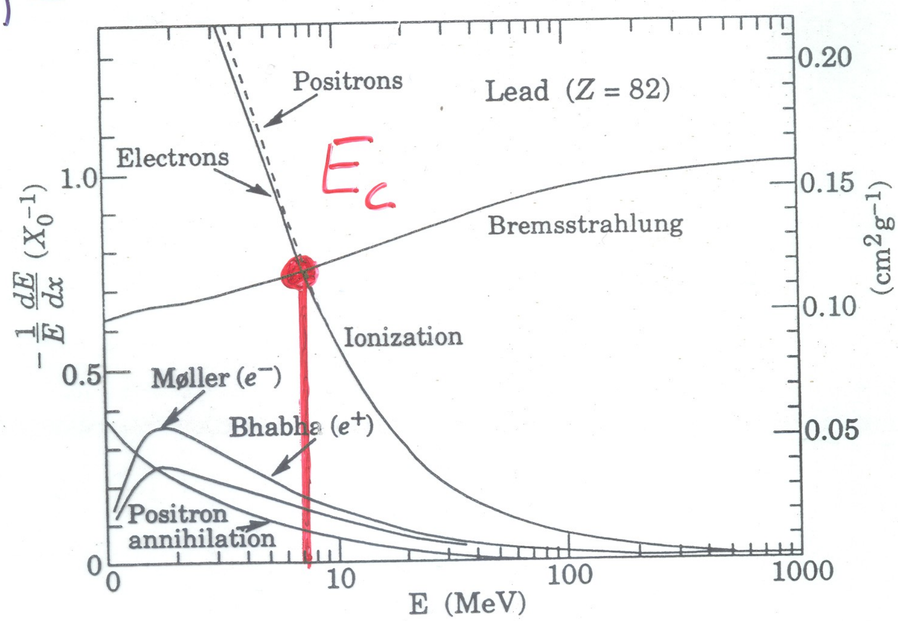
$$\left(\frac{dE}{dx}\right)_{trki} = \left(\frac{dE}{dx}\right)_{rad}$$

$$E_c \sim \frac{800\ \mathrm{MeV}}{Z + 1{,}2} \qquad (24)$$

Pb ~ 9,5 MeV   Al ~ 51 MeV

RPP '96   plini ⟷ tekočine + trdne (I !)

drugačna definicija (→16b)
- plin $710\ \mathrm{MeV}/(Z + 0{,}92)$
- T + T. $610\ \mathrm{MeV}/(Z + 1{,}24)$ Pb ~ 7,5 MeV



16b

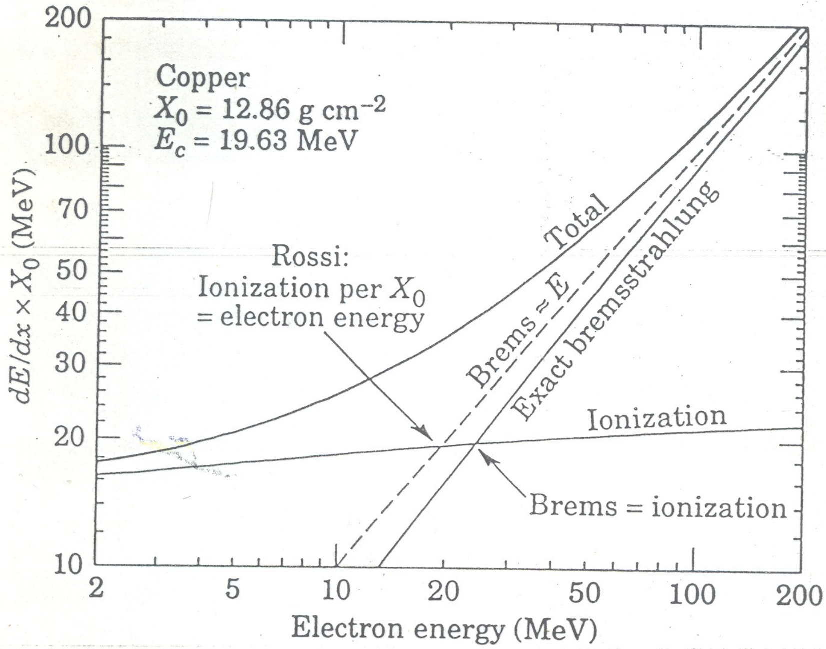


Figure 22.6: Two definitions of the critical energy E_c.

"nova" (Rossi 1952!) definicija E_c

$$\left(\frac{dE}{dx}\right)_{ion} \cdot X_0 = E \; (\equiv E_c)$$

definiciji identični za $\left(\frac{dE}{dx}\right)_{zavorno} = \frac{E}{X_0}$

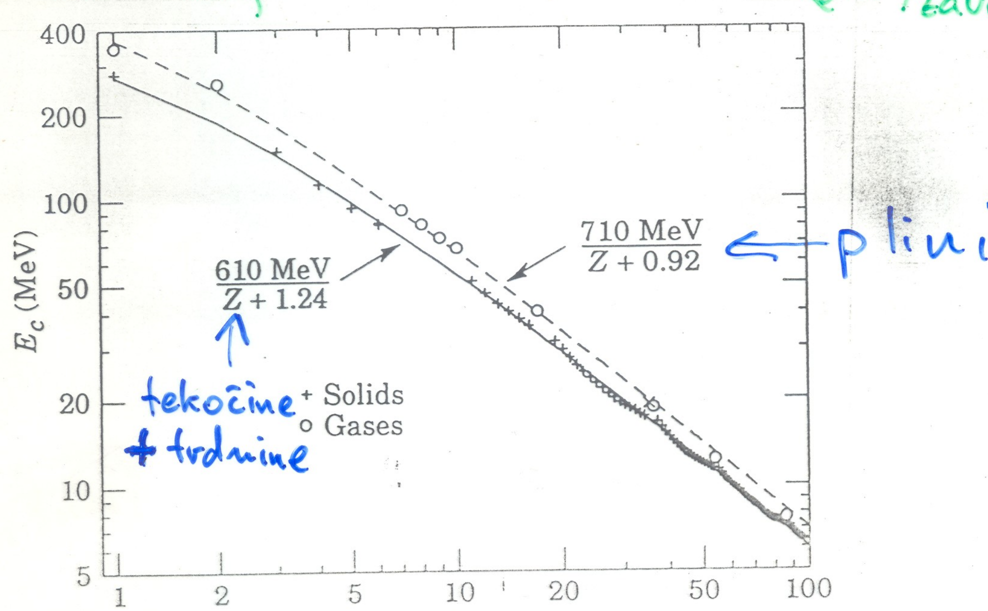


Figure 22.7: Electron critical energy for the chemical elements, using Rossi's definition [1]. The fits shown are for solids and liquids (solid line) and gases (dashed line). The rms deviation is 2.2% for the solids and 4.0% for the gases. (Computed with code supplied by A. Fassó.)

16 c

LPM – Landau-Pomeranchuk-Migdal

Pri visokih energijah potekajo majhni prenosi energij preko več atomov, pomembno če

$$E_\gamma < E^2/(E + E_{LPM})$$

kjer je

$$E_{LPM} = \frac{(m_e c^2)^2 \alpha X_0}{4\pi \hbar c \rho} = (7.7\ \mathrm{TeV/cm}) \times \frac{X_0}{\rho}$$

Formacijska dolžina – interferenca med sevalci (atomi) na tej dolžini (destruktivna)

Senčenje v snovi !

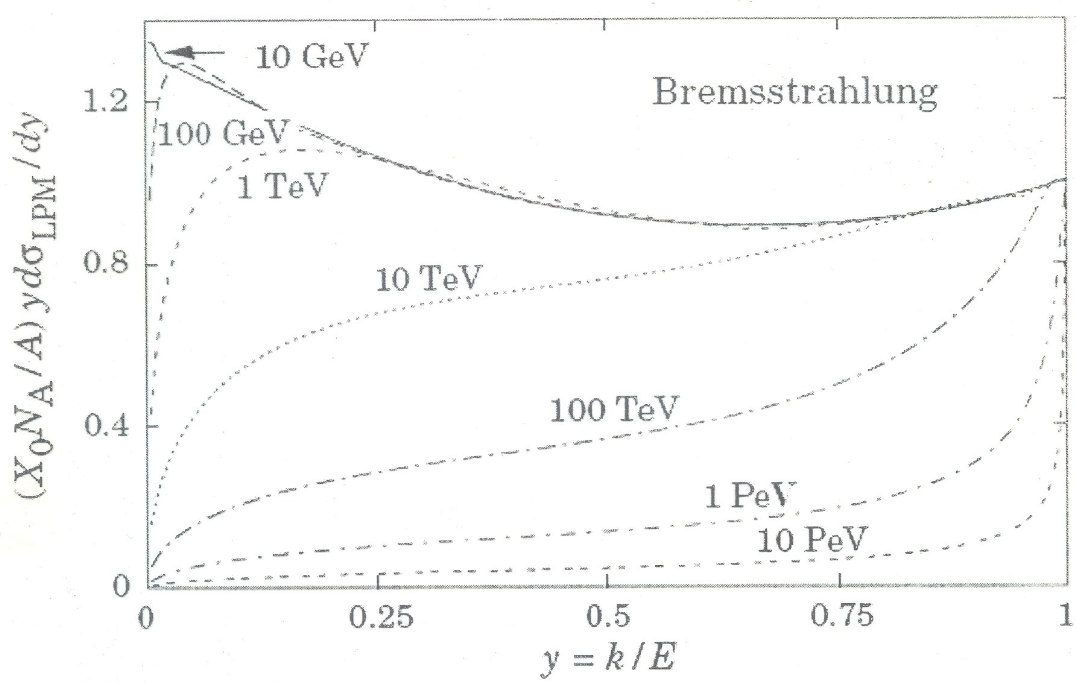


Figure 27.11: The normalized bremsstrahlung cross section $k\, d\sigma_{LPM}/dk$ in lead versus the fractional photon energy $y = k/E$. The vertical axis has units of photons per radiation length.

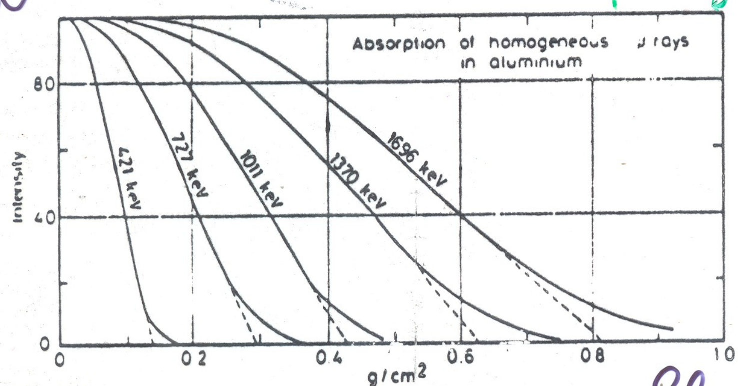
Doseg elektronov

- veliko stresanje zaradi sipanja
- za β-razpad efektivno $I = I_0 e^{-\mu x}$

spekter & doseg

le dovoljeni β-prehodi

N



ρl

◄ Fig. 2.11. Range number-distance curves for electrons (from *Marshall* and *Ward* [2.15])

Fig. 2.12. Range curves for electrons in several materials as calculated in the continuous slowing down approximation (data from [2.16])

Fig. 2.13. Absorption curves for beta decay electrons from ^{185}W (from *Baltakmens* et al. [2.17])

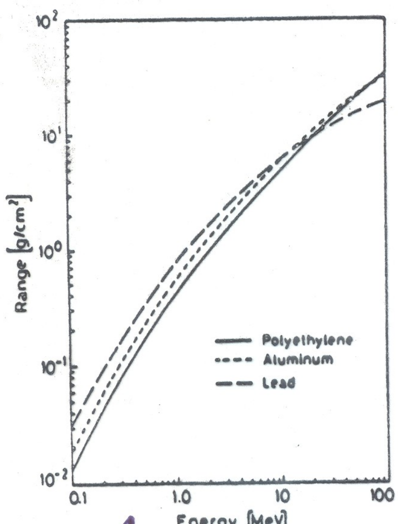


Fig. 2.12

N

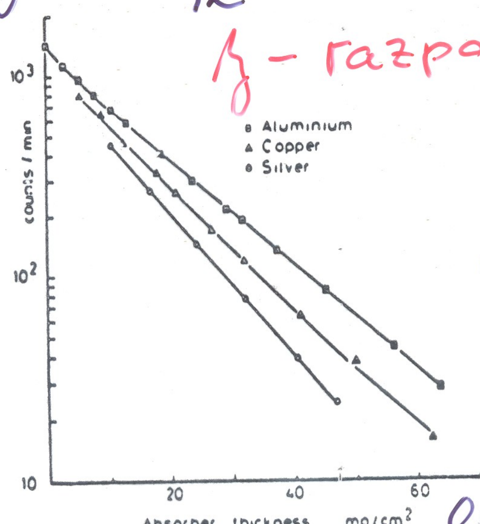


Fig. 2.13

ρl

β - razpad

$$N = N_0 e^{-\frac{x}{x_0}}$$

$$R = \int_{E_0}^{0} \frac{dE}{dE/dx}$$

d) Večkratno Coulombsko sipanje

sipanje na jedrih $M_j >> M$

Rutherford: $\frac{d\sigma}{d\Omega} = z_1^2 z_2^2 r_e^2 \frac{mc}{4\beta p \sin^4\frac{\vartheta}{2}}$

$\Rightarrow$ majhni koti

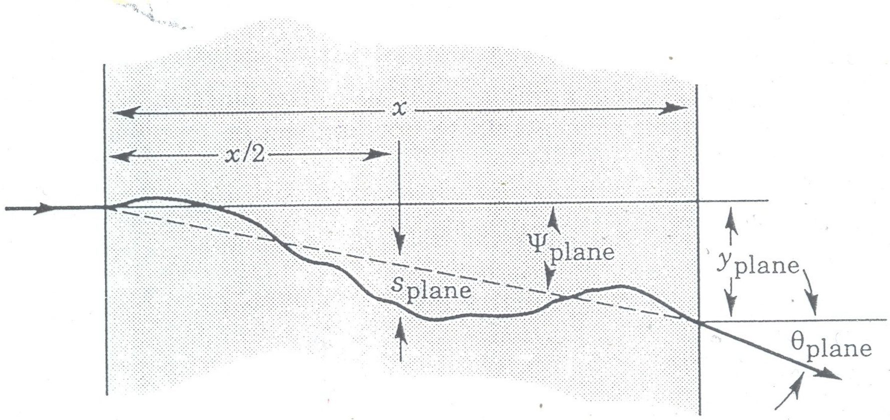
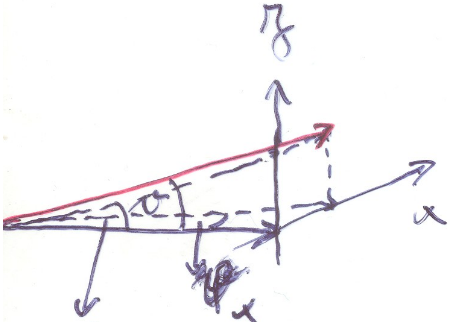


Figure 10.5: Quantities used to describe multiple Coulomb scattering. The particle is incident in the plane of the figure.

trije režimi:
- tanek absorber
  $N_s < 1$; Rutherford
- vmesno področje
  $1 < N_s < 20$ ni statistično
- večkratno sipanje
  $N_s > 20$ statistično



Moliere

$$P(\vartheta)\,d\Omega = \eta\,d\eta\left(2e^{-\eta^2} + \frac{F_1(\eta)}{B} + \frac{F_2(\eta)}{B^2} + \dots\right) \quad (25)$$

$$\eta = \frac{\vartheta}{\vartheta_1\sqrt{B}} \qquad \vartheta_1 = 0{,}4\left(\frac{zZ}{p\beta}\right)\sqrt{\frac{\varrho L}{A}}$$

B : $\ln B - B + \ln \gamma - 0{\cdot}154 = 0$

$\gamma = 8{\cdot}831 \cdot 10^3 \dfrac{q\, z^2 Q L}{\beta^2 A \Delta}$; $\Delta = 1{\cdot}13 + 3{\cdot}76 \left(\dfrac{z_z}{137\beta}\right)^2$

$$F_k(\eta) = \frac{1}{k!} \int J_0(\eta y)\, e^{-\frac{y^2}{4}} \left[\frac{y^2}{4} \ln \frac{y^2}{4}\right]^k y\, dy$$

$$Q = \begin{cases} \sqrt{z(z+1)} & e^- \\ z & \text{ostalo} \end{cases}$$

$$q = \begin{cases} (z+1)\, z^{1/3} & e^- \\ z^{4/3} & \text{ostalo} \end{cases}$$

GEANT računa VCS do F_2!

oblika → Gaussova sredica (98%)

+ repi

Gaussova sredica

$$P(\vartheta)\, d\Omega = \frac{2\vartheta}{\vartheta_0^2}\, e^{-\frac{\vartheta^2}{\vartheta_0^2}}\, d\vartheta \qquad (26)$$

$\vartheta_0 = \sqrt{\langle \vartheta^2 \rangle}$ $\qquad \vartheta_0 \sim \vartheta_1 \sqrt{B}$

empirično

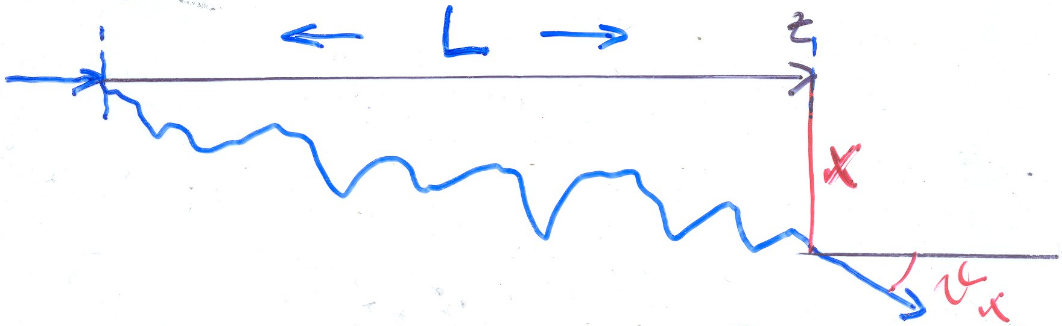
$$\vartheta_0 = z_p \frac{20\, \text{MeV}}{p\, \beta\, c} \sqrt{\frac{L}{X_0}} \left(1 + \frac{1}{9} \log_{10} \frac{L}{X_0}\right) \qquad (27)$$

projekciji na ravnino : ϑ_x, ϑ_y

$$P(\vartheta_x)\, d\vartheta_x = \frac{1}{\sqrt{2\pi}\,\vartheta_0'} e^{-\frac{\vartheta_x^2}{2\vartheta_0'^2}} \qquad (28)$$

enako za ϑ_y ; majhni koti $\rightarrow$ $\vartheta^2 = \vartheta_x^2 + \vartheta_y^2$

ϑ_x, ϑ_y neodvisna $\Rightarrow$ $\vartheta_0^2 = 2\vartheta_0'^2$ (RPP: $\vartheta_0 \to \vartheta_0'$)

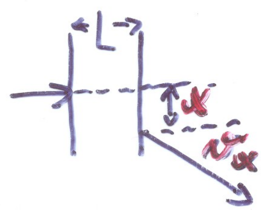


$$P(x)\, dx = \frac{1}{\sqrt{2\pi}\, x_0} e^{-\frac{x^2}{2x_0^2}} dx \qquad (29)$$

$$x_0 = \frac{L\,\vartheta_0'}{\sqrt{3}}$$

x in ϑ_x korelirana $\left(\rho = \frac{\sqrt{3}}{2}\right)$

MC recept



$z_1, z_2 \in N(0,1)$

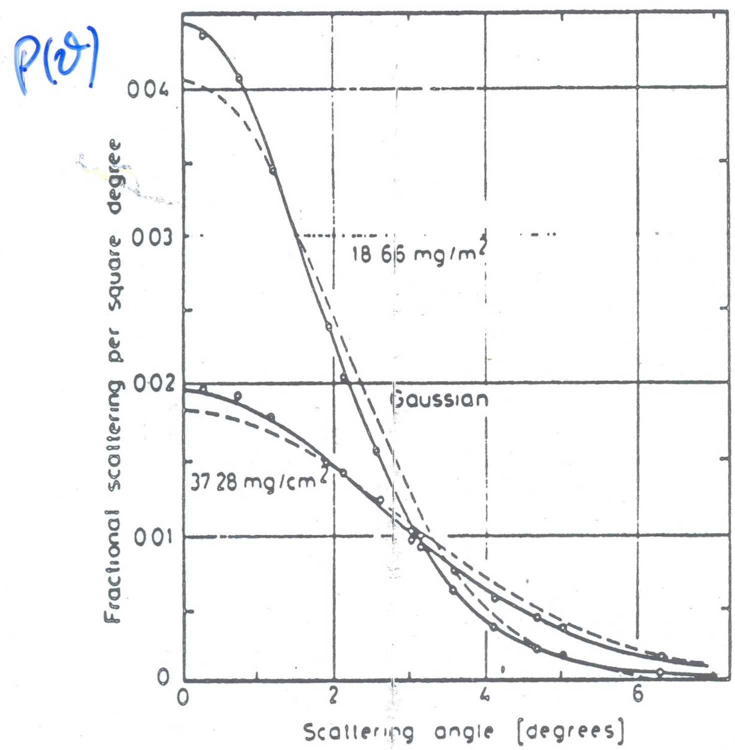
$x = z_1 L \vartheta_0' / \sqrt{12} + z_2 L \vartheta_0' / 2$

$\vartheta_x = z_2 \vartheta_0'$

enako za y, ϑ_y

povratno sipanje

nekaj elektronov nazaj $- f(\vartheta, E, Z)$



VCS

na zlatu

albedo – verjetnost za sipanje nazaj

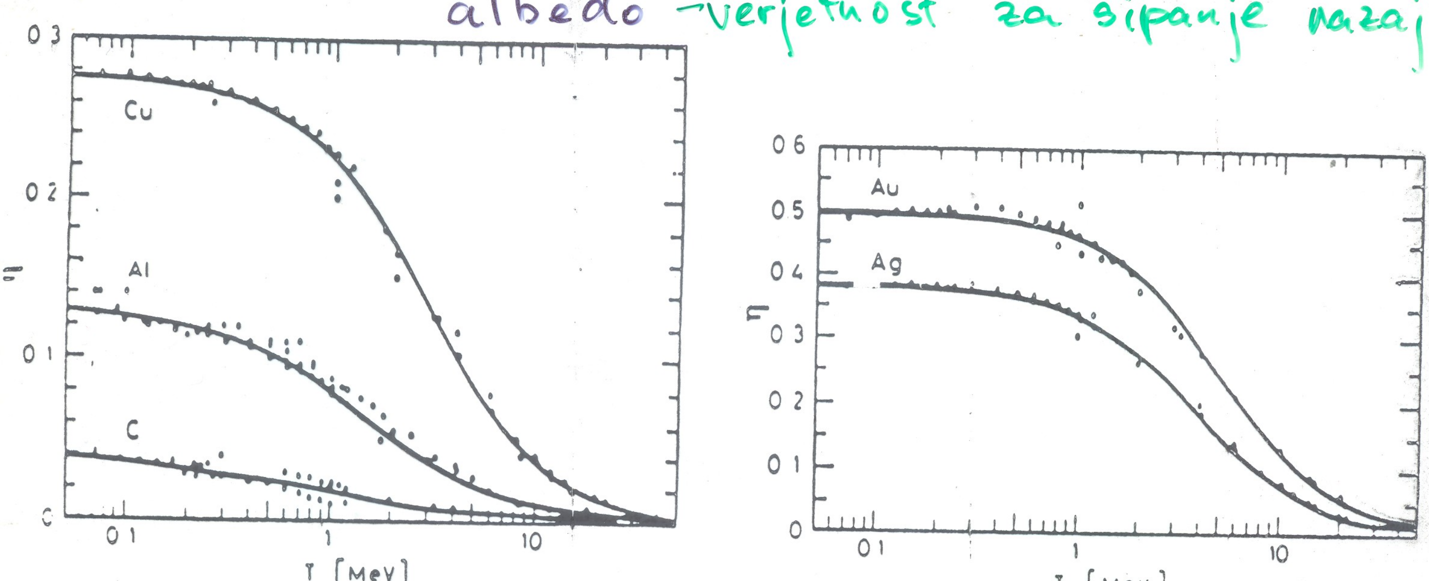


Fig. 2.17. Some measured electron backscattering coefficients for various materials. The electrons are perpendicularly incident on the surface of the sample (from *Tabata* et al. [2.24])

c) Energijsko stresanje

Bethe Bloch poda le srednjo vrednost dE/dx

stresanje dosega $\Leftrightarrow$ stresanje en. izgub

debeli absorberji $\rightarrow$ izrek o srednji vrednosti $\rightarrow$ Gauss

tanki absorberji $\rightarrow$ malo trkov

parameter $\quad \kappa = \dfrac{\bar{\Delta}}{T_{max}}$ $\qquad \bar{\Delta}$ – Bethe Bloch

$T_{max} \approx 2 m_e c^2 \beta^2 \gamma^2$

$\kappa > 10$ $(\sim > 1)$ in $\dfrac{\Delta}{E_0} \ll 1$ debeli absorberji

$$P(\Delta) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{(\Delta - \bar{\Delta})^2}{2\sigma_0^2}} \tag{30}$$

$$\sigma_0^2 = \frac{1 - \frac{1}{2}\beta^2}{1 - \beta^2} \cdot \underbrace{0.1569}_{4\pi N_a r_e^2 (m_e c^2)^2} \cdot \frac{\rho z}{A} \cdot L \quad [\mathrm{MeV}^2] \tag{31}$$

zelo debeli absorberji $\dfrac{\Delta}{E_0} \rightarrow 1$

Gauss ne velja $\rightarrow$ numerična integracija

tanki absorberji $\quad T_{max} > \bar{\Delta}$

rep proti visokim dE/dx (δ-elektroni!)

$$\bar{\Delta} \neq \Delta_{mp}$$

zelo tanki absorberji $\varkappa < 0.01$

Landau:
- $T_{max} \to \infty$ $(\varkappa \to 0)$
- prosti elektroni $(\delta_1 > E_{at})$
- v konstantna
- $\bar{\Delta}$ brez ln člena

$$\left.\begin{aligned}
&\bar{\Delta} \to \xi = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \left(\frac{z_p}{\beta}\right)^2 d \\
&P(\Delta) = \phi(\lambda)/\xi \\
&\phi(\lambda) = \frac{1}{\pi} \int_0^\infty e^{-u \ln u - u\lambda} \sin \pi u \, du \\
&\lambda = \frac{1}{\xi}\left[\Delta - \xi(\ln \xi - \ln \varepsilon + 1 - C)\right]
\end{aligned}\right\} \quad (32)$$

C – Eulerjeva konstanta = 0.577

$\ln \varepsilon = \ln \frac{(1-\beta^2) I^2}{2 m c^2 \beta^2} + \beta^2$ minimalni transfer (el. prosti)

$\phi(\lambda)$ – tabelirana, neodvisna od d.

$$\underline{\Delta_{mp} = \xi \left[\ln(\xi/\varepsilon) + 0.198 - \delta \right]} \quad (33)$$

srednje debeli absorberji $0.01 < \varkappa < 10$ (1)

porazdelitev Vavilova $\to$ Landau + T_{max}

limiti $\varkappa \to 0$ Landau

$\varkappa \to \infty$ Gauss

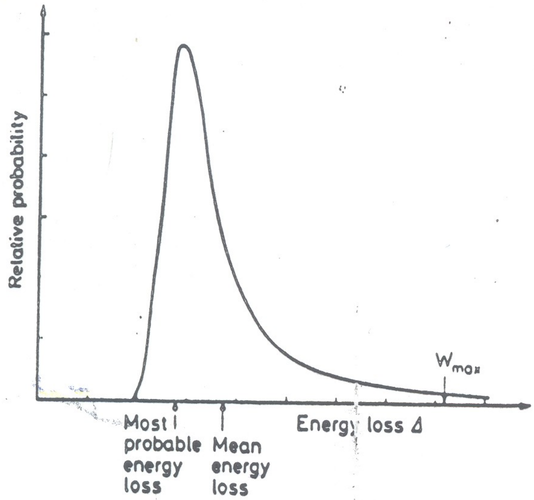


Fig. 2.18. Typical distribution of energy loss in a thin absorber. Note that it is asymmetric with a long high energy tail

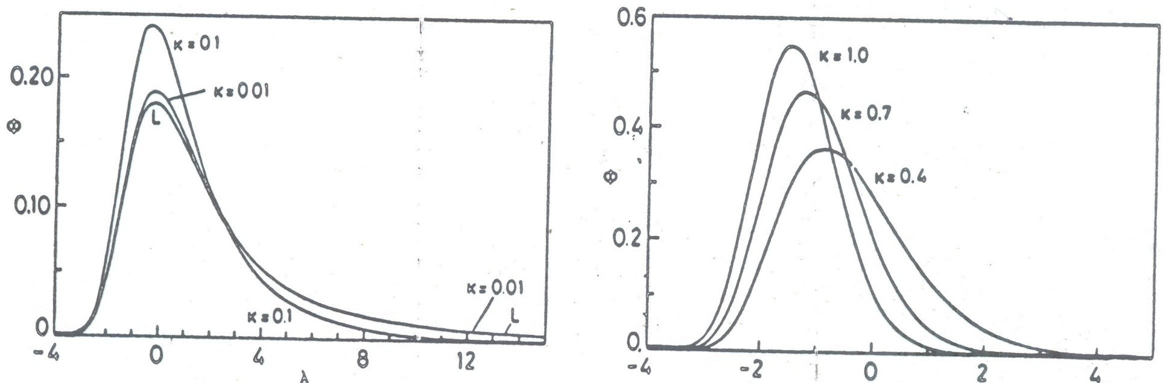
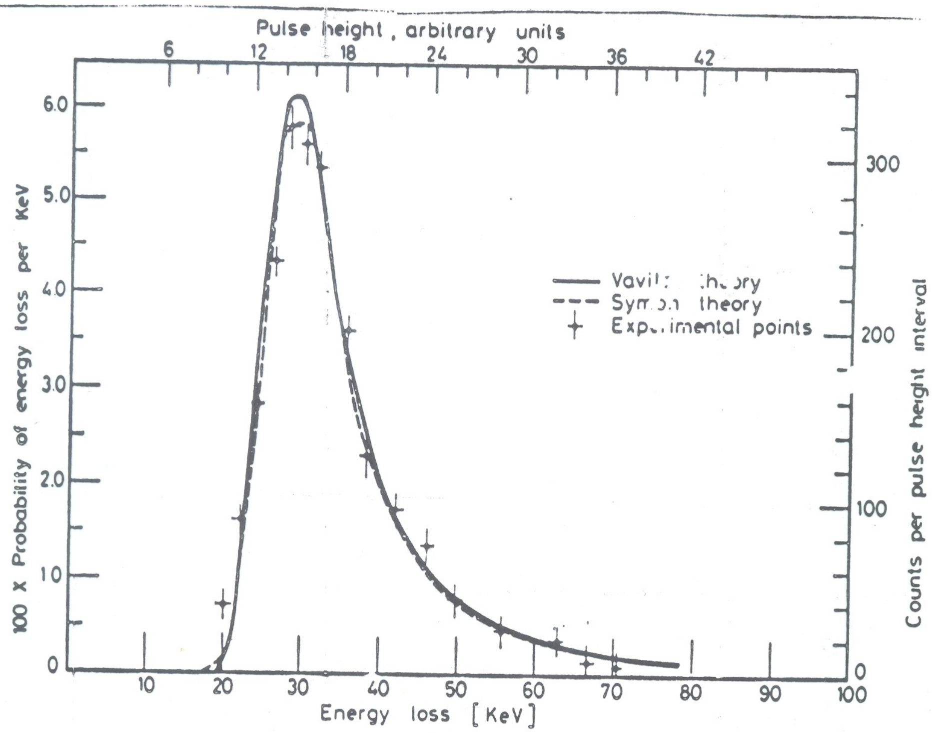


Fig. 2.19. Vavilov distributions for various κ. For comparison, Landau's distribution (denoted by the L) for $\kappa = 0$ is also shown (from *Seltzer* and *Berger* [2.29])



f) energijske izgube visokoenergijskih mionov

$$-\frac{dE}{dx} = a(E) + b(E)E \qquad (33a)$$

$a(E)$ – ionizacija $\sim 2\,\mathrm{MeV/g\,cm^{-2}}$ $\sim$ konst

$b(E)$ – zavorno sevanje
– tvorba parov
– fotojedrske reakcije
$\Big\}$ $\sim$ konst. za $E_\mu >$ TeV

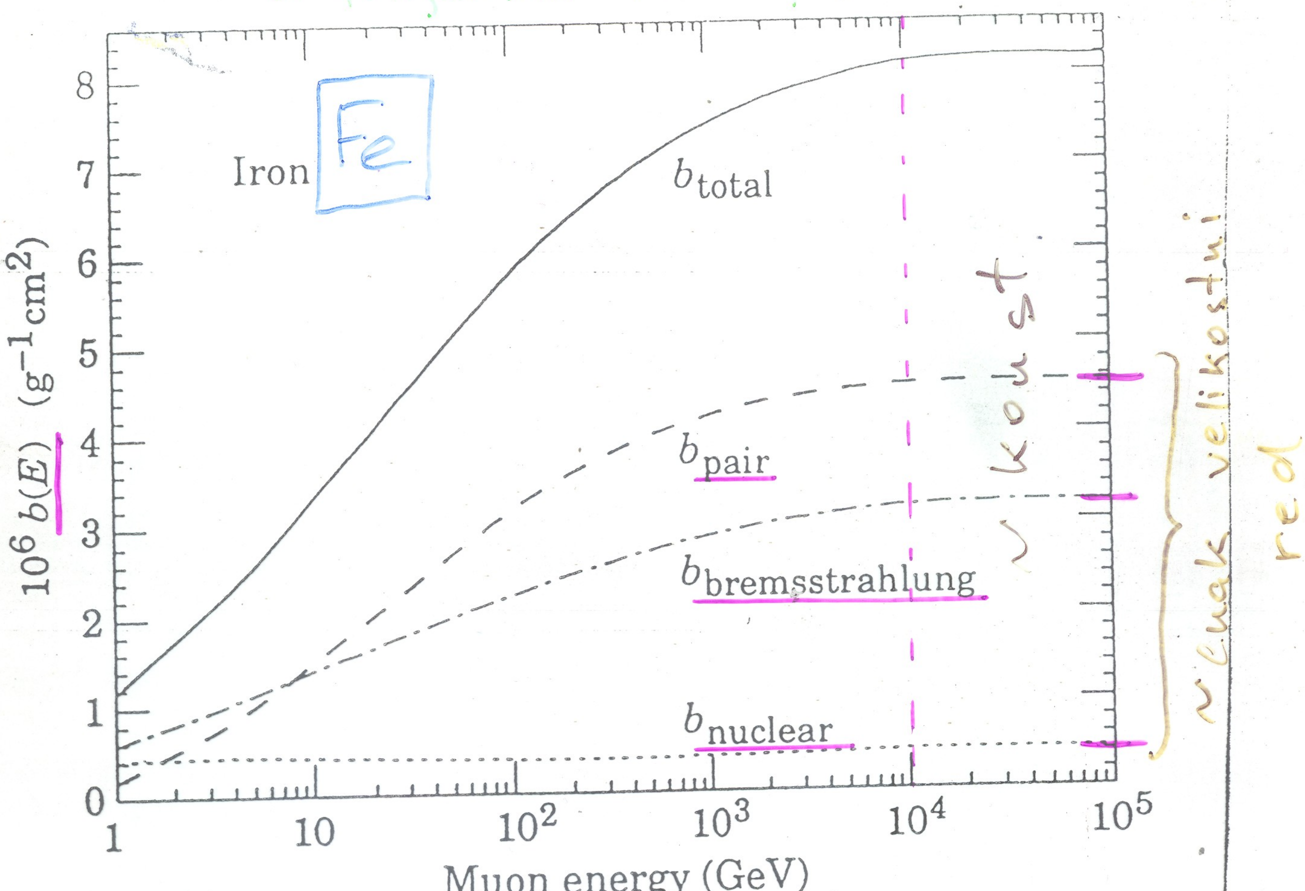


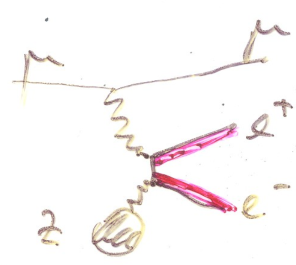
Figure 22.10: Contributions to the fractional energy loss by muons in iron due to e^+e^- pair production, bremsstrahlung, and photonuclear interactions, as obtained from Lohmann *et al.* [39].

zavorno sevanje – kot za elektrone z $E_e = E_\mu \left(\frac{m_e}{m_\mu}\right)^2$

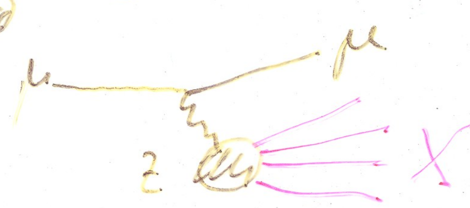
2.5 MeV ↔ 1 TeV

tvorba parov

(enako za e, le
zav. sevanje x 40000)



fotojedrske reakcije



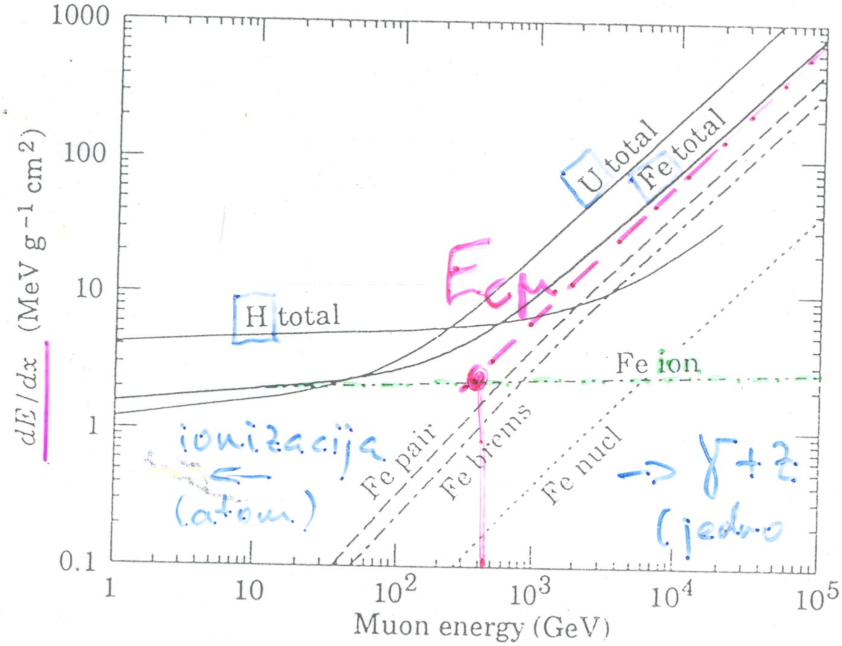


Figure 22.11: The average energy loss of a muon in hydrogen, iron, and uranium as a function of muon energy. Contributions to dE/dx in iron from ionization and the processes shown in Fig. 22.10 are also shown.

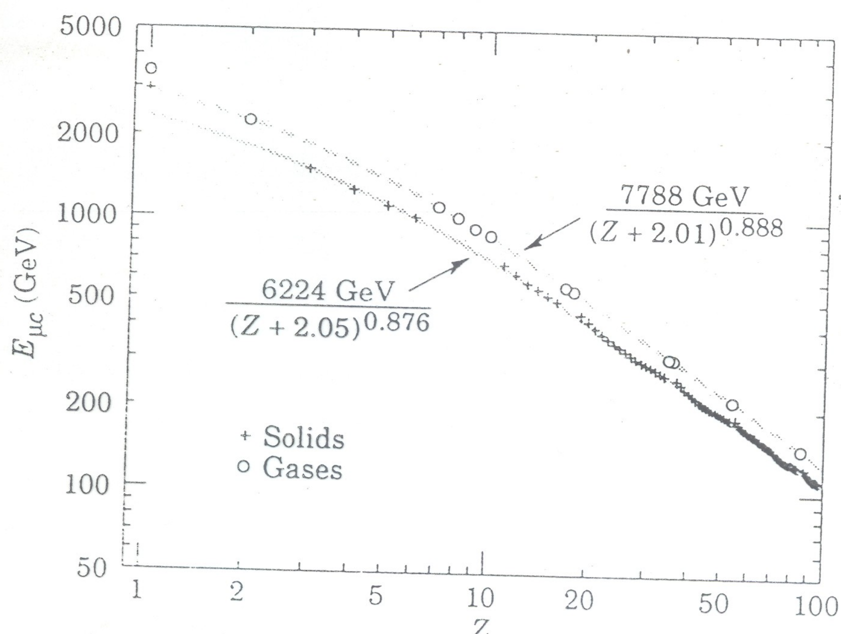


Figure 22.12: Muon critical energy for the chemical elements, defined as the energy at which radiative and ionization energy loss rates are equal. The equality comes at a higher energy for gases than for solids or liquids with the same atomic number because of a smaller density effect reduction of the ionization losses. The fits shown in the figure exclude hydrogen. Alkali metals fall 3–4% above the fitted function for alkali metals, while most other solids are within 2% of the function. Among the gases the worst fit is for neon (1.4% high). (Courtesy of N.V. Mokhov, using the MARS code system [48].)

$E_{\mu c}$

$$E_{\mu c} = \frac{a(E_{\mu c})}{b(E_{\mu c})} \sim \frac{a}{b}$$

$a, b \sim$ konst

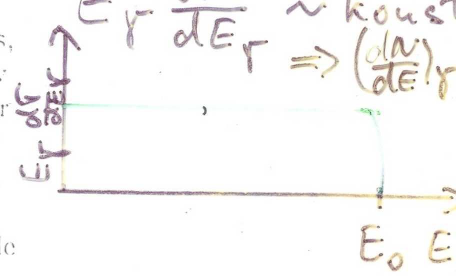
doseg

$$x_0 \sim \frac{1}{b} \ln\left(1 + \frac{E_0}{E_{\mu c}}\right)$$

$$E_0 \ll E_c : \; x_0 = \frac{E_0}{a}$$

zavorno sevanje

$$E_\gamma \frac{d\sigma}{dE_\gamma} \sim \text{konst} \Rightarrow \left(\frac{dN}{dE}\right)_\gamma \propto \frac{1}{E_\gamma}$$



stresanje dE_μ/dx

(+) $\frac{dE}{dx}_{pari} > \frac{dE}{dx}_{\gamma} > \frac{dE}{dx}_{fotojed.}$

(–) spekter produktov
- zelo trd — fotojedrske
- trd — zavorno sevanje
- mehkejši — tvorba parov

$\Rightarrow$ velik $\frac{dE}{dx}$ posledica redkih
- trdih fotonov
- fotojedrskih reakcij

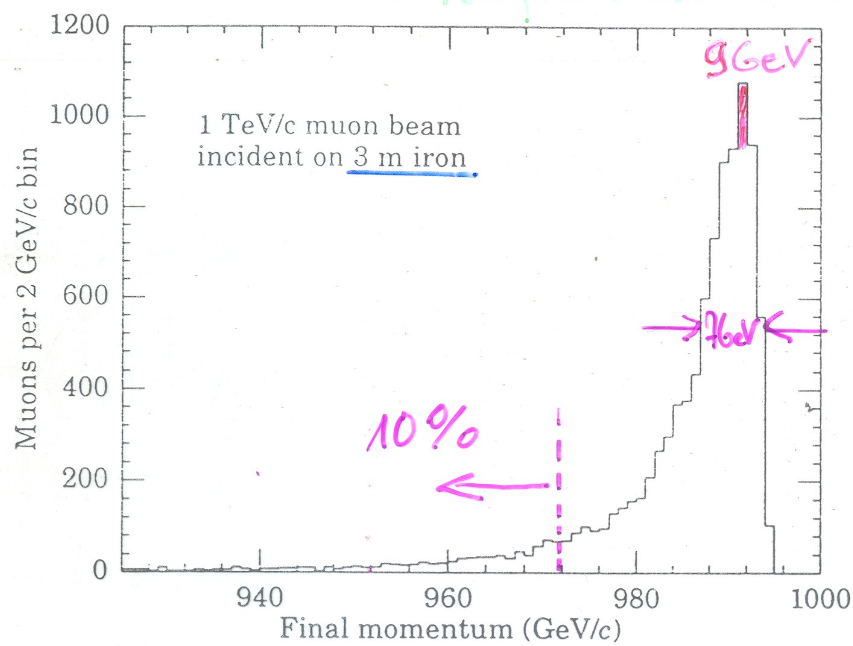


Figure 22.13: The momentum distribution of 1 TeV/c muons after traversing 3 m of iron, as obtained with Van Ginniken's TRAMU muon transport code [50].

3 m Fe ~ 18 λ_I ~ material pred mionskimi komorami LHC detektorjev

1 TeV μ : $\Delta E_{mp} = 9$ GeV (~1%)
FWHM = 2 GeV (~1%)

$\boxed{\text{rep}}$: 10% $\Delta E > 28$ GeV ~ zavorno sevanje
3,3% $\Delta E > 100$ GeV ~ fotojedrske
{+ kalorimeter
{+ sledilnik
!

II Prehod fotonov skozi snov

- fotoefekt
- comptonski pojav
- tvorba parov

razlika proti delcem

ni energijskih izgub – $\sigma \ll \sigma_{trki}$

foton izgine – "doseg" večji

– manjša se fluks

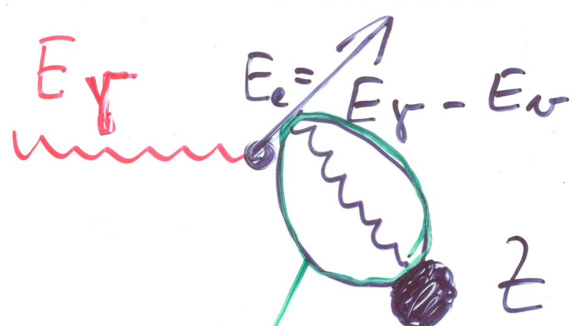
$$I = I_0 \, e^{-\mu x} \tag{34}$$

μ – absobcijski koeficient

$\lambda = \frac{1}{\mu}$ – atenuacijska dolžina

a) fotoefekt

vezani elektroni v atomu



velika verjetnost le za K-elektrone

presek narase za ~2 reda, ko E_γ preseže E_v za K-elektrone – K rob

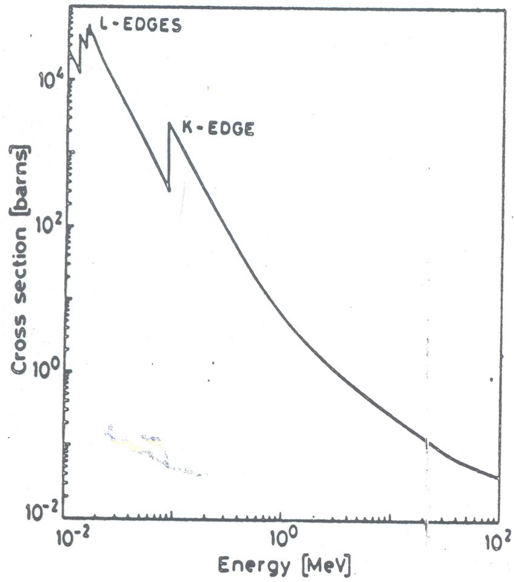


Fig. 2.21. Calculated photoelectric cross section for lead

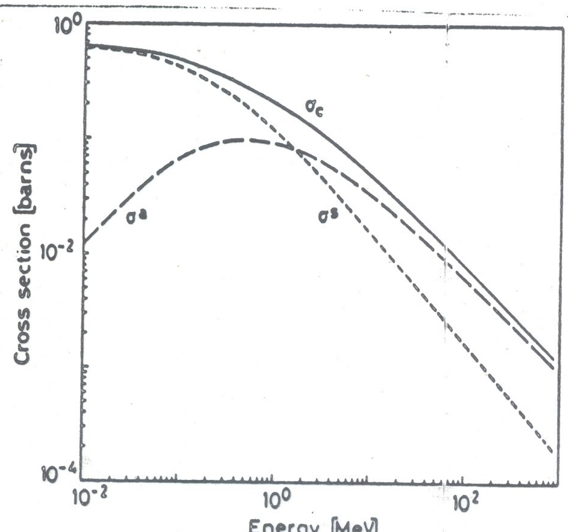


Fig. 2.23. Total Compton scattering cross sections

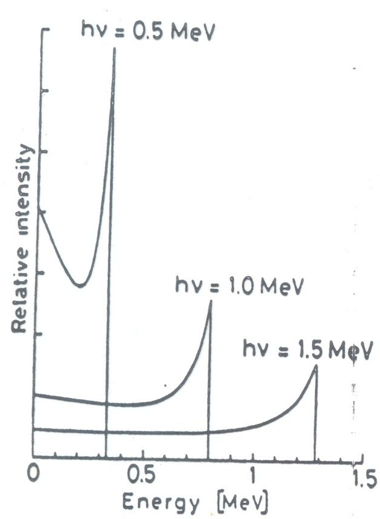


Fig. 2.24. Energy distribution of Compton recoil electrons. The sharp drop at the maximum recoil energy is known as the *Compton edge*

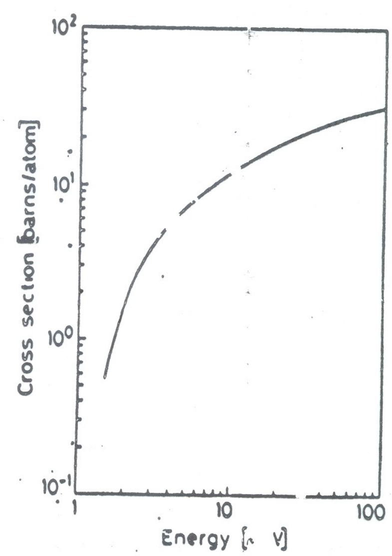


Fig. 2.25. Pair prod. tion cross section in lead

odvisnost za σ nad K-robom

$E_K < E_\gamma << m_e c^2$ nerel. račun

$$\sigma = \frac{32\pi}{3} r_e^2 \sqrt{2}\; Z^5 \alpha^4 \left(\frac{m_e c^2}{E_\gamma}\right)^{7/2} \propto \frac{Z^5}{E_\gamma^{7/2}} \qquad (35)$$

s $\sigma_{Th} = \frac{8\pi}{3} r_e^2 = 6,65 \cdot 10^{-25}\ \mathrm{cm}^2$

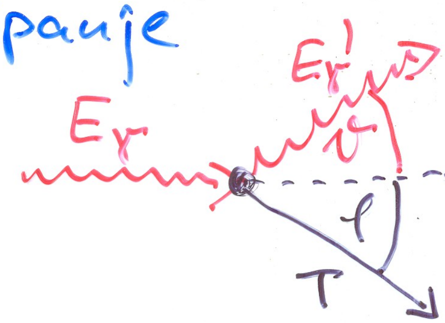
$$\sigma = 4\sqrt{2}\; \sigma_{Th}\; Z^5 \alpha^4 \left(\frac{m_e c^2}{E_\gamma}\right)^{7/2} \qquad (35a)$$

$E_\gamma >> m_e c^2$ ultrarel. račun

$$\sigma = \frac{3}{2} \sigma_{Th}\; Z^5 \alpha^4 \left(\frac{m c^2}{E_\gamma}\right) \qquad (36)$$

b) Comptonsko sipanje

prost elektron



$$E_\gamma' = \frac{E_\gamma}{1 + \gamma(1-\cos\vartheta)} \qquad (37)$$

$$\gamma = \frac{E_\gamma}{m_e c^2}$$

$$T = E_\gamma - E_\gamma' = E_\gamma \frac{\gamma(1-\cos\vartheta)}{1+\gamma(1-\cos\vartheta)} \qquad (38)$$

presek – Klein-Nishina (QED)

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \frac{1}{[1+\gamma(1-\cos\vartheta)]^2} \left(1+\cos^2\vartheta + \frac{\gamma^2(1-\cos\vartheta)^2}{1+\gamma(1-\cos\vartheta)}\right) \tag{39}$$

Integralni presek

$$\sigma_c = 2\pi r_e^2 \left\{ \frac{1+\gamma}{\gamma^2}\left[\frac{2(1+\gamma)}{1+2\gamma} - \frac{1}{\gamma}\ln(1+2\gamma)\right] + \frac{1}{2\gamma}\ln(1+2\gamma) - \frac{1+3\gamma}{(1+2\gamma)^2}\right\} \tag{40}$$

$$\sigma_c \propto \frac{1}{E_\gamma} \quad (\text{na elektron; } \frac{Z}{E_\gamma} \text{ na atom})$$

$$\sigma_c = \sigma_s + \sigma_a \quad ; \quad \frac{d\sigma_s}{d\Omega} = \frac{E_\gamma'}{E_\gamma}\frac{d\sigma}{d\Omega} \tag{41}$$

za velike energije : $\sigma_a \to \sigma$ (absorbcija)
za majhne energije : $\sigma_s \to \sigma$ (sipanje)

elektroni

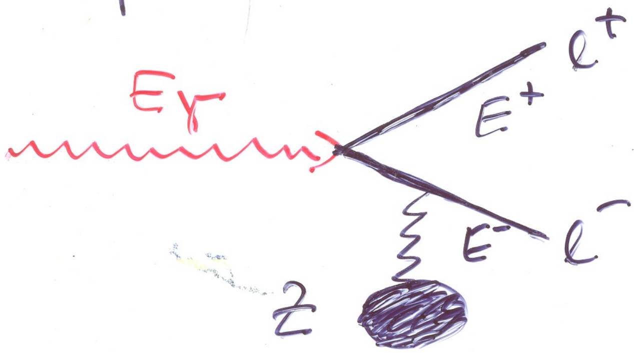
$$\frac{d\sigma}{dT} = \frac{\pi r_e^2}{m_e c^2 \gamma^2}\left[2 + \frac{s^2}{\gamma^2(1-s)^2} + \frac{s}{1-s}\left(s - \frac{2}{\gamma}\right)\right] \tag{42}$$

$$s = \frac{T}{E_\gamma} \quad ; \quad T_{max} = E_\gamma \frac{2\gamma}{1+2\gamma}$$ Comptonski rob

Thompson – prost elektron v klasični limiti
Rayleigh – koherentno sipanje na atomu kot celoti
$E_\gamma' = E_\gamma$ elastično ; zanemarljivo za X in γ

c) tvorba parov

na jedrih - obraten proces k zavornemu sevanju



senčenje parameter $\xi = \frac{100\, m_e c^2\, E_\gamma}{E^+ E^-\, Z^{1/3}}$

$E_\gamma \gg 137\, m_e c^2\, Z^{-1/3} \to \xi \to 0$

popolno senčenje

presek konstanten

$$\sigma_p = \frac{A}{N_a \rho}\, \frac{7}{9 X_0}\, \alpha\, Z^2 \qquad (43)$$

$$N = N_0\, e^{-\frac{7x}{9X_0}} \qquad (44)$$

elektroni vključeni v X_0 ($Z^2 \to Z(Z+1)$)

$$\lambda = \frac{9}{7} X_0$$

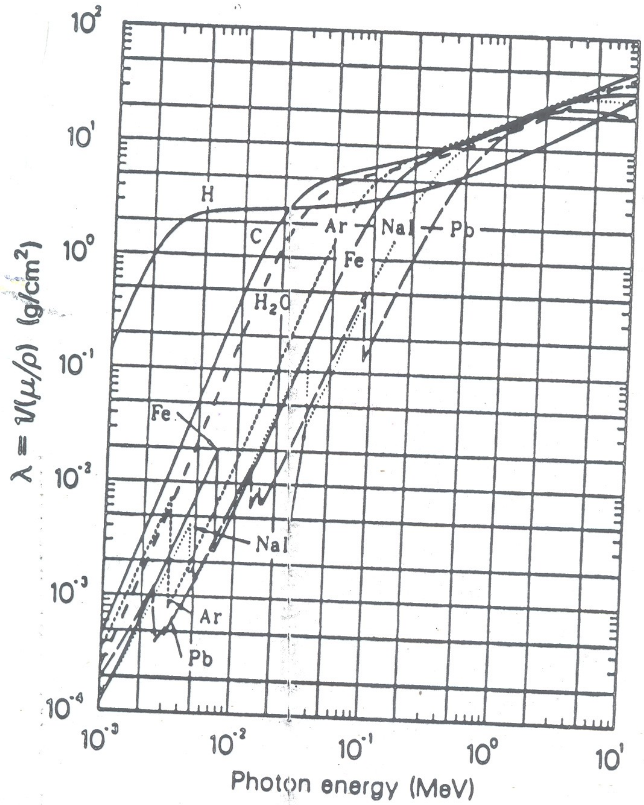
atenuacijska $\sim$ radiacijska dolžina

skupaj $\sigma_{tot} = \sigma_{ph} + Z\sigma_c + \sigma_p \qquad (45)$

$$\mu = N\sigma = \left(\frac{N_a \rho}{A}\right)\sigma \qquad (46)$$

PHOTON AND ELECTRON ATTENUATION

Photon Attenuation Length



Photon Attenuation l h (Hi h Energy)